w/a

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Val Q ~~[illegible]~~ { result of Val P and Val Q ~~[illegible]~~ Val P.

1   14 × 40
     16 . 40
gotbook
13 February

Mathematical Physics

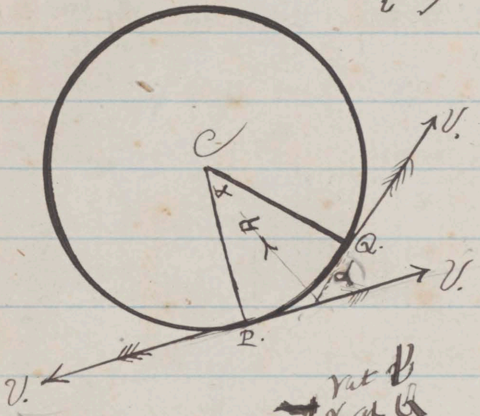
~~To make a body m describe any curve~~
I

To find what force is required to make a body move round a ⊙ of radius a with a uniform vel. V.:

$s = vt.$
$t = \frac{s}{v}$

Find the acceleration of P.

After the interval $\frac{PQ}{v}$ the velocity is still v but it has changed its direction. The increment of velocity is ∴ the resultant of V at Q. And a velocity in direction opposite of P. This resultant acts towards C & = $2V\sin\alpha$.



V at P
V at Q

NB The resultant of V at Q and V at P = what must be added to V at R to get V at Q.

Acceleration of P = $\frac{\text{change of vel.}}{t}$ {when time is very small

$= \frac{2V\alpha}{\frac{PQ}{V}} = \frac{V.\frac{PQ}{r}}{\frac{PQ}{V}} = \frac{V^2}{r}$

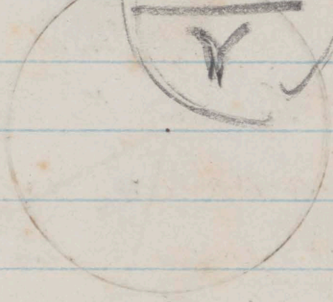
And acts towards C
∴ forces req^d^ to produce motion
$= M\frac{V^2}{r_0}$ Dynamical units acting towards C

$V = ft$

Hence the force necessary for keeping a body in a circ path without changed ~~velocity~~ velocity

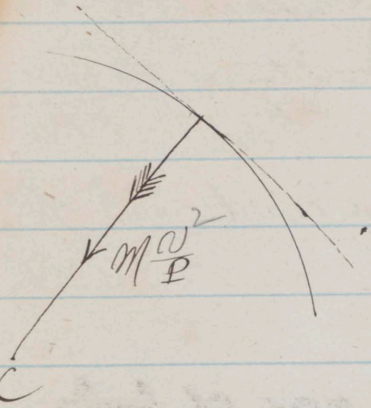
= is a force of $\frac{v^2}{r}$ directed towards the Centre.

∴ if m is mass of body then centl force $= \frac{mv^2}{r}$



II

To make a body M describe any curve with uniform velocity v. a force = $m\frac{v^2}{\rho}$ must act normal to it inwards where ρ is radius of ○ of curvature at the point



III.

Find the amount of attraction which the earth exercises on the moon: (r of orbit) = 240 000 miles & goes round in 28 days.

The ratio of attraction which bodies ~~ex~~ act on each other is with ~~inverse~~ inversely as the square of the distance. The earth attracts a body of unit mass 4 000 miles from centre with force g.

+ directly as the masses

$\therefore\ g\left(\frac{\text{radius of earth}}{r}\right)^2$ dynamical units

= force required to keep moon going round in a ○.

The work required to produce a velocity (v) in a mass (m) originally at rest = $\frac{1}{2} mv^2$. #

Simple Pendulum.

The amplitude is the distance thro which a simple pendulum travels in moving from its lowest position to its furthest position on either side.

The complete period:

is the time from its passing through any given position to its next passing through the same position in the same direction.

Sometimes called a double vibration

Isochronism.— For small vibrations, the time of vibration is independent of the amplitude:

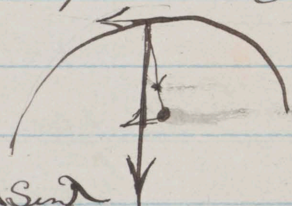
$m\left(g - \frac{v^2}{r}\right)$ = Force of push.

if $g = \frac{v^2}{r}$. The apparent wt. of gravity at the Equator would be nil.

This state of things would exist if [illegible] rotation were 17 times as fast as at present.

111

Show that the total intensity of centrifugal force due to the earth's rotations, at a place in latitude λ, is $\mu^2 R\cos\lambda$ μ denoting $\frac{2\pi}{T}$ and R the earth's radius; that the ~~other~~ vertical component (tending to diminish gravity) is $\mu^2 R\cos^2\lambda$ and the horizontal component (directed from the pole towards the equator)



is $\mu^2 R\cos\lambda\sin\lambda$,

N.B.; The earth is an oblate spheroid & true gravity does not pass through the centre ∴ only nearly; however, the corrections thus required are $m g$ of second order of small qntities.

The part of (2) which changes direction of G. (i.e.) of vert^l at P)

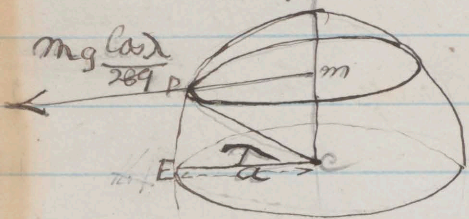
$$= \frac{mg}{289}\cos\lambda\sin\lambda;$$ and the change

∠ in the direction of the vertical $= \frac{\cos\lambda\sin\lambda}{289} = \frac{\sin 2\lambda}{2\cdot 289}$ in ⊙ mean

Ex 101

February 14th 1883 Art 103 / 104

(at the equator the weight of a body is diminished by $(\frac{1}{17})^2$ owing to centrifugal force [read)



(Centrifugal force)

(α) The motion of P round a ⊙ with PM as radius in a day makes the body pull outward with a force $\frac{mv^2}{PM(r)}$ [for we have seen that to sustain this motion a force = to this acting along PM must be applied to it].

(β) Thus the force acting on P are
(1) its weight mg along PC. (Free S, V)
(2) the force $\frac{mv^2}{a\cos\lambda}$ along MP = $\frac{M(\text{vel of } E)^2\cos^2\lambda}{a\cos\lambda} = \frac{Mg}{289}\cos\lambda$

(γ) The part of (2) wh. diminishes the weight of the body is its vertical component wh = $\frac{mg}{289}\cos^2\lambda$ = the fraction ~~mg~~ $\frac{\cos^2\lambda}{289}$ of its weight

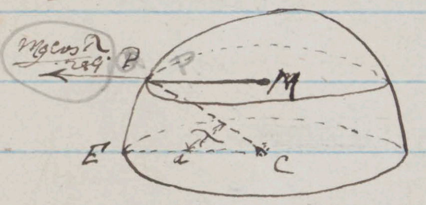
(P)

if the force of gravity were = to centrifugal

i.e. $g = \frac{V^2}{a}$

The apparent weight of bodies would be nil :— i.e. if $\frac{g}{r}$ were 17 times as great as it is at present

101



The motion of P round a O with PM as radius in a day makes the body pull outward with a force $\frac{mV^2}{PM}$. [for we have seen that to sustain this ~~force~~ motion a force = to this acting along PM must be applied to it]

Thus the forces acting on P are :—

(1) its weight mg acting along PC.

2. A force $\frac{mv^2}{a\cos\lambda}$ acting along MP.

$$= m \cdot \frac{\text{vel}^2 \text{ of } E \cos^2\lambda}{a\cos\lambda} = \frac{m \cdot g}{289}\cos\lambda \qquad \frac{V^2}{a} = \frac{g}{289}$$

The part of two which diminishes the weight of the body is its vertical component $\frac{mg \cdot \cos^2\lambda}{289}$

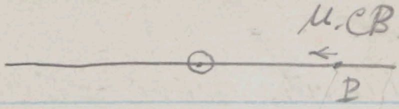
= the fraction $\frac{\cos^2\lambda}{289}$ of its weight

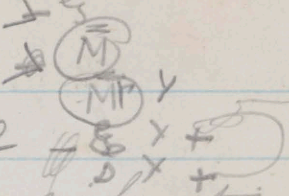
The part of (2) which changes the direction of G i.e. of vertical at P.

$= \frac{m \cdot g}{289} \cdot \cos\lambda \cdot \sin\lambda$, & the change $\angle$ in the direction of the vertical $= \frac{\cos\lambda \sin\lambda}{289} = \frac{\sin 2\lambda}{2 \cdot 289}$ in circular measure

W. H. Moore.

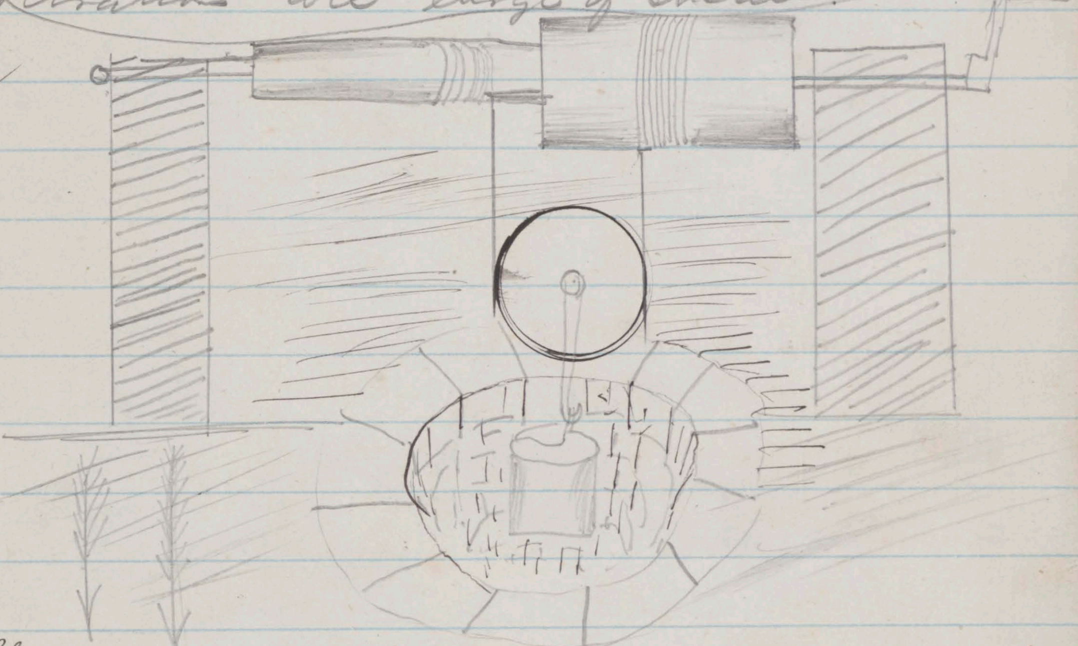
 Ch. x I

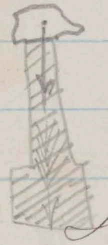




will vibrate round C

The time of a complete vibration is $\frac{2\pi}{\sqrt{\mu}}$: this is the same whether the vibrations are large or small.





The axle of the diff. wheel & axle consists of two parts one stouter than the other — The rope connecting the moveable pulley (& weight) with the axle is wound round the thicker part. which of course raises the weight as at every revolution of the axle the rope from the thinner part is unwound while at the same time, (i.e. revolution) the length of the Circumference of the thicker ^part^ axle is wound up.

And every revolution of the axle raises the weight a distance = $\frac{1}{2}$ difference of circumferences of larger & smaller part of axle.

(β). Let c = Circumference described by power

~~a~~ = " " " = larger pt

~~a~~b = " " = smaller pt.

Then $cP = \frac{(a-b)}{2} W$.

& the mechanical advantage $= \frac{W}{P}$

$$= \frac{2c}{(a-b)}.$$

(γ).

: Let P = power
Q = Wt.
Radii = p & q.
t = time s = distance. v = vel.

1. part of Example at other side—

$x = v\cos\alpha\, t.$

$y = v\sin\alpha\, t - \frac{1}{2}gt^2.$

$\frac{y}{x}$ dividing $\tan\beta = \tan\alpha - \frac{gt}{2V\cos\alpha}.$

$\therefore t = \underline{\tan\alpha - \tan\beta}\,.\frac{2V\cos\alpha}{g}.$

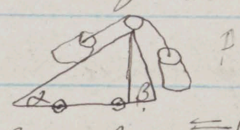
When Projectile has not ascended to top of Path:—

Supposing it were descending you would have to change the sign of $\tan\beta$.

$\therefore t = \frac{\tan\alpha + \tan\beta}{\&}\,\frac{2V.\cos\alpha}{g}.$

A projectile is discharged with a vel V at an elevation α. Find the time it will be moving at an inclinⁿ β:—

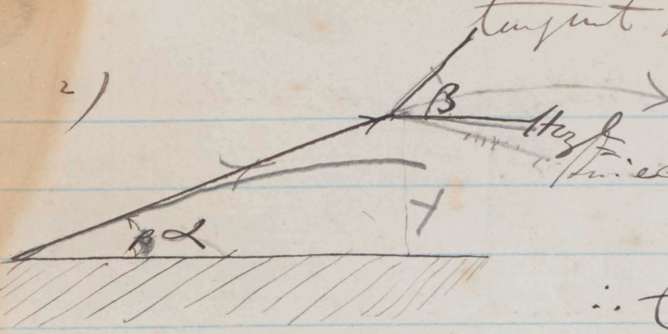
Second case of 3. if plane were not right ∠ed.



accelⁿ $= \left(\frac{P\sin\beta - Q\sin\alpha}{P+Q}\right)g.$

$\therefore$ reqᵈ force $= \frac{(P\sin\beta - Q\sin\alpha)}{P+Q}\left(Q\cos\alpha + P\cos\beta\right).$

2)



$\frac{1}{2}$ time the vel = f. time (for ascent)

$\therefore \frac{t}{2}$ in this case $= \frac{V}{f} \sin\beta$.

$\therefore f = g$. (here)

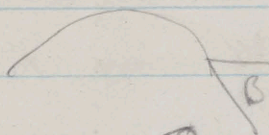
~~$\therefore$ total time it will be moving $= \frac{2V\sin\beta}{g}$.~~

$x = v\cos\alpha\, t$

$y = v\sin\alpha\, t - \frac{1}{2}gt^2$

Divide $\therefore \tan\beta = \tan\alpha - \frac{gt}{2V\cos\alpha}$

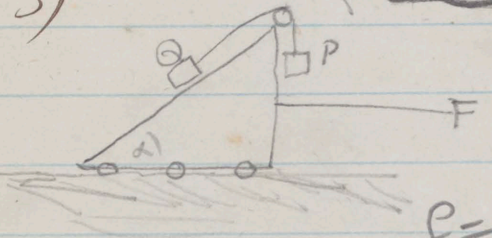
$\therefore t = \frac{2v\cos\alpha}{g}(\tan\alpha - \tan\beta)$



$t = \frac{v\cos\alpha}{g}(\tan\alpha + \tan\beta)$ when $\tan\beta$ is the numerical value –

The wedge is held firm by a force F –

3)

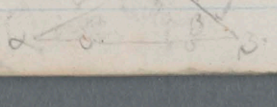


The acceln of P or Q is

$f = \frac{F}{M}$

$\frac{(P - Q\sin\alpha)g}{P + Q}$

Thus, the whole horizontal effect produced is to make Q move with a horizontal acceln $f\cos\alpha$ & the force required to do this is $\cancel{=}$ $Q f\cos\alpha$. dynamical units $= Q\cos\alpha . \frac{P - Q\sin\alpha}{P + Q}$.



$$\therefore t = \frac{4g\, W_1 W_2 W_3}{\left\{ W_2 W_1 + W_3 W_2 + 4 W_1 W_3 \right\}}$$

$$= \frac{28 \cdot g}{2 + 6 + 12} = \frac{32.2 \cdot \cancel{28}\,14}{\cancel{20}\;10}$$

$$\begin{array}{r} 64.4 \\ 450.8 \end{array}$$

$$\therefore t = 45.08.$$

$$\therefore f = \cancel{\frac{gR - 2t}{}}\ \frac{2t - W_2 g}{W_2}$$

$$= \frac{2t}{W_2} - g. \quad = 45 - 32 = 13 \text{ feet per sec}$$

Writing out 3 equations.

$$+ \quad f W_2 = 2t - W_2 g.$$

$$+ \ (f' + f) W_3 = g W_3 - t.$$

$$+ \ (f' - f) W_1 = t - W_1 g.$$

$$W_1 \times \quad \cancel{W_1 f' W_3} + f W_3 W_1 = g W_3 W_1 - t W_1$$

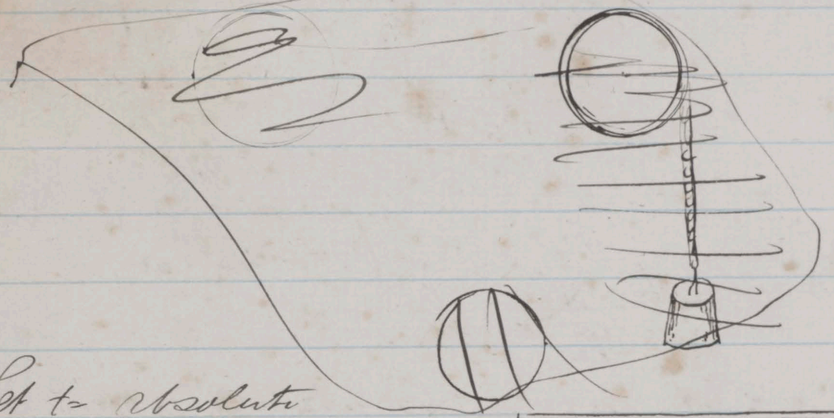
$$W_3 \times \quad \cancel{W_3 f' W_1} - f W_1 W_3 = t W_3 - W_1 g W_3$$

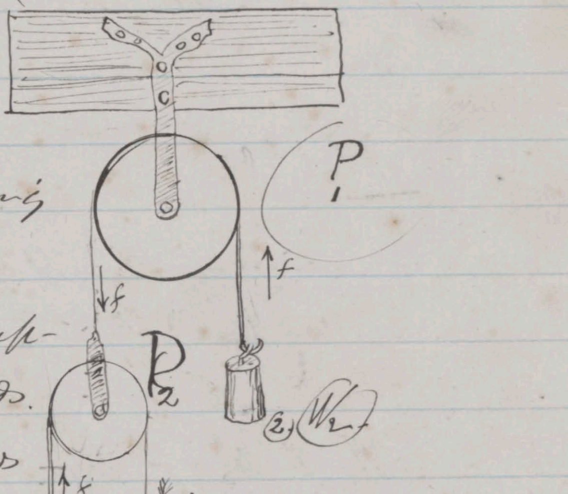
Subtract $\cancel{f W_1 W_3}$ $f(2 W_1 W_3) = g W_3 W_1 - t(W_1 + W_3) + W_1 W_3 g.$

$$+ \quad \cancel{f \cdot 2 W_1 W_2 W_3} = g\, W_1 W_2 W_3 - t\, W_2 (W_1 + W_3) + W_1 W_2 W_3 g.$$

$$f \cdot 2 \cancel{W_1 W_2 W_3} = 4t\, W_1 W_3 - 2 W_1 W_2 W_3 g.$$

$$W_2 t (W_1 + W_3) \quad 4t\, W_1 W_3 = 4\, g \cdot W_1 W_2 \cdot W_3$$





Let t = absolute tension measure of Chord passing over P_2. Then $2t$ = abs. measure of cord passing over P_1 —

Let f = acceln of W_1 upwards. (W_1 W_3) downwards.

f' acceln of W_3 downwards & W_1 upwards —

Then the total acceln of W_3 downwards = $(f+f')$

W_1 upwards = $(f'-f)$

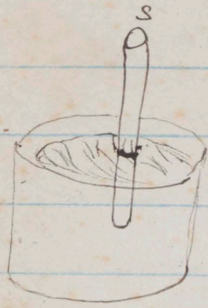
of W_2. The forces acting on W_2. W_3. W_1 are.

$\frac{(2t - W_2 g)}{2}$ $\frac{(gW_3 - t)}{3}$ $\frac{(t - W_1 g)}{1}$

$f\,W_2 = 2t - W_2 g.$

$(f'+f)\,W_3 = gW_3 - t.$ $(f'-f)\,W_1 = t - W_1 g.$

5.



1 —— Density of water

D .. Density of brine

Let V c.c. be immersed when in water
∴ its weight is V gr. :

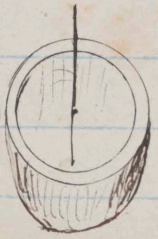
also (v-sh) c.c. be immersed when in brine of Density D.
∴ its weight (v-sh) D gr

$V = (v-sh)D$; $v = \frac{shD}{D-1}$

6. The C.G. may be defined thus.

The resultant of a system of ∥^{el} forces acting on a rigid body passes through a fixed point the position of which is independent of the direction of the forces —— If the forces be the weights of the several elements of the body the ~~abo~~ fixed point is termed the Centre of Gravity

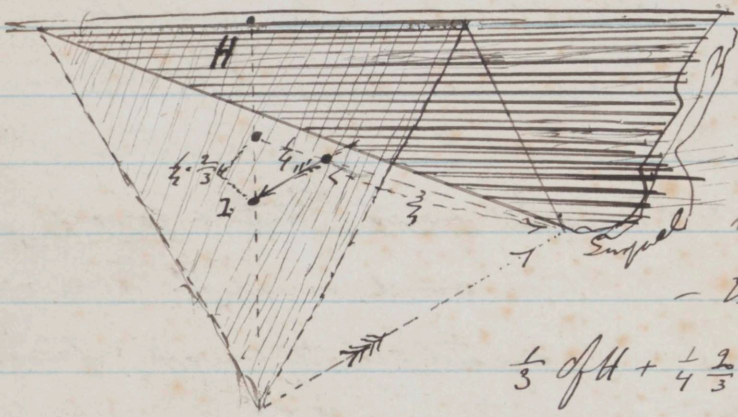
∴ the point of application of the resultant ~~is in~~ of a system of ∥^{el} forces is independent of their direction



Question 8. (Univ^y Ent^ce Paper - Honor Arts 82)

Determine the magnitude and point of application of the resultant fluid pressure of one side of a bar plate whose base is in the surface of the fluid:—

otherwise.



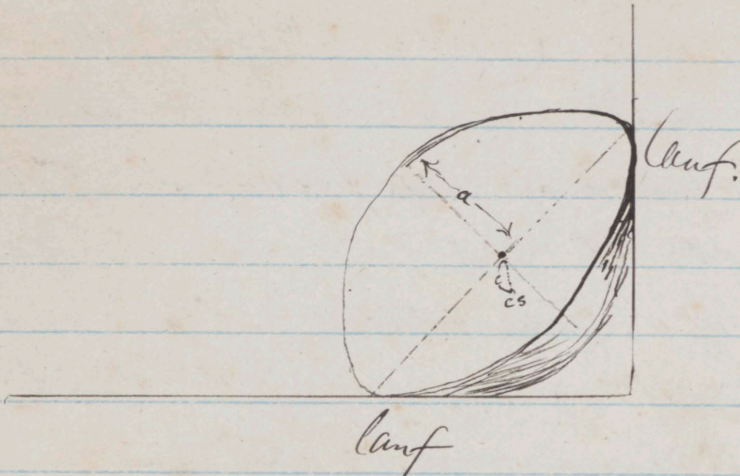
The moment of inertia of plane of plate with regard to line in which it intersects the surface.

$\frac{1}{3}$ of $H + \frac{1}{4}\frac{2}{3}H$

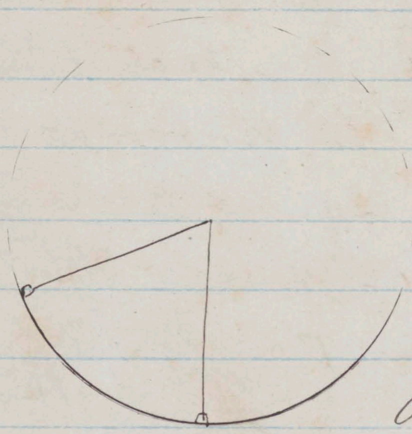
$\frac{1}{3}$ of $H + \frac{2}{12}H = \frac{6}{12}H =$

$\frac{1}{2}H$. or middle point of line joining middle point of base with vertex of Δ.

The particles of fluid all act $\perp$ to plane of plate from this point in form of resultant



See further on done in Examples on friction:

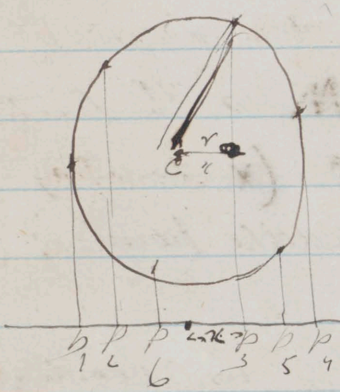


In order that a force should move a body with uniform vel in a curved path from previous information we know that $f = \frac{v^2}{r}$

& If we denote time of revol^n by T. the distance moved in this $s = 2\pi r$.
$s = vt$.
hence $v = \frac{2\pi r}{T}$.

& an expression for the intensity of the force is $\left\{\left(\frac{2\pi}{T}\right)^2 \times r\right\}$.

Q. If a mass M is moving North with v. V
another mass M' is moving E. – – V'
They strike & stick find the
vel afterwards



Suppose a pt. to travel with a uniform vel. round a ⊙, ~~its projection p from~~ & from its successive positions ⊥ers be drawn to a fixed straight line in the plane of the ⊙

Then while P travels round the ⊙ its projection executes a complete vibration.

acceln of P towards centre $= \left(\frac{2\pi}{T}\right)^2 r$. The component of this acceleration $\|^{el}$ to the line of motion of p is $= \frac{x}{r}\left(\frac{2\pi}{T}\right)^2 r = \left(\frac{2\pi}{T}\right)^2 x$ = acceln of p $= \mu x$.

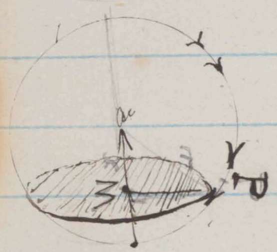
$\therefore \mu = \left(\frac{2\pi}{T}\right)^2$ & T (time of complete vibn) $= \frac{2\pi}{\sqrt{\mu}}$

$vt = S = 2\pi r$ 11

$\therefore \frac{v^2}{r} = \left(\frac{2\pi}{T}\right)^2 r$ $\frac{v^2}{r}$

continued

Let P be the particle the acceln of P is always directed towards the centre of O & is $= \left(\frac{2\pi}{T}\right)^2 r$.



The component of this acceln $\parallel$ to line of ~~direction~~ motion of M is the fraction $\frac{x}{r}$ of the whole acceln (x denoting distance of P from middle point of its path) & is :

$$\left(\frac{2\pi}{T}\right)^2 x.$$

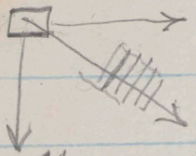
This is accordingly the acceln of M and for brevity is denoted by μx. To compute the periodic time T of a complete vibration we have $\mu = \left(\frac{2\pi}{T}\right)^2$ wh gives $T = \frac{2\pi}{\sqrt{\mu}}$:—

for the motion of a pendulum in a small arc we have acceln $\frac{g}{l} \cdot s$. we must $\therefore$ put $\mu = \frac{g}{l}$. & we have. $T = \sqrt{\frac{l}{g}} \cdot 2\pi$. for double vibration

MB

$T = 2\pi\sqrt{\frac{l}{g}}$.

$\boxed{1 \cdot \frac{7}{12} \cdot 2}$ 202



rails are inclined So that the resultant Should press ⊥arly on them.

11. Work done for a second = vel × force exerted by men.

14. Time of a ~~half~~ vibration is $t = \pi\sqrt{\frac{l}{g}}$

{half vib. occurs in a second}

{if g vary} {which is cnst} ~~$\delta t = \frac{\pi}{2}\sqrt{\frac{l}{g^3}}\,\delta g$~~

better in this case take log.

$$\log t = \log\pi + \tfrac{1}{2}\log l - \tfrac{1}{2}\log g$$

Diff. $\frac{\delta t}{t} = -\frac{1}{2}\frac{\delta g}{g}$

$$-\frac{1}{640} = \frac{1}{2}\frac{\delta g}{g}$$

$$\therefore \frac{\delta g}{g} = \frac{1}{320}$$

The mass reqd $= 80\frac{g}{g+\delta g} = 80\left(1-\frac{\delta g}{g}\right) = 80\left(1+\frac{1}{320}\right) = 80\frac{1}{4}$

If l be measured by δl. & g by δg, ? will be the alteration in t.

$$\log t = \log \pi + \tfrac{1}{2}\log l - \tfrac{1}{2}\log g$$

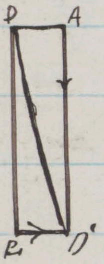
it is δt where $\rightarrow$ $\dfrac{\delta t}{t} = \dfrac{\delta l}{2l} - \dfrac{\delta g}{2g}$ #

This differential relation is accurate if ~~dg d~~ δt δl, δg are infinitely small or are very nearly accurate if they are any small quantities whatever.

Ex

Page 202 Ex 1.

2. What ought to be the diff^ce of level of the two rails to bring the whole pressure $\perp$ to plane of rails. The distance between them being 57 inches $\therefore$



DE represents 5 tons & we find below DA to be = .18 tons owing to circular vel.

$\therefore$ ~~(DD' = ...)~~

$$\begin{array}{r} 57 \\ \times .18 \\ \hline 5 \,)\, 10.26 \\ \hline 2.05 \text{ inches} \end{array}$$

1 The outward force on the rails = force req^d to constrain the carriage to move in a $\odot$

$$\therefore = \frac{Mv^2}{gr} \text{ tons.}$$

$V = \frac{88}{3}$ ft per sec.

$$= \frac{5\left(\frac{88}{3}\right)^2}{32 \cdot 3 \cdot 2\cancel{50}} \text{ tons}$$

as v is in ft per sec $g = 32$ we need not reduce to lbs.

$$\frac{\overset{22}{\cancel{88}} \times 11 \times \cancel{8}}{\cancel{8} \times \cancel{4} \times 3 \times 50 \times 9}$$

$$= \frac{11}{4 \cdot 9 \cdot 50} \text{ tons}$$

$$\frac{22 \times 11 \times 112}{3 \cdot 9 \cdot 50}$$

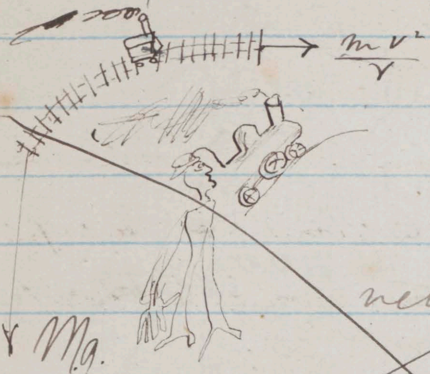
$$= \frac{11 \cdot \cancel{2} \cdot \cancel{112}}{\cancel{4} \cdot 9 \cdot 5\cancel{0}} = \frac{44 \cdot 28}{90} \text{ lbs}$$

$$= 13.7 \times \frac{88}{3}$$

401·4

2. The Rails are inclined so that the resultant pressure should press + on them

"Example 1" page 202.



$(5\text{ tons}) = 5 \times 20 \times 112$

$= 11200$ lbs.

$(\text{radius}) = 1550$ feet.

velocity per minute $= \frac{20}{60}$ [illegible]

$= \frac{1}{3} \times \frac{1}{60} = \frac{5280}{180} = \frac{88}{3}$ feet per sec.

Michael O'Shaughnessy

Michael O'Shaughnessy

$$\begin{array}{r} 88 \\ 88 \\ \hline 704 \\ 704 \\ \hline 7744 \end{array}$$

$\frac{88 \cdot 11}{32 \cdot 27 \cdot 50} \cdot \frac{112 \cdot 20}{31}$

616

One component $= \frac{7744 \cdot 11200}{9 \cdot 1550} = \frac{7744 \cdot 224}{9 \cdot 31}$

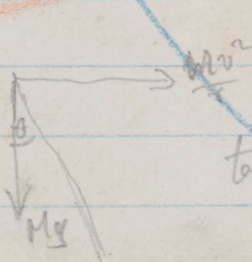
Other component

$= Mg = 11200$ lbs

$$\begin{array}{r} 7744 \\ 224 \\ \hline 30976 \\ 15488 \\ 15488 \\ \hline 1734656 \\ \hline 279 \end{array}$$

$$\begin{array}{r} (112)^2 \\ 112 \\ \hline 224 \\ 112 \\ 112 \\ \hline 12544 \end{array}$$

125440000.



$\tan\theta = \frac{v^2}{gr}$

$W = 5$ tons $\qquad F$ 80 lbs. Resistance

$f = \frac{F}{11200 + 80} = \frac{8}{1125} = \frac{1}{140}$ = acceleration

$s = \frac{1}{280} \times \frac{3600}{\frac{90}{7}} = 128$

No. 11

$F = f M$

$f = \frac{M}{F} = \frac{5 \times 20 \times 112}{30}$

$= \frac{1120}{3} = \cancel{373}$

$= \frac{3}{1120}$

$\cancel{f^{t^2}} = vt.$

$S = \frac{1}{2} f t^2.$

$= \frac{3}{\cancel{2240}}$ 34

$\cancel{3600}$ 60

$S = \frac{1}{2} f t^2$

$= \frac{560}{3}$ 3360 191600 63833

180
10800
102 $\quad$ 318
60
34
260
272

f acceleration $= \frac{5 \times 20 \times 112 - 50}{80}$

1 $\qquad$ 11200 $\quad$ 50 $\quad$ 11150 $\quad$ 8 $\quad \frac{1}{139}$

$S = \frac{1115}{8.2}$ $\quad$ 1800 $\quad$ 3600 $\quad$ 250

$\frac{8}{1115}$ $\quad$ 223 $\quad$ 3600 $\quad$ 28800 $\quad$ 5760 $\quad$ 223 $\quad$ 446 $\quad$ 1300 $\quad$ 2

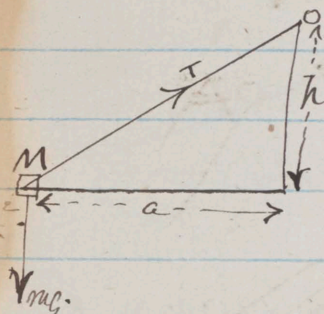
139. $\quad$ 3600

1800 $\quad$ 139 $\quad$ 410 $\quad$ 1

$\frac{1}{139}$

~~Work done for a sec = Vel × force exerted by men.~~

(General proof) for examples Exercise XII.



The mass M describes a horizontal ⊙ of radius a.

A force $\frac{mv^2}{gr}$ lbs must act on it towards O to make it do this.

i.e. it pulls outwards with a force $\frac{mv^2}{g.r}$ lbs. :—

Thus, the forces acting on M are:–

I. its weight m lbs.

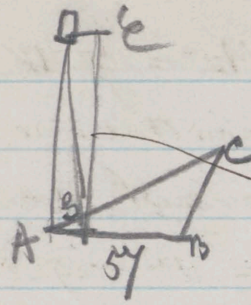
II. The tension of string T lbs.

III. This pull outwards $\frac{mv^2}{gr}$ owing to our motion.

∵ These do not cause it to move in or out from O ∴ They balance

21 Feb

2.



~~DA represents 5 tons~~
~~measure as in previous~~
~~example that DE = .07 $\frac{73}{1}$~~
~~.18 tons nearly.~~

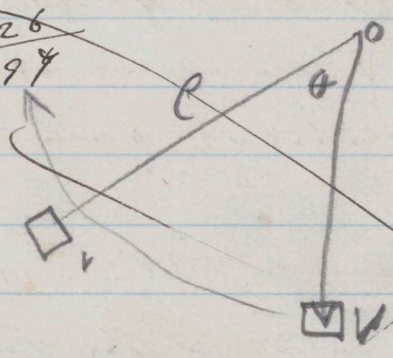
we have to find CB. ~~CB = 57 tons~~

~~$= 57 \times \frac{DE}{AD} = 57 \times \frac{.18}{5} = $~~ 57
18

5 | 10.26

= 2.05 nearly

57
3
17 1
10 26
1197



~~$\frac{1}{2}MV^2 - \frac{1}{2}Mv^2 = gMh$~~
~~as the tension~~
~~does no work.~~

~~$\therefore V^2 - v^2 = 2gl(1-\cos\theta)$~~

~~(1) $\therefore v^2 = V^2 - 2gl(1-\cos\theta)$~~

~~To find T.~~

~~The outward pull due to the rotation~~
~~in the ⊙ at P is $\frac{mv^2}{gl}$ lbs.~~
~~The component of wgt of m acting outward~~
~~is $m \cos\theta$ lbs~~

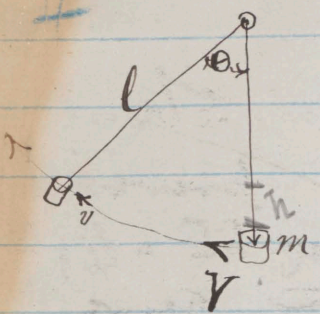
Thus to contrast both these

$\frac{mv^2}{r} > mg.$ or

$\therefore T = \frac{mv^2}{gl} + m\cos\theta$ in wh v is known from 1

(continued)

work done = $\frac{1}{2}$ ÷ $l(1-\cos\theta)$



$\frac{1}{2} MV^2 - \frac{1}{2} Mv^2 = gMh$

as the tension does no work.

$\therefore \quad (V^2 - v^2) = 2gl(1-\cos\theta)$

$\therefore \quad (1) \quad v^2 = V^2 - 2gl(1-\cos\theta)$

To find T tension on string

The outward pull due to the motion of the o at P is $\frac{mv^2}{gl}$ lbs.

The component of the weight of m acting outwards is $m \cos\theta$ lbs.

Thus to contrast both these are is

$\frac{Mv^2}{r} > mg$

$\therefore \quad T = \frac{mv^2}{gl} + m\cos\theta$

in which v is known from (1)

In order that the particle should go round in a circle at the top the centrifugal force there must be at least > than the weight

$\frac{M}{gl}(V^2 - 2gl)$ is taken great as M

$\therefore \quad V^2$ is taken greater ~~than~~ $5gl$.

G H Darrow

in order there it should go round in the ⊙

$\frac{mv^2}{r}$

mg

The centrifugal force at the top must must be at least larger than the weight.

$$\frac{M}{gl}(v^2 - 2gl) \text{ is twice great as } M$$

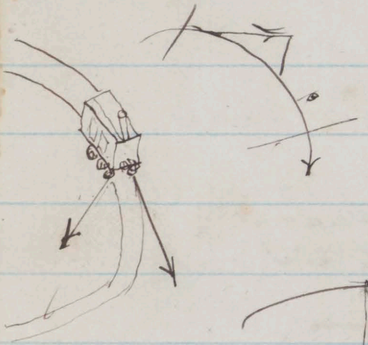
$$\therefore v^2 - 3gl$$

not needed

Question 12. The result of W + Centrif^l force > than weight ∴ pendulum goes faster.

Pendulum is carried outwards

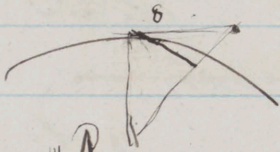
$$\sqrt{\left(\frac{121}{2}\right)^2 - 60^2} = \text{Comp}$$



60 $\frac{5280}{88}$ feet per sec.

121
242
121
$\frac{14641}{4} - \frac{14400}{4}$

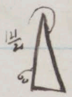
8



1 $\frac{241}{4}$ 141 $\sqrt{\frac{16}{4}}$

for 2:

= 2 / fr



8r = 80
88
704 11
7744
960
1320

8 represents

v

45 3.141

v^2/r

f = $\left(\frac{2\pi r}{t}\right)$ 362/60²

$\frac{28800}{48}$ 800 fr

8 = 362

Question 14

The attraction of any body by another varies inversely as the square of the distance —

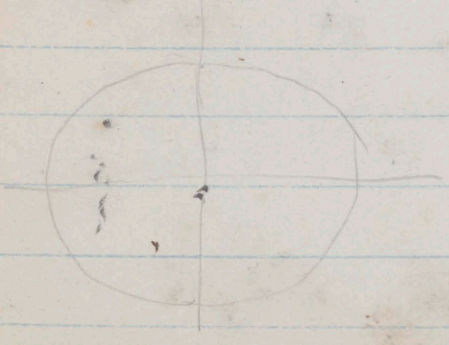
∴ The surface of the earth is nearer the centre in the Northern sphere than the ~~Southern~~.

∴ The intensity of the Tension on the String varies as $\left(\frac{641}{640}\right)^2$

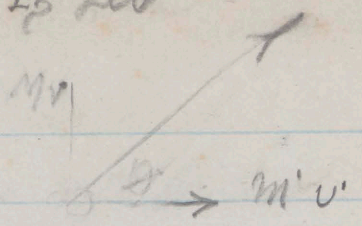
641
641
641

640
2560
3840
~~410240~~
5128
641

2564
3846
4 1 0 8 8 1 ÷ 82
51360·1
5128 80½ nearly
or 80p



2b Feb



No external force act. & ∴ the momentum is unaltered.

Thus the momentum of the whole mass $M+M'$ after the collision is the resultant of the momenta Mv north & $M'v'$ East.

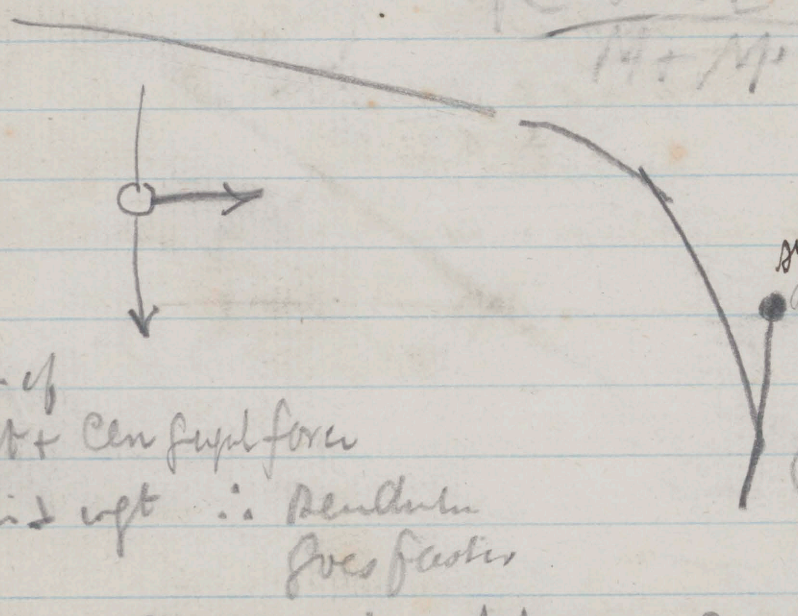
∴ the final momentum is

$$\sqrt{(Mv)^2 + M'v'^2}$$

& the direction of motion is given by $\tan\theta = \frac{Mv}{M'v'}$

$$= \frac{\sqrt{(Mv)^2 + (M'v')^2}}{M+M'}$$

12)



result of wgt + Centrifugal force

Third wgt ∴ pendulum goes faster

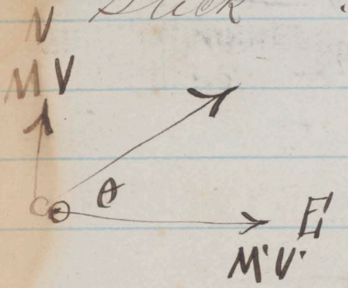
prove formula for single pend.

Chap. about Machines AA 121. Ex 3-4.5 Ex 5.8

Exercise 22.23.18.19

"Larmors own question"

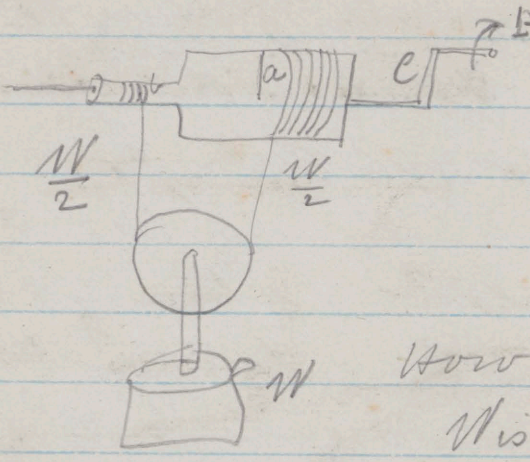
A mass M is moving North with a velocity V, another mass M' is moving east with a velocity V'. They strike and stick ? the velocity afterwards.



No external force acts & ∴ the momentum is unaltered. Thus the momentum of the whole mass $(M+M')$ after the collision is the resultant of the moment MV north. + $M'V'$ East. Thus the final momentum is

$$\sqrt{(MV)^2+(M'V')^2}$$ and final direction of motion is given by $\tan\theta = \frac{MV}{M'V'}$

$$= \frac{\sqrt{(MV)^2+(M'V')^2}}{M+M'}$$



b & a
In turning this
an angle θ
the power P does work
$P \times c\theta$ —

How much in W?
W is raised thro $\frac{1}{2}(a-b)\theta$
∴ the work done is $W\frac{1}{2}(a-b)\theta$
These are equal

The ratio of power to resistance when in equilibrium is = their "velocity ratio" when in motion if you can neglect friction

The moment of P round this axle balances the moments of the two parts of the String. $Pc = \frac{W}{2}a - \frac{W}{2}b$

(Avoid those pulleys & Diff^l. Pulley & ~~Screw~~ ~~by~~ method of last solution as example only —)

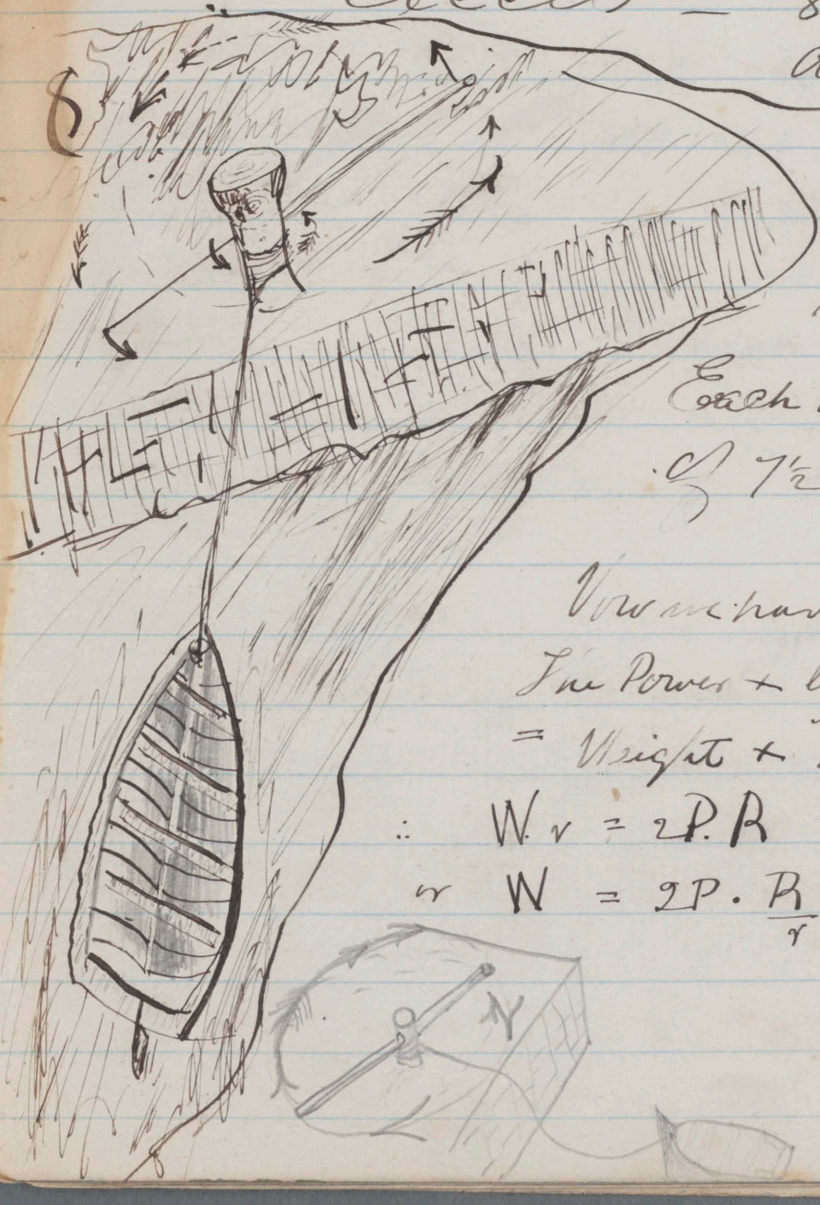
m06. Try when balanced at some other point outside the middle

The force · g = Mass × acceleration

$$\frac{2 \cdot 32}{8} = \text{acceleration}$$

acceln = 8 lbs. stupid

dynamic units



levers 12' long

radius of axle 18"

Each horse cart weight of 7½ cwt..

Now we have.

The Power × by its radius

= Weight × radius of axle

$$\therefore\ W \cdot r = 2P \cdot R$$

$$\text{or}\ W = 2P \cdot \frac{R}{r} = 16P.$$

$$= 16 \times 7\tfrac{1}{2}\ \text{cwts.}$$

$$= 6\ \text{Tons.}$$

5,224 00 lbs

~~F = f 5.22400 +~~

$F \; a_{car} = \frac{80}{2\,112050}$

$\frac{1120\overset{00}{50}}{112050}$

$f = vt \quad s = \frac{1}{2} f t^2$

$= \frac{4 \quad 3600}{11205}$

$\frac{110}{4} \; .71$

13.

Since in the screw

$P C = W \cdot d.$

where C = circum of P. & d. distance between consecutive threads

$\frac{21}{2} \times \frac{22}{4} = \frac{441}{4}$

$\therefore P = \frac{110}{\frac{441 \cdot 4}{4}} = \frac{22}{88} = \frac{1}{4}$

$7 : 22 :: 21$

$= \frac{22 \cdot 21}{7}$

$\therefore \frac{P\ d}{\frac{22 \cdot 21}{7}} = \frac{110}{22 \cdot 3 \cdot 4 \cdot 2} = \frac{110}{264} = \frac{22}{53}$

$= (\div 4\ 22)\ \frac{5}{24}$ lbs.

{find power to be applied to the end of lever}

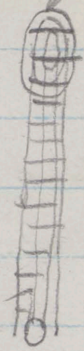
M. M. O'Shaughnessy.

When power goes round the whole circumference the weight is raised 1/4 inch

Work done expended = Kinetic En. acquired

= march

When W descends thro' any space.
P must descend thro' 2 m times the same space where n = No. of pulleys



P = 18

W = 112

W will descend with vel. v

P " ascend " " 6V as there would

when W has descended thro' a height h

the whole work done = $Wgh - Pg.6h$

This must be = the Kinetic energy gained

wh is $\frac{1}{2}WV^2 + \frac{1}{2}P(6V)^2$

$\therefore \left(\frac{1}{2}W + 18P\right)V^2 = (W - 6P)gh.$

$\therefore \cancel{V^2 =} \; 2\left(\frac{W-6P}{W+36P}\right)gh = \sqrt{\frac{2}{19}}\,g$

This is the force $V^2 = 2gh$.

& $\therefore$ it shows that weight W descends with the

uniform accel^n $f = \dfrac{W - 6P}{W + 36P}\,g$

& $\therefore$ the time descent

$= h = \frac{1}{2}ft^2$

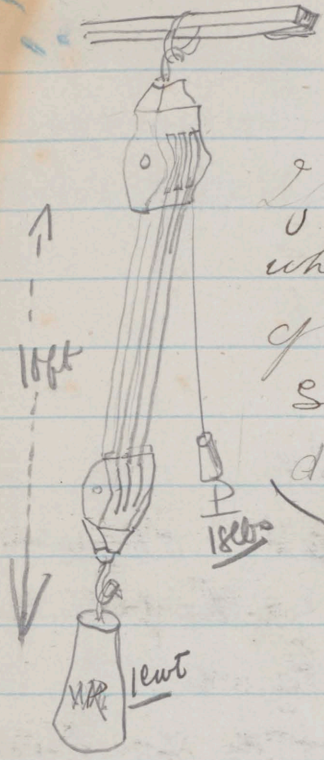
19/ If the system were at rest the beam would have to support P+W

& P is moving with acc^{en} 6f

& W " down " f

$\therefore$ The beam has to support $P + \dfrac{6f}{g}.P + \left(W - \dfrac{f}{g}W\right)$

(H)



Wt. of 1cwt is let down 10ft.

If there are n pulleys in a block when the Weight rises 1 foot each of the $2n$ parts of the string is Shortened 1 foot & ∴ P must descend $2n$ feet:—

$\therefore W = 2n \cdot P$

$$f\text{ accel}^n = \frac{\text{Force}}{\text{Mass}} = \frac{18}{130}$$

$S = \frac{1}{2} f t^2$

$$t = \sqrt{\frac{2s}{f}} = \frac{\cancel{40}^{20} \times 130}{\cancel{18}\ 9}$$

$$v = \sqrt{2f \cdot s} = \sqrt{\frac{18}{\cancel{45}\ 15} \times \cancel{10}\ 2}$$

$$= \sqrt{\frac{36}{15}} = \frac{6}{4} \text{ nearly} = 1\tfrac{1}{2}$$

Wc

$Pg\, 2nh - Wg.h = \frac{1}{2} P V^2 + \frac{1}{2} W v^2$

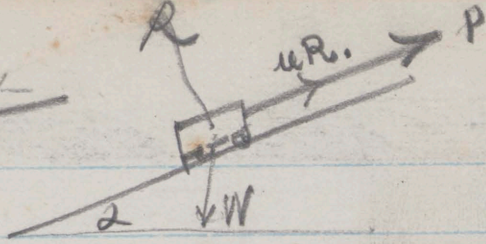
when $\frac{V}{v} = \frac{2n}{1}$

$$V = 2\left(\frac{112 - 108}{112 + 648}\right) g \cdot 10$$

$$\frac{4}{760} = \frac{10}{95} g$$

2 March

(1)

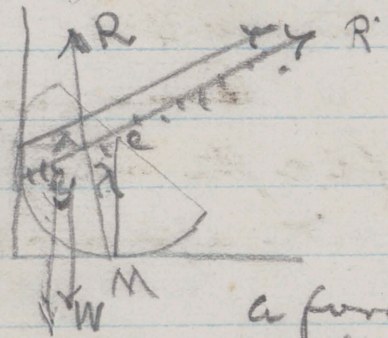


P is the force wh can only just prevent motion, & ∴ all the friction possible is called into play, wh is μR.

Tangent of ∠ of repose = μ

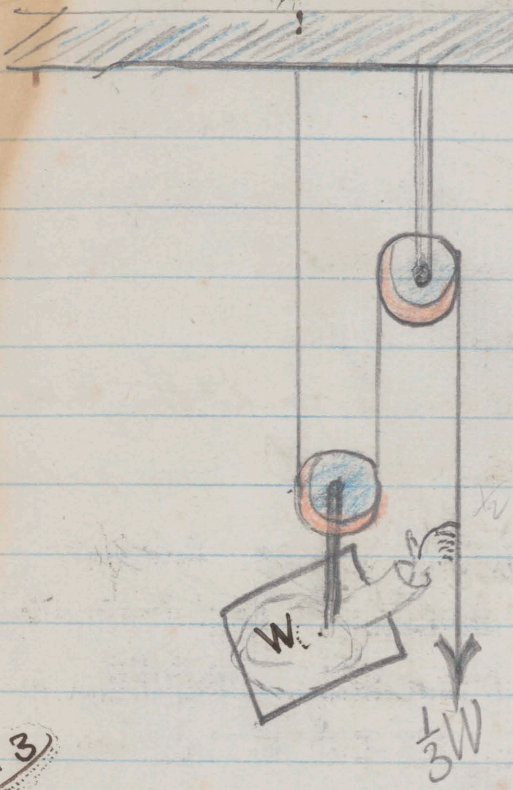
from R Sety.

(3)



When just about to slip the resultant reaction at M is a force R at an ∠ λ with the normal at N

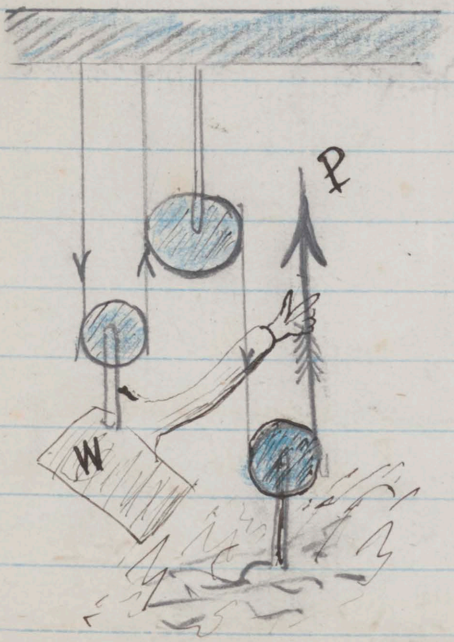
+ the other forces acting is wgt W



In the single movable pulley. The Power need only = $\frac{1}{2}$ the Weight that it might raise the latter.

Consequently, he (i.e., the man) must pull with a force = $\frac{1}{3}$ his weight.

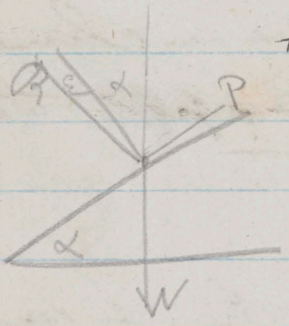
(2-3)

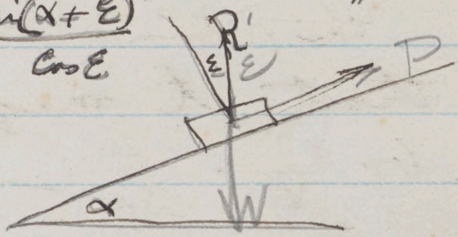


Evidently he must pull upwards with a force = his own weight.

(β) If the question be to find the force that will just move it ~~upwards~~

$$P = W\frac{\sin(\alpha+\varepsilon)}{\cos\varepsilon}$$





(α) When about to slide down the reaction of the plane will be a force R' at an ∠ ε with the normal to the plane, when tan ε = μ

When about to slide down the reaction of a plane will be a force R' at an ∠ ε with the normal to the plane, when tan ε = μ

We have ∴ Three forces [illegible] in equilibrium

$$\therefore \frac{P}{\sin R W} = \frac{W}{\sin R'P}$$

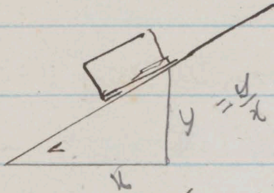
$$\therefore P = W\frac{\sin(\alpha-\varepsilon)}{\cos\varepsilon}$$

∴ when α varies P is [illegible] when $\alpha - \varepsilon = \frac{\pi}{2}$

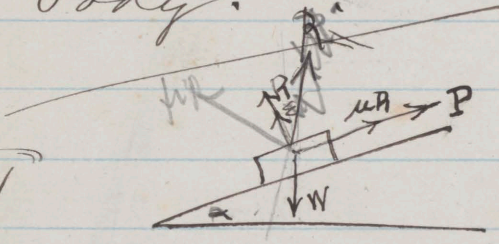
So that the force P continually increases till α becomes ∠ [illegible]

{1. 2. 3. 5. 8 9. (230)}

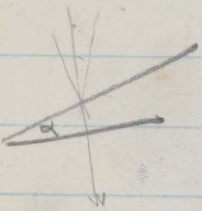
The co-efficient of friction is the tangent of the limiting ∠ of repose:



and the limiting ∠ of repose = the greatest ∠ at which the plane must be inclined that it might support the body.



"No. 1"

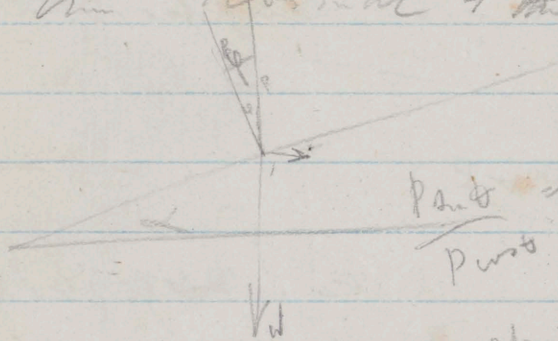


P is the force which can only just prevent motion, & ∴ all the friction possible is called into play which is ∴ μR, μ being tangent of ∠ of repose

$\sin(\theta - \phi)$

Now we must have a const. force to support the body (on the plane whatever its inclination), whose variable components are P & μR, and it is clear when the latter is comparatively small (as in the present case) the former must be greatest.

When two bodies are pressed together in such a manner that the direction of their mutual pressure is not normal to the surface of contact, the pressure can be resolved into two parts one normal + the other ~~tangential~~.



$\frac{P \sin\theta}{P \cos\theta} = \tan\theta = \text{ratio}$

Coeff of statical friction = tangent of limiting angle of friction

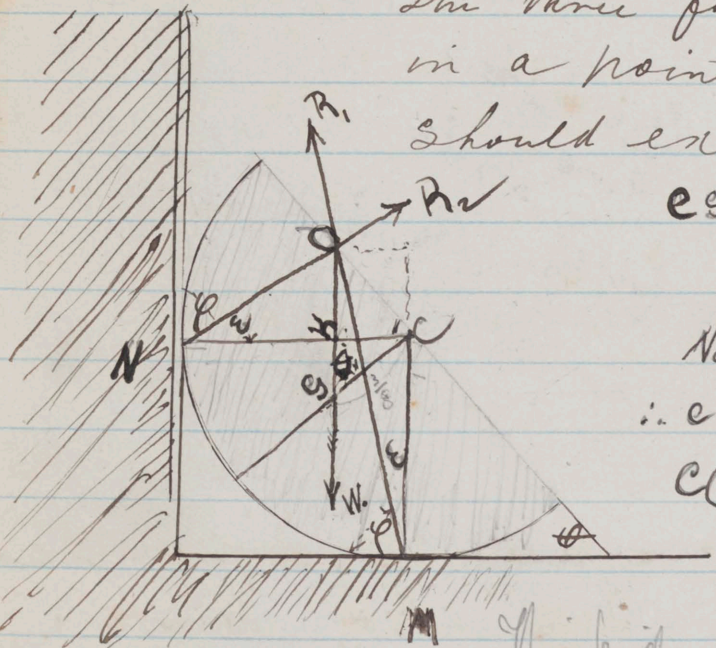
Commense.
Monday March 12

2.

θ Evidently the angle must be = the angle of repose — ∵ frictional forces are produced only when components of the acting force are in the plane of contact; and the resistense of friction can only lie in the plane of contact.

3)

The three forces must meet in a point that equl^m should exist —



$CG = c, \quad CN = a, \quad CK = c\sin\theta.$

$KO = (a - c\sin\theta)\tan\epsilon.$

Now $CK = (CM + OK)\tan\epsilon.$

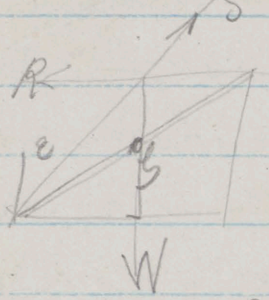
$\therefore c\sin\theta = \{a + (a - c\sin\theta)\tan\epsilon\}\tan\epsilon$

$c(1+\tan^2\epsilon)\sin\theta = a\tan\epsilon\,(1+\tan\epsilon)$

$\sin\theta = \frac{a}{c}\,\frac{\mu(1+\mu)}{1+\mu^2}$

This fixes the highest position in which it can remain without slipping

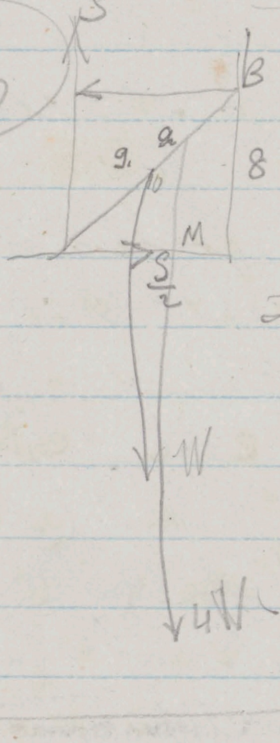
1 way



$\tan \epsilon = \frac{1}{2}$

G is CG of man and ladder together when the ladder is about to slip

2nd way



Resolve vertically

$S = 5W$

Resolve horizontally

$R = \frac{S}{2}$

Take moments about A.

$8\frac{5}{2}W = 3W + W.AM$

$\therefore 17 = 4AM$

$AM = 4\frac{1}{4}$

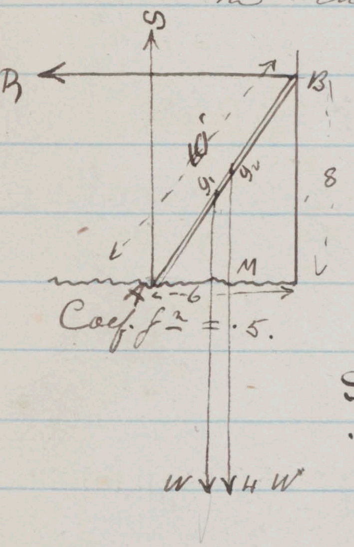
$\therefore AG_2 = \frac{5}{3}\cdot\frac{17}{4} = 7\frac{1}{12}$ feet

Friction increases with roughness of surfaces in Contact

Resistance of friction is ∝ to ~~motion~~ pressure of Surfaces in Contact.

1. The resistence of friction in rolling motion is always smaller than in sliding motion.—

2. The resistence is directly proportional to the normal pressure, & inversely as the radius of the rolling body.



Coef. $f^n = .5$.

Resolve horizontally $R = \frac{S}{2}$

Resolve vertically $S = 5W$.

Taking moments about A.

$8 \times \frac{5}{2}W = 3.W + 4W.AM$.

$4\,AM = 20 - 3 = 17$

$\therefore\ AM = 4\frac{1}{4}$

$S = 2R$.

$\therefore W = \frac{2}{5}.R$.

$\therefore R = \frac{5}{2}W$.

$\cancel{6} : \cancel{10} :: 4\frac{1}{4} : AG_2$
$\phantom{\cancel{6}}3 \quad 5$

$\frac{21\frac{1}{4}}{3} = 7\frac{1}{12}$

Friction increases with roughness of surfaces in contact)

Resistance of ") is $1:^l$ to mutual pressure of surfaces in Contact.

Friction increases with roughness of surfaces in Contact

Resist. of Friction is $1:^l$ to mutual pressure of surfaces in Contact

Kinetic energy of W when string breaks

$$= \frac{1}{2} W v^2 = \frac{1}{2} W \cdot 10g \cdot \frac{3-12\mu}{3+12}$$

$$+ \; v^2 = 2fs.$$

This kinetic energy makes W move thro' 4 feet against the friction μW : this friction does work $4 \cdot \mu W$

$$\therefore 4\mu(Wg) = \frac{1}{2} W \cdot 10g \; \frac{3-12\mu}{3+12}$$

$$\therefore 4\mu = \frac{1}{2} \cdot 10 \cdot \frac{3-12\mu}{15}$$

$$60\mu = 15 - 60\mu$$

$$\therefore 120\mu = 15$$

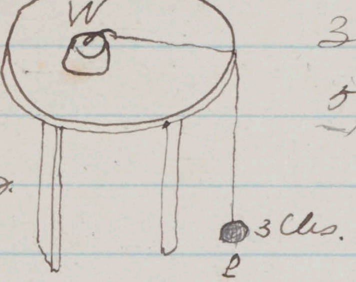
hence $\mu = \frac{3}{24} = \frac{1}{8}$

6 7 8 9 MC

Ballyhahill Farm

Total acerage of surveyed land.

5



$W = 12$ lbs. When the weight 3 lbs has ascended through 5 feet the thread breaks. & W moves thro 4 feet more & comes to rest. What is coef. of friction?.

$$\text{Acceleration} = \frac{\text{Force}}{\text{mass}} \cdot g.$$

$(\mu = \text{coef. of friction})$

$$= \frac{g(P - \mu W)}{P+W}. \quad \left(g\frac{P-\mu W}{P+W}\right)$$

$$= g\left(\frac{3-12\mu}{P+W}\right).$$

$$\text{and } u^2 = 2fs$$
$$= 2gs \cdot \left(\frac{3-12\mu}{P+W}\right)$$

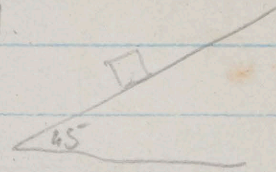
Kinetic energy accumulated at beginning of the motion $= \mu s = \frac{1}{2}u^2 m = \frac{1}{2} 2gs\left(\frac{3-12\mu}{P+W}\right).12$

$$\cancel{15\mu = 36 \times 32 - 12^2 \times 32\mu}$$

$$\cancel{349\mu = 26}$$

$$\cancel{133\mu = 32}$$

$$\cancel{\mu = \frac{32}{133}}$$



$\mu = \tan 45^\circ = 1$

If about to slide down

$$\frac{T}{W} = \frac{\sin(45-\alpha)}{\sin(45-\beta)}$$

If about to slip up

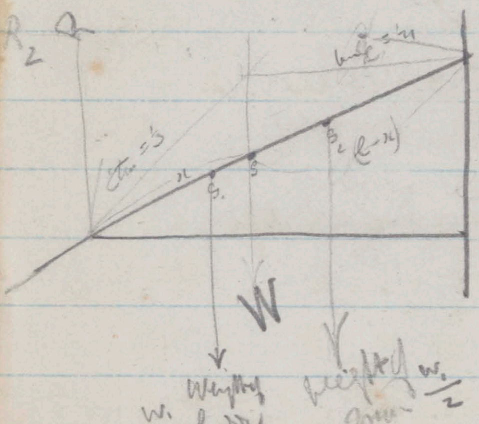
$$\frac{E}{W} = \frac{\sin(45+\alpha)}{\sin(45+\beta)}$$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{4}$

$\frac{1}{\sqrt{2}}\ \frac{3}{2}W \cdot = W \cdot \frac{1}{3\sqrt{2}} + \frac{3}{2}W \cdot x\sqrt{2}$

$\frac{3}{2} - \frac{1}{3} = \frac{7}{6} = \frac{x\sqrt{2}}{2}$

$\frac{14}{6\sqrt{2}} = x$



$R_1 = \frac{S}{3}$

$R_2 = \frac{S}{4}$

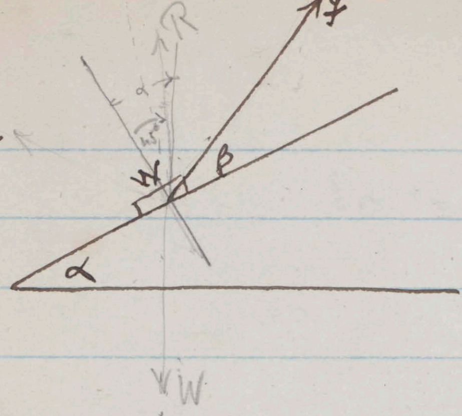
$\frac{R}{4} = \frac{S}{3}$ & $S = \frac{3}{2}W$

$3R = 4S$

Resolvment $S =$

$\frac{CA}{CO} = \frac{x}{y} \cdot \frac{CO}{CB} = \frac{y}{l-x} \therefore \frac{AC}{CB}\ \frac{x}{l-x}$

6.

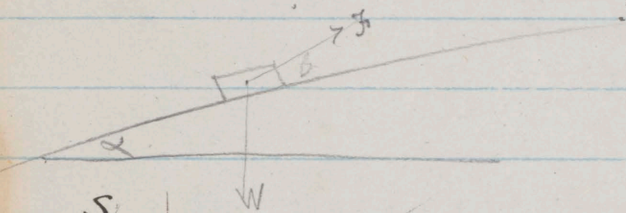


$W = 100$ lbs.

F = Sustaining force inclined at $\angle \beta$.

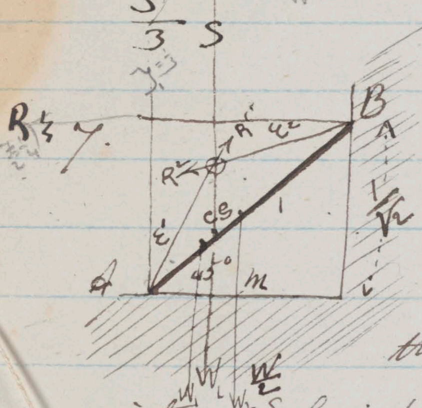
greatest $\angle$ at which body would rest $= 45^\circ$

& limits between wh. F must lie.



$$\frac{F}{\sin(45^\circ - \alpha)} = \frac{W}{\sin 45^\circ - \beta}$$

7.



CG of ladder $= \frac{1}{3}$ length from foot.

Coef of friction for plane $\frac{1}{3}$. wall $\frac{1}{4}$.

If a man whose weight $= \frac{1}{2}$ the weight of the ladder ascend it, ∴ height at which he will go before the ladder commences to slide —

The three forces must meet in a point O.

Taking moments about A.

$$\frac{3W}{2} \times \frac{l}{2} = W \times \frac{l}{3} + \frac{W}{2} \times x$$

$$\frac{3l}{4} - \frac{l}{3} = \frac{x}{2}$$

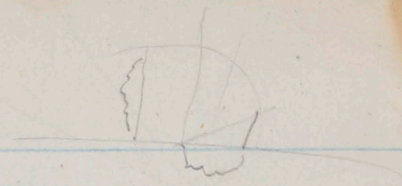
$$\frac{9l - 4l}{12} = \frac{10}{12} \cdot \frac{5}{6}$$

$$\frac{AC}{CO} = \frac{\frac{l}{3}}{x}$$

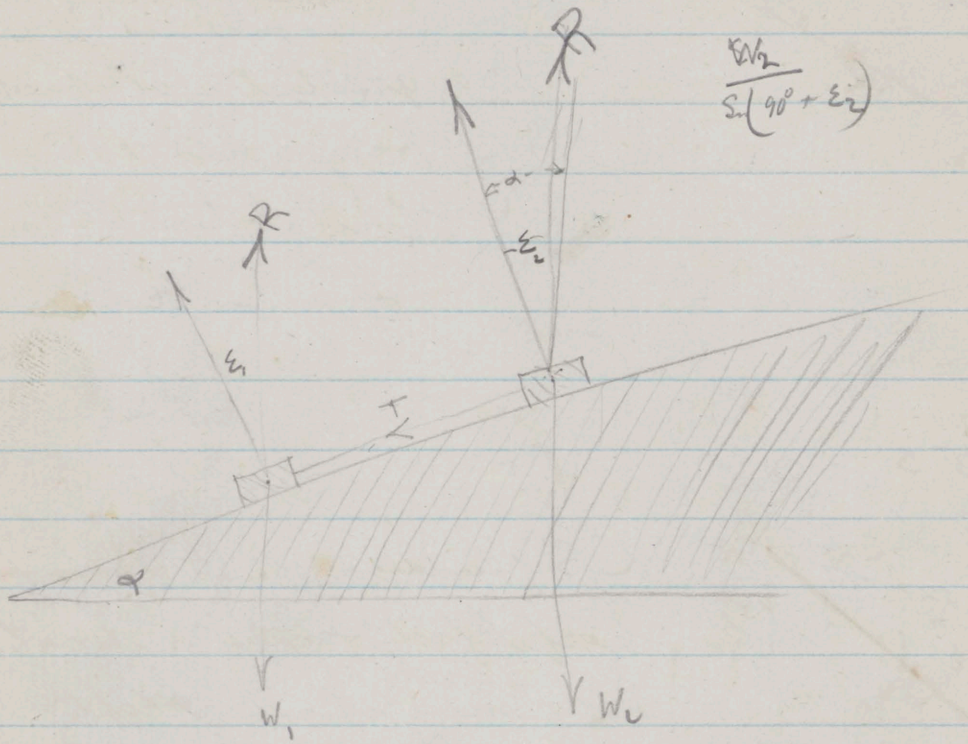
$$\frac{CO}{CB} = \frac{AC}{CB} = \frac{\frac{l}{3}}{\frac{2}{3}l} = \frac{l}{3} \cdot \frac{3}{2l} = \frac{l}{2}$$

$\frac{2l}{3}$

Mult



$$\frac{W_2}{\sin(90^\circ + \varepsilon_2)}$$

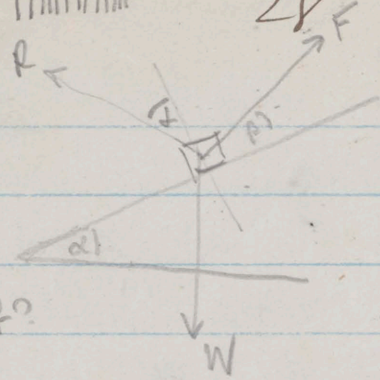


For upper body: $$\frac{T}{\sin(\alpha - \varepsilon_2)} = \frac{\text{Tension } T}{\sin \angle \text{ betw } W, R} = \left(\frac{T}{S(\alpha - \varepsilon_2)}\right) = \frac{W_2}{\cos \varepsilon_2}$$

Similarly. $$\frac{T}{\sin(\alpha - \varepsilon_1)} = \frac{W_1}{\cos \varepsilon_1}$$

Then $\therefore$ $$\frac{S(\alpha - \varepsilon_2)}{S(\alpha - \varepsilon_1)} = \frac{W_1 \cos \varepsilon_2}{W_2 \cos \varepsilon_1}$$

Given $\mu = \tan\alpha$



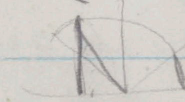
$$\frac{F}{W} = \frac{\sin 2\alpha}{\cos(\alpha-\beta)}.$$

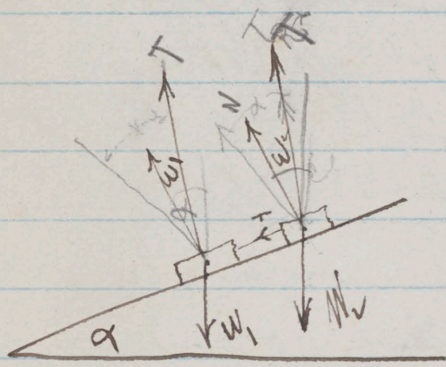
What value of β makes F least?

$\beta = \alpha$

then $F = W.\sin 2\alpha.$

$\sin(90 + \alpha - \beta)$





9 Two rough bodies rest on an inclined plane, & are connected by a string ‖l to plane. If the coef. of fricⁿ be not the same for both find the greatest inclination of the plane which is consistent with eqm. Solution: Let T = tension.

The upper body is acted on by 3 forces and is just about to slip $\therefore \frac{T}{\sin(\alpha-\epsilon_2)} = \frac{W_2}{\cos\epsilon_2}$

Similarly the lower one is acted on by 3 forces & about to slip

$$\therefore \frac{T}{\sin(\alpha-\epsilon_1)} = \frac{W_1}{\cos\epsilon_1}$$

Then dividing $\frac{\sin(\alpha-\epsilon_2)}{\sin(\alpha-\epsilon_1)} = \frac{W_1\cos\epsilon_2}{W_2\cos\epsilon_1}$

which gives an equation for $\tan\alpha$.

Intensity of pressure
$= \text{total amount of Pr.} \div \text{area}$

A fluid in a closed vessel perfectly transmits thro' its whole substance whatever pressure we apply ~~at~~ to any part.

The name "centre of buoyancy" is given to the centre of gravity of the liquid displaced

Every body immersed in a liquid is subjected to a resultant pressure = weight of liquid displaced & acting vertically upwards thro' "centre of buoyancy"

Hydrostatics

Definition :— any body that presses when at rest only ⊥arly to the surfaces in contact with it, is called a fluid.

For any fluid at rest, whether a perfect fluid or not, the pressure is always ⊥ to the sides of the containing vessel —: & this is ~~the~~ a great distinction between the theory of fluids at rest and in motion.

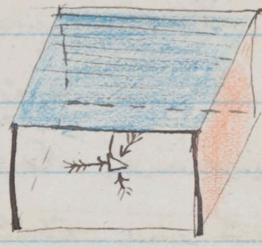
The pressure at a point is the pressure per unit area at that point i.e. if we take a very small area at the point, the total pressure on it divided by its area is the intensity of the pressure at that point and this pressure is the same no matter how the small area may be turned round.

Thus in a liquid the pressure on a point is the same in all directions:

Theorem of Archimedes.

Every body immersed in a liquid looses a portion of its weight = weight of liquid displaced

Suppose we consider a small prism :— whose ends are right sections



Then the pressures on its sides are ⊥ to those sides.

In the case of this very small element the pressures are ∷ to the lengths of the sides, & the weight ∷ to to the squares of those sides & ∴ may be neglected when the element is infinitely small.

The result is that the △ar element is in eqilibrium under the 3 forces ⊥ to its sides & acting at their middle points

∴ each pressure must be proportional to the side on which it acts

∴ the pressure per unit area is the same on all the sides. And generally the pressure per unit area at depth h in any direction is wh.

where W is weight of unit ~~mass~~ volume.

The whole pressure of a liquid on any surface is equal to the weight of a column of liquid, the area of whose base = that ~~of the C.S. of the~~ of the surface, & whose height = the depth of the C.S of the surface.

Let the surface be divided into a great number of very small rectangular areas as a_1, a_2, a_3, a_4 .... and let the depths of the centres of gravity of these areas be h_1 h_2 h_3 h_4.

The pressure on them will be

$w a_1 h_1 . w a_2 h_2 . w a_3 h_3$ ....

and ∴ the whole pressure is

$$w(a_1 h_1 + a_2 h_2 + a_3 h_3 + \text{etc.})$$

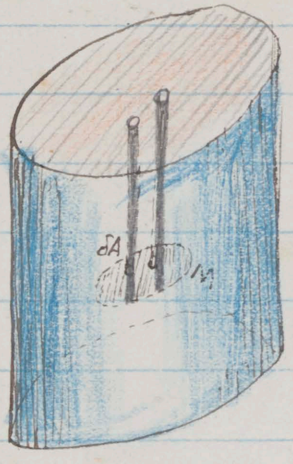
Now let H be the depth of the C.S of the surface, & let A be its area then by a proposition in Statics.

$$AH = a_1 h_1 + a_2 h_2 + a_3 h_3 \text{ etc.}$$

Hence the whole pressure is W.H.A.

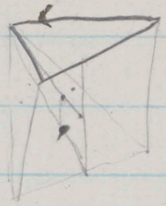
magnitude of pressure $= \bar{W}.h.A.$

※ Now let us consider a large area M. Let it be divided into a number of small elements as δA. (infinitesimals) The p.r. on a large plane area is $\perp$ to it and $= \Sigma W.\delta A.h = W.\Sigma h.\delta A. = W.\bar{h}.A.$ This finds the magnitude of the pressure on the given area :- where $\bar{h}$ = ? depth of $\underline{\delta A}$



The point of application of this is called the Centre of pressure.

The pressure on δA = weight of cylinder of the liquid, whose base is δA. and height $\underline{h}$, pressing down on it : hence the rule- at each point of the Area draw a $\perp$ = in length to the depth at that point. ~~The sum of~~ These form a solid figure, and the total pressure = weight of this solid. and it acts in a line thro' the CG of the solid: Thus the problem of Centre of Pressure is reduced to find the CG of a solid figure —

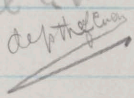


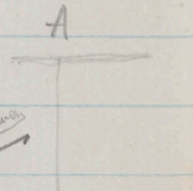
$\frac{3}{4} : \frac{1}{4} :: x : \frac{2}{3}$

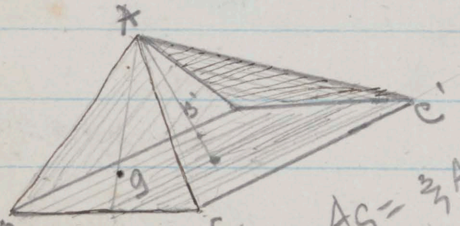
$\therefore x \times \frac{1}{4} = \frac{6}{12}$

Draw CC' $\perp^r$ to plane ABC and equal to the depth of C

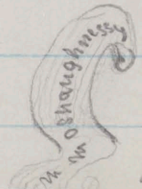
BB' and CC' are each $\perp^r$ to plane ABC and equal to th







$AG = \frac{2}{3} AH$

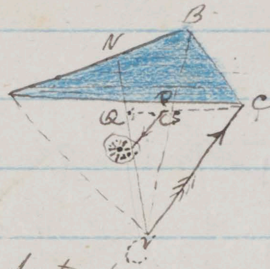


✓

2/3

#

If a Δ have its base in the surface of the liquid to find the magnitude and direction of the resul$^{\underline{tnt}}$ pressure.



$$\begin{cases} C'P : C'Q :: CO : CQ \\ \frac{3}{4} : 1 :: x : \frac{2}{3} \end{cases}$$

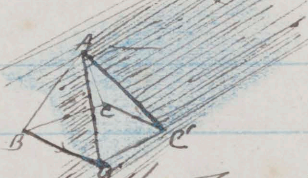
The magnitude of the pressure is (as was demonstrated before) $= W\bar{h}A$.
where $\bar{h}$ is depth of centre of G of Δ —
Since the pressure acts $\perp$ to the plane of the Δ. ~~By projecting from C to the surface~~ you have the req^{d} solid, a Δ ar pyramid. The projection of whose CG gives point of application of the pressure.

$$C'P = \tfrac{3}{4}\,C'Q = \tfrac{3}{4} \text{ of } \tfrac{2}{3}\,CN = \tfrac{1}{2}\,CN. \quad (\because \text{pr. bisects } CN.$$

#

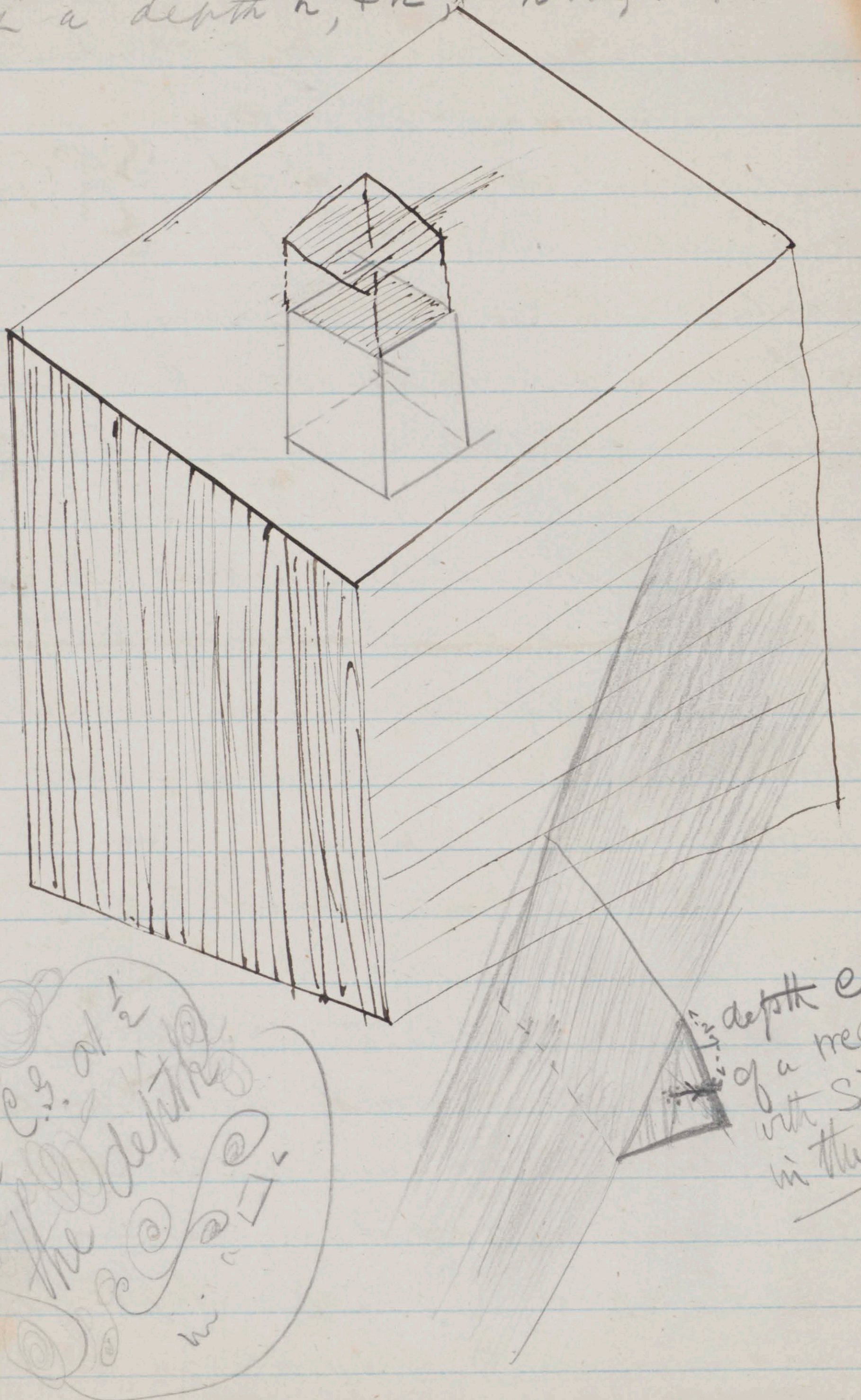
wah = pr.

If the vertex be in the surface of the liquid and the base $\parallel^{\underline{el}}$ to surface then we have a solid figure — a pyramid on a rect$\angle$ar base — whose CG is the intersection of [illegible] centre of pressure, $\therefore$ if h = depth of BC P passes through G, where $AG = \frac{3}{4}h$



Total pr $= \frac{2}{3} A.h.W$, where A = area of $\square$ base.

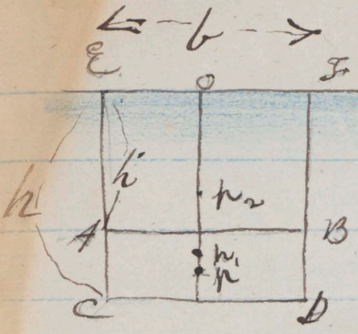
To find C.P of a rectangle, having its sides immersed at a depth h, + h', h being $> h'$.



depth CP of a rect^{L} with side in the surface

The C.G. at $\frac{1}{2}$ the depths

14 March



surface.

pr on A = W.a.h. Acting at $\bar{z}$

$EC = h \quad EA = h' \quad EF = b$

The pressure on ED $= bh.\frac{h}{2}W = b\frac{h^2}{2}W.$

& it acts at p_1 where $Op_1 = \frac{2}{3}h$.

C.P. of $\square$ with side in surface $= \frac{2}{3}$ depth.

The pressure on EB $= b\frac{h'^2}{2}W.$

and it acts at p_2 where $Op_2 = \frac{2}{3}h'$

The pr on AD is the resultant of the first and the second reversed

The magnitude of the resultant $= \frac{b}{2}(h^2 - h'^2)\,W$

& its point of application is p, where

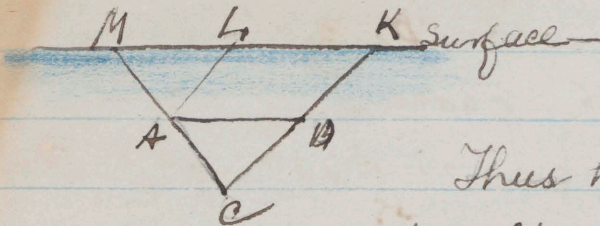
$$\frac{b}{2}(h^2 - h'^2)\,W\,Op = \frac{b}{2}h^2 W\,Op_1 - \frac{b}{2}h'^2 W\,Op_2$$

by taking moments around O

$$(h^2 - h'^2)\,Op = h^2.\frac{2}{3}h - h'^2\frac{2}{3}h'$$

$$Op = \frac{2}{3}\frac{h^3 - h'^3}{h^2 - h'^2}$$

$$= \frac{2}{3}\left(\frac{h^2 + hh' + h'^2}{h + h'}\right).$$

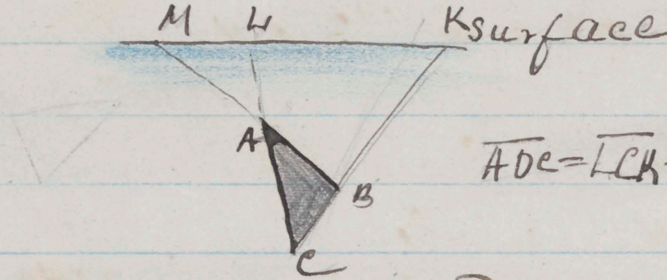


$\overline{ABC} = \overline{MCK} - \overline{MAL} - \overline{LB}$

Thus the pr on $\overline{ABC}$ is the resultant of that on $\overline{MCK}$; together with that on $\overline{MAL}$, and LB reversed:—

Thus, to find depth of (C)(P) of $\overline{ABC}$, take moments round MK —

[illegible] simple only work out



$\overline{ADC} = \overline{LCK} - \overline{MBK} + \overline{MAL}$

The (CP) of a Δ is same as CG of 3 weights at middle points of sides :: to the depths of those points xx ∴ Taking moments about D'B' we have

if $\bar{z}$ = depth of CP of the square

$$\bar{z}\left\{\frac{2\sqrt{2}a^3}{3} - 2\frac{a^3\sqrt{2}}{12}\right\}w = \frac{2\sqrt{2}a^3}{(2)\,3}w.\frac{a\sqrt{2}}{2} - 2\frac{a^3\sqrt{2}}{12}w.\frac{a\sqrt{2}}{4}$$

$$\bar{z} = \cancel{\frac{\frac{11}{24}\sqrt{2}\cdot a}{\frac{1}{2}}} \quad \frac{11\sqrt{2}}{12}\cdot a \qquad \frac{7}{6}\sqrt{2}\cdot a$$

?? Ans $\frac{7}{12}\sqrt{2}\cdot a$

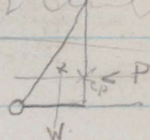
$\frac{7}{6}\sqrt{2}$

No. It depends on position of the Δs as well

$x = a\cot\alpha$

$a\,\frac{a\cot\alpha}{3}\,\sigma w$

Then take moments about O.



Wall will be overturned when moment of P round O > that of W

$h \times \frac{h}{2}$

$P = h\frac{h}{2}w$ per foot of length of the wall $= \frac{h^2}{2}w.$

?? $W = a\,\frac{a\cot\alpha}{3_2}.\sigma.w$

∴ The water will overturn the wall when the moment of P round O becomes = that of W ∴ when

$$h\cdot\frac{h}{2}w.\frac{h}{3} = a\frac{a\cot\alpha}{3}\,\sigma.w.\,\frac{2a\cot\alpha}{3}$$

$$h = a\sqrt[3]{\tfrac{4}{3}\sigma\cot^2\alpha}$$

The p.c. in an immersed □ acts at $\frac{1}{3}$ height from base.

W. a. L

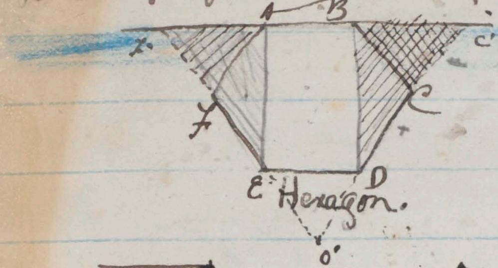
for lozged $\triangle$ $8\, a\sqrt{2} + a\sqrt{2}$

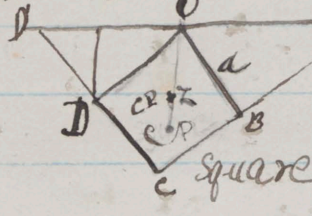
$2a\sqrt{2} + \ldots = $

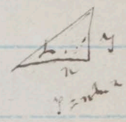
$\frac{a\sqrt{2}}{2} + \frac{a}{2}\sqrt{2}\ \&\ \frac{a\sqrt{2}}{6}$

acts at $\frac{1}{2}$ depth

Find C.P. of each of the following







$\overline{ABCDE} + \overline{AEF} - \overline{AFF'}$

Thus, pressure on hexagon = is resultant of that on $\overline{C'OF}$ & 3 times AFF' reversed

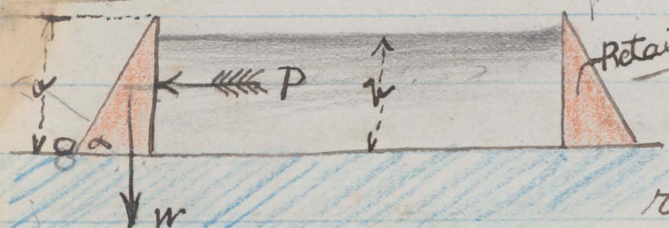
$\overline{BCDO} = \overline{B'CD'} - 2\overline{ODD'}\ \overline{OBB'}$

$\frac{2}{3}$ w.a.h.

Pr of $CD'B = 2a^2\left(\frac{a\sqrt{2}}{3}\right)W$ & acts at Depth $\frac{a\sqrt{2}}{2}$

" " $DD'O = \frac{a^3\sqrt{2}}{12}W$ " " $\frac{a\sqrt{2}}{4}$

" " $OBB' =$ same



The masonry of an embankment is supposed rigid & Solid, find height of water that will overturn it

SG = 2.

~~The water acts ⊥ to surface of retaining wall.~~

~~The C.G. of retaining wall is $\frac{1}{3}$ its height.~~

~~∴ if retaining wall were of same S.G. as liquid in contact, any height of water above $\frac{1}{3}$th would cause unstable eq^m, & consequently the overturning of retaining wall~~

~~∵ since S.Gy of wall is twice that of water, it will take a height of water something over $\frac{2}{3}$ height of wall to overturn latter.~~

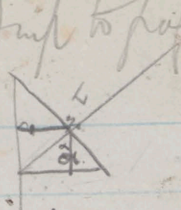
No

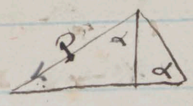
Next term

Ans $h = \frac{W}{P} \cdot 2a \cot\alpha$

Resolving to prof. (96)

$\frac{h}{\sin\alpha} L = \frac{h}{\sin\alpha}$





$L\sin\alpha = h$.

The wall will be overturned when the moments of P round O exceeds that of W.

The hor.l component of P is $P\sin\alpha$: its moment is $P\sin\alpha\,\frac{h}{3}$.

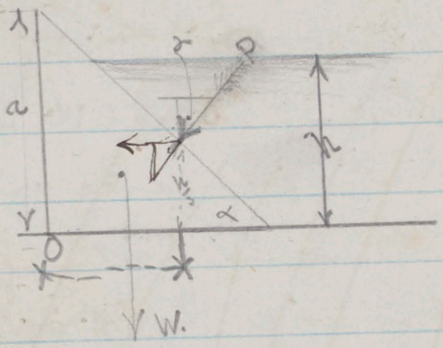
∴ Vertical = = = = $P\cos\alpha$: = = $-P\cos\alpha\left(a-\frac{h}{3}\right)\cot\alpha$

The moment of W is $W\,\frac{2a}{3}\cot\alpha$.

$\sin\alpha\,\frac{h}{3} = \frac{\cos^2\alpha}{\sin\alpha}\left(a-\frac{h}{3}\right) + \frac{2}{3}a\cot\alpha\left(\frac{a\alpha^2}{h^2}\cos\alpha\right)$ ch. pulls h

∴ $h = 3a\cot\alpha - 2\frac{a^3}{h^2}\sigma\cot\alpha$.

$h^3 - 3ah^2\cos^2\alpha - 2\sigma a^3\cos^2\alpha = 0$ (a cubic eq.n for h which you can solve by Horner's method)



The pr. acts at $\frac{h}{3}$ from base

$P = \frac{h^2}{2\sin\alpha}$. { hor.l comp.t $= P\sin\alpha\,\frac{h}{3}$ } red.s $= \frac{h^3}{6}$

Vert comp.

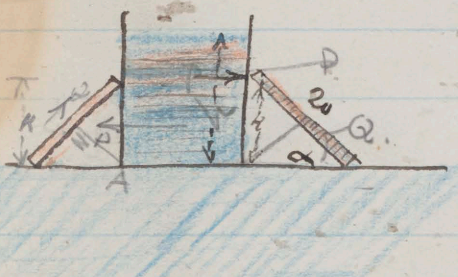
$\frac{W}{P} = \frac{a^2\cos\alpha W\sigma}{2\sin\alpha} \times \frac{2\sin\alpha}{h^2 W}$

h = depth of water.
its pr on wall is a force P acting at $\frac{2}{3}h$ depth from top

If you neglect the strength of the foundations the water will overturn the embankment when it rises to such a height the moment of P becomes > that of W round O.

And the question is to find the height of the water when it overturns;:

Where $\underline{P = \frac{h}{\sin\alpha}\frac{h}{2}\cdot w}$ per foot of length of the wall. The wt. of 1 ft. of length of the wall $= \frac{a^2}{2}\cot\alpha \cdot w\sigma$ where σ is S.G.



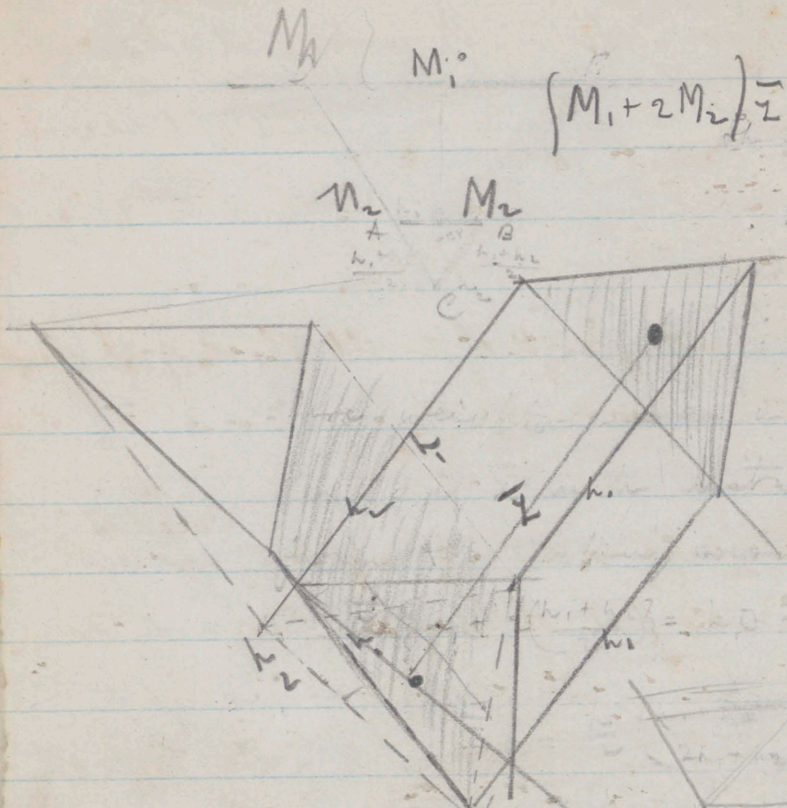
(Coffer dam) a prop is placed at every 20 feet of length — find thrust on any one of them — height of water being h. also that of top of prop $= k$.

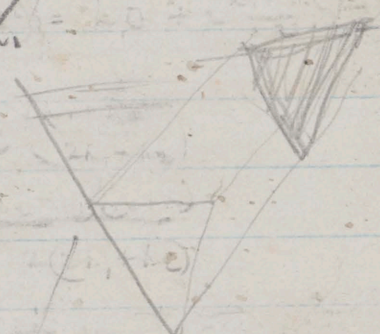
Required to find thrust on prop:—
The moment of the water pressure round A is balanced by that of Q round A:—

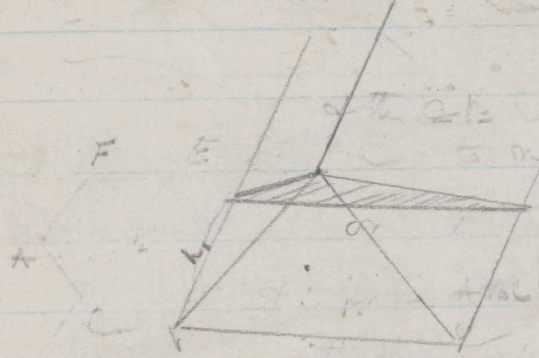
$$h\cdot\frac{h}{2}\cdot w\frac{h}{3} = Q\cdot AN$$
$$= Q\cdot \cancel{20}\,l\cdot\cos\alpha\,\sin\alpha \qquad (20 = l)$$
$$\therefore\ Q = \frac{h^3\cdot w}{120\sin\alpha\cos\alpha} \text{ where } w = 62\tfrac{1}{2}\text{ lbs.}$$

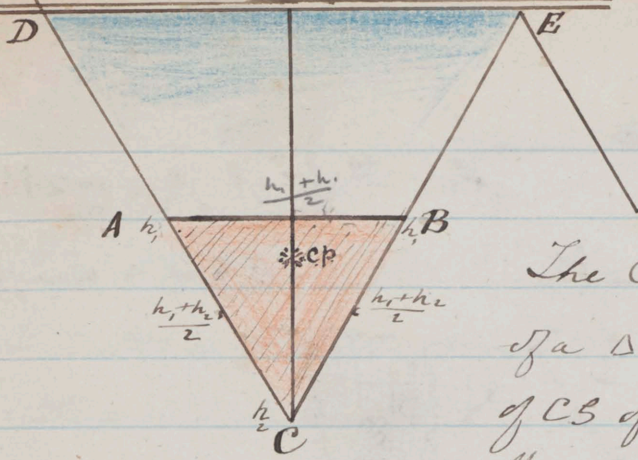
= value of the thrust.

$$(M_1 + 2M_2)\bar{x} = M_1 a_1 + 2M_2 a_2$$









The Centre of Pressure of a Δ is the same as that of CG of three weights at the middle points of its sides :: to their depths.

In this case the weights are as in the Fig; - Thus if $\bar{z}$ be the distance of their C.G. from AB. (Taking moments with respect to AB):- $\bar{z}\left\{\frac{h_1+h_1}{2}+2\left(\frac{h_1+h_2}{2}\right)\right\} = h_1 0 + 2\left(\frac{h_1+h_2}{2}\right)\frac{p}{2}$; (where p is $\perp$ from C on AB)

Hence $\bar{z} = \frac{p}{2}\left(\frac{h_1+h_2}{2h_1+h_2}\right). = \frac{(h_2-h_1)(h_2+h_1)}{2(2h_1+h_2)}$. and the (C.P.) lies on the straight line joining C to middle of AB.

$\bar{z}\left(h_2 +\right.$

$\bar{z}\left(2h_1+h_2\right) = \left(h_1+h_2\right)\frac{p}{2}$

∴ remo

$\therefore \quad \bar{z} = \frac{(h^2-h_1^2)}{2(2h_1+h_2)}$

$$h_2 - h_1 \,\big)\, h_2^4 - h_1^4 \,\big(\, h_2^3 + h_1 h_2^2 + h_1^2 h_2 + h_1^3$$

$$\underline{h_2^4 - h_1 h_2^3}$$

$$h_1 h_2^3 - h_1^4$$

$$\underline{h_1 h_2^3 - h_1^2 h_2^2}$$

$$h_1^2 h_2^2 - h_1^4$$

$$\underline{h_1^2 h_2^2 - h_1^3 h_2}$$

$$h_1^3 h_2 - h_1^4$$

$$\underline{h_1^3 h_2 - h_1^4}$$

$h_2 - h_1)$

$$h_1^2 - h_2^2$$

$$\underline{h_1 - 2h}$$

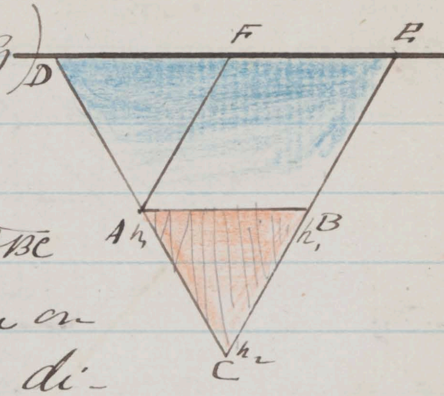
$$h_1^2 - h_1 h_2$$

$$\frac{h_2^4 - h_1^4 - 2h_1^3(h_2 - h_1)}{2h_2^3 - 2h_1^3 - 6h_1^2(h_2 - h_1)}$$

$$\frac{h_2^3 + h_1 h_2^2 + h_1^2 h_2 \cancel{+ h_1^3} - \cancel{2} h_1^3}{2(h_2^2 + h_1 h_2 - 2h_1^2)}$$

(another Solution of preceding)

or



$\Delta\ \overline{ABC} = \Delta\overline{DCE} - \Delta\overline{DAF} - \overline{FG}$

and ∴ the pressure on $\overline{ABC}$ is the weight of the pressure on this figure with proper direction for each. Now pressure on $\overline{DCE}$

on $\overline{DCE} = DE \cdot \frac{h_2}{2} \frac{h_2}{3} w$. & it acts at a depth $\frac{h_2}{2}$.

pr. " $\overline{DAF} = DF \cdot \frac{h_1}{2} \cdot \frac{h_1}{3} . w$. " " " " $\frac{h_1}{2}$.

" " $\overline{FA} = FE \cdot h_1 \frac{h_1}{2} w$. " " " " " " $\frac{2h_1}{3}$.

[We know that $DE : DF : FE = h_2 : h_1 : (h_2 - h_1)$.]

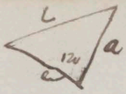
If $\bar{z}$ be the depth of c.p. on ABC. we have (taking moments with respect to DE).

$$\bar{z}\left\{DE\frac{h_2^2}{6} - DF\frac{h_1^2}{6} - FE\frac{h_1^2}{2}\right\}w. = \left\{DE\frac{h_2^3}{12} - DF\frac{h_1^3}{12} - FE\frac{h_1^3}{3}\right\}w$$

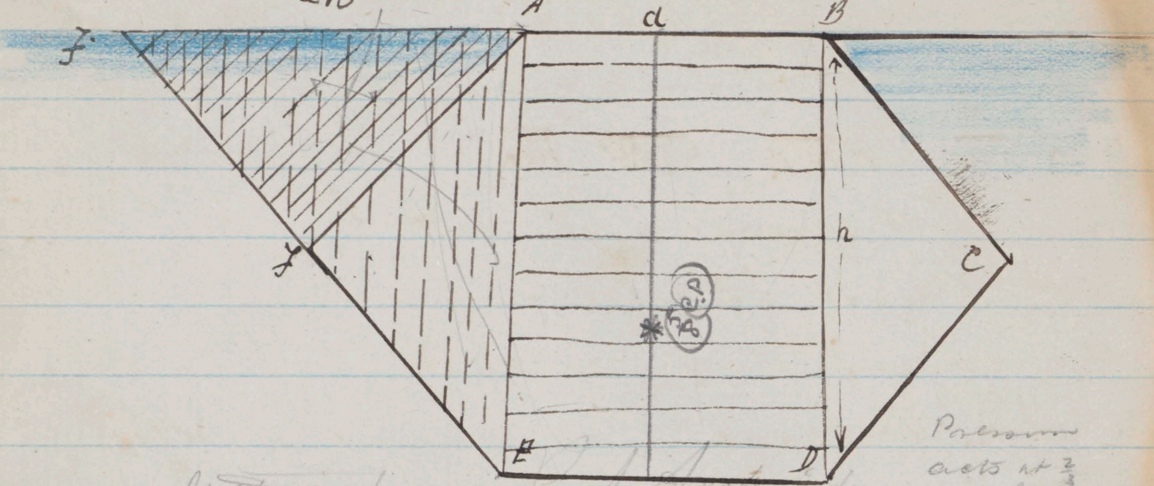
$$\therefore \bar{z} = \left(\frac{\frac{h_2^4}{12} - \frac{h_1^4}{12} - \frac{h_1^3(h_2-h_1)}{6}}{\frac{h_2^3}{6} - \frac{h_1^3}{6} - \frac{h_1^2(h_2-h_1)}{2}}\right)$$

which when reduced will agree with former result.

II



$= 2a^2 - 2a^2\cos 120° = 2a^2 + a^2 = a\sqrt{3}$



~~meta centre~~ C.P. of Steamer. Pressure acts at $\frac{2}{3}$ depth of $\square$

at apex from force in rotating fluid.

~~$h^2 = 2a^2$~~

$h \cos 30° = a$

$h = \frac{2a}{\sqrt{3}}$

$$\overline{ABDE} + 2\overline{AEF'} - 2\overline{AFF'} = \overline{BCDEF}$$

The pressure on $ABDE = w.h.a = \frac{a\sqrt{3}}{2} w.\frac{2a^2}{\sqrt{3}}$. + acts at p_1 where $op_1 = \frac{2a}{3}\sqrt{3}$

" " " $2AEF' = w\,h\,a = \frac{2a}{\sqrt{3}} w \cdot 3a^2 \cdot op_2$: where $op_2 = \frac{a}{2}\sqrt{3}$

" " " $2AFF' = w\,h\,a = \frac{a}{\sqrt{3}} w. \frac{3a^2}{2} \cdot op_3$: $op_3 = \frac{7a}{4\sqrt{3}}$

" " " $BCDEF = w\,h\,a = w.\cancel{\tfrac{2}{1}}\, a^2(\sqrt{3} + \frac{3}{2})$ alg. sum ∴ $\bar{z}$

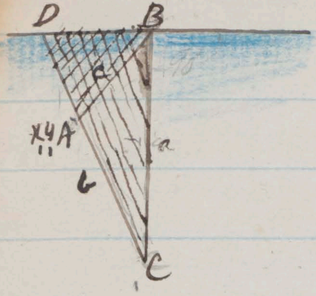
∴ Taking moments about AB we have :-

$$\cancel{w}\cdot\bar{z}\left\{a^2(\sqrt{3}+\tfrac{3}{2})\right\} = \cancel{w}\, 2a^3 + \cancel{w}\,\tfrac{3}{2}a^3\sqrt{3} - \cancel{w}\,\tfrac{21}{8}\frac{a^3}{\sqrt{3}}$$

$$\bar{z} = \frac{2a + \frac{3}{2}a\sqrt{3} - \frac{21}{8}\frac{a}{\sqrt{3}}}{\sqrt{3} + \frac{3}{2}}$$

$$\frac{5}{8}a\sqrt{3} = \bar{z}$$

Quod Erat Demonstrandum



Find the distance of C.P. of a Δ having a vertex at the surface & a line or side forming this vertex $\perp^r$ to surface.

$$\overline{BCA} = \overline{BCD} - \overline{BAD}.$$

Pressure on $BCD = w \cdot h \cdot a = W \cdot \frac{a^2 \cdot BD}{6}$. acting at depth $= \frac{a}{2}$

" " $BAD =$ " $= \frac{W \cdot y_1^2 \cdot BD}{6}$ " " " " $= \frac{y_1}{2}$

" " $BAC = \frac{2}{3}$ " $= \frac{2}{3} w a^2 x_1 \frac{3}{4}$ " " " $= \frac{3}{4} a$

Taking moments about BD we get :-

$$\cancel{W}\, \overline{Z} \left\{ \frac{a^2 \cdot \cancel{BD}}{6} - \frac{y_1^2 \cancel{BD}}{6} - \frac{2(a^2 - y_1^2)\cancel{BD}}{3 \cdot 6} \right\} = \frac{\cancel{W} a^3 \cdot \cancel{BD}}{12} - \frac{\cancel{W} y_1^3\, \cancel{BD}}{12} \frac{2(a^2 - y_1^2)}{6 \cdot 3}$$

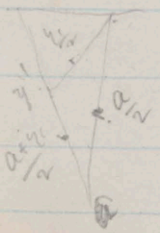
$$\therefore \overline{Z} \left\{ \left(\frac{a^2}{6} - \frac{1}{9}a^2\right) - y_1^2 \left(\frac{1}{6} - \frac{1}{9}\right) \right\} = \frac{a^3}{12} - \frac{y_1^3}{12} - \left(\frac{a^2 - y_1^2}{9}\right) \overline{Z}.$$

$$\therefore \overline{Z} \left\{ a^2 \left(\frac{1}{6} - \frac{1}{9}\right) - y_1^2 \left(\frac{1}{6} - \frac{1}{9}\right) + \frac{a^2 - y_1^2}{9} \right\} = \frac{a^3 - y_1^3}{12}$$

$$\therefore \overline{Z} \left\{ \left(\frac{1}{18} + \frac{2}{18}\right) a^2 - \frac{1}{6} y_1^2 \right\} = \frac{a^3 - y_1^3}{12}$$

$$\therefore \overline{Z} = \frac{a^3 - y_1^3}{12} \times \frac{6}{a^2 - y_1^2} = \frac{a^2 + a y_1 + y_1^2}{2(a + y_1)}.$$

Q.E.D.

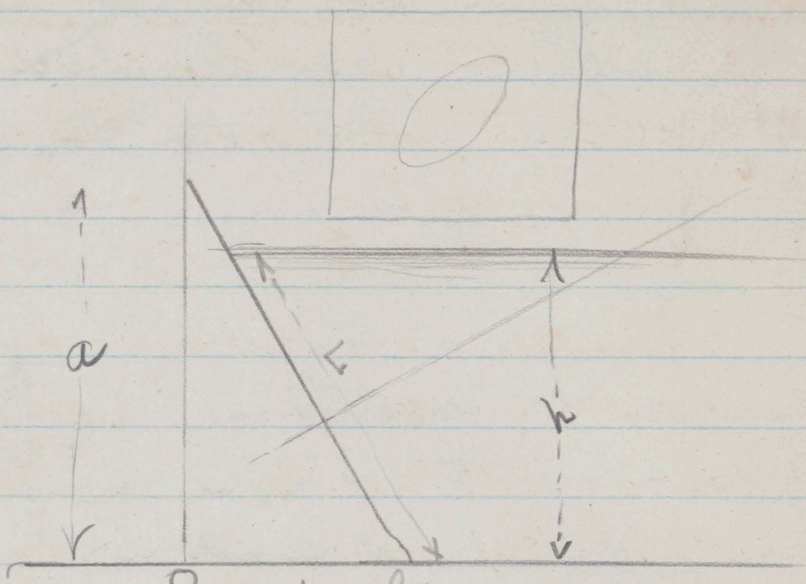


$$\frac{2(a+y_1)}{2} \cdot \overline{Z} = \left(\frac{a}{2}\right)^2 + \left(\frac{y_1}{2}\right)^2 + \left(\frac{a+y_1}{2}\right)^2$$

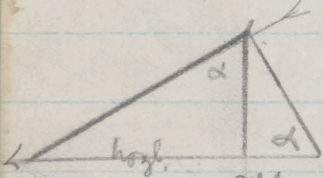
$$\overline{Z} = \frac{a^2 + a y_1 + y_1^2}{2(a + y_1)}$$

Substituting for $\overline{Z}$ ...

$$= \frac{\frac{3a^2}{4} + \frac{a^2}{2} + \frac{3a^2}{4}}{3 a \sqrt{3}}$$



$L = \frac{h}{\sin\alpha}$



Moment of P becomes $\frac{1}{3}$rd the thrust W.

$\therefore P = \frac{h}{\sin\alpha}\cdot\frac{h}{2}W = \frac{Wh^2}{2\sin\alpha}$.

$W = \frac{a^2\cot\alpha}{2}\rho W$. Where ρ ? 89

The hozl Compt. of P = vert.l Compt + Wt of wall

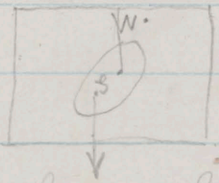
$\therefore \frac{Wh^2}{2\sin\alpha}\cdot\cancel{\sin\alpha}\,\frac{h}{3} = \cancel{\frac{Wh^2}{2\sin\alpha}\cos\alpha}\left(a - \frac{h}{3}\right)\cancel{\cot\alpha} + \cancel{\frac{a^2\cot\alpha}{2}}\rho$

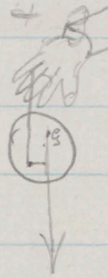
$\therefore \frac{h^3}{6\cot^2\alpha} = h^2 a - \frac{h^3}{3} + \rho\frac{a^3}{3}$

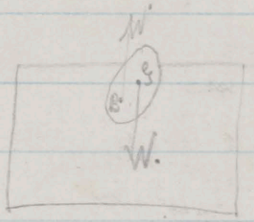
$\therefore h^3(1+2\cot^2\alpha) - h^2 a\,6\cot^2\alpha = 2\rho a^3\cot^2\alpha$ = Answer

The pressure on an immersed body
= weight of water displaced by the body.
The CG of body + CG of Displacement
must be in the same vertical

Suppose a piece of lead
to be attached to a pumper
plate. W' = W. + g must be lowest.







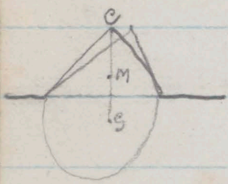
B must come
round under G
vertically.

For a body

~~W.~~ $\frac{2}{3}$ ~~a cot α~~

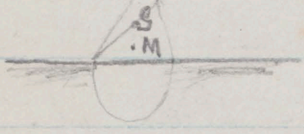
Angwin L.A. 6
You can find the height of water capable of
overturning the wall.

The equilibrium is always stable when the CS of the floating body is above The CS of the displaced liquid.

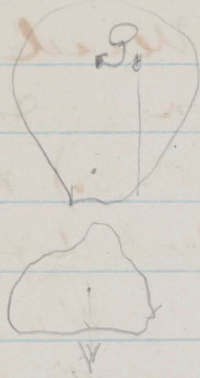


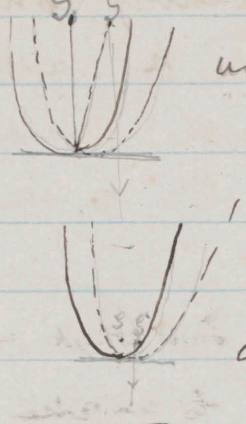
Find what weight must be placed at C to make it float on its Side:-

If the section of a ship be oar, find condition of Stability:- (a) if unstable find what weight of ballast of density σ if laid uniformly in the bottom, will make it stable:- i.e. float upright

(a) 

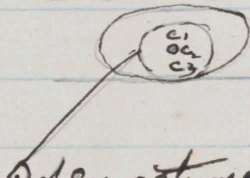
The weight of the body = W. & MS = h _ r = a





unstaple

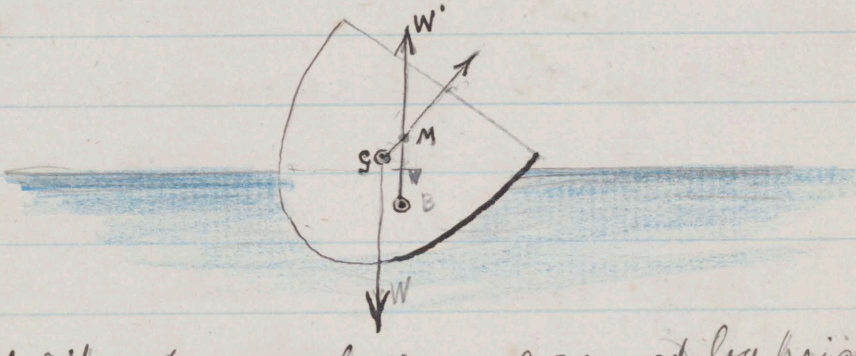
Staple.

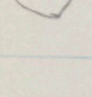


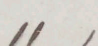
curve

CG at C_1. unstaple
CG at C. Staple eqlm
CG at C_2. Staple.

O of curvature





The stability of a vessel is measured by height of M above G.  In fact if the ship roll a small $\angle \theta$, the "righting couple" is $W \cdot GM \cdot \theta$ since $GV = GM \cdot \theta$.

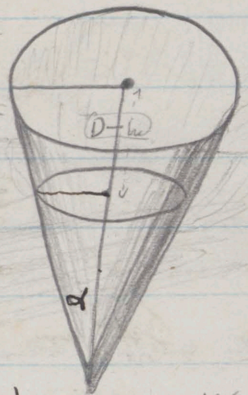
A thin heavy conical vessel floats with its vertex h feet down : find how much Mercury of SG ($\sigma = 13{\cdot}6$) must be placed in it to make it sink a feet further

$$W = \tfrac{1}{2} h^3 \tan^2\alpha \cdot g\gamma$$

$$W + W' = \tfrac{1}{3}(h+a)^3 \tan^2\alpha \cdot \gamma$$

to find W'

$$W' = \frac{(h+a)^3 - h^3}{h^3} \, W$$



$$W = h^3 \tan^2\alpha \cdot g\gamma$$

$$W + W' = (h+a)^3 \tan^2\alpha \, \rho g \gamma$$

$r = h \tan\alpha$

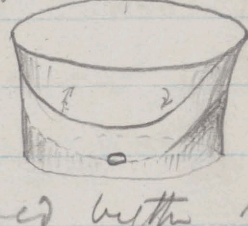
$\tan\alpha = \frac{r}{h}$

$$W' = (\overline{h+a}^3 - h^3) \tan^2\alpha \, g\gamma(\rho - 1)$$

If you make a hole in the bottom of the liquid find how much will remain.

Need read Boundys, Gaylussis

read Annelsons.



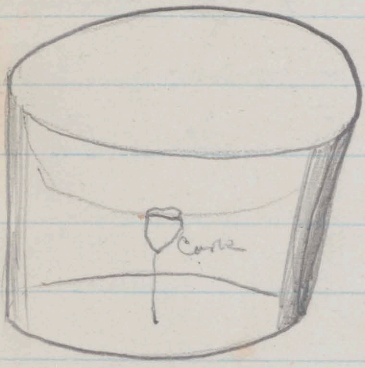
the Quantity of water bounded by the base & surface curve of paraboloid tangential to the centre of the base when the orifice is.

In Deschanel. Exs 4-8-13-16-17- 20

Godfrey's Astronomy

{Michael M O'Shaughnessy}

at the free surface the force acting on the particles is $\perp$ to the surface.

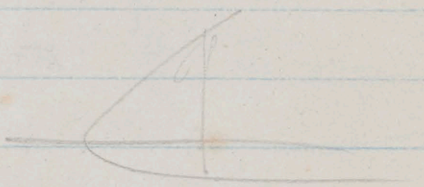


(~~The pressure inward~~)

The cork will tend inwards, owing to greater pressure by water than centrifugal force:

In the Case of a rotating liquid find pressure at point P.

Vol parabola —

$=$ 

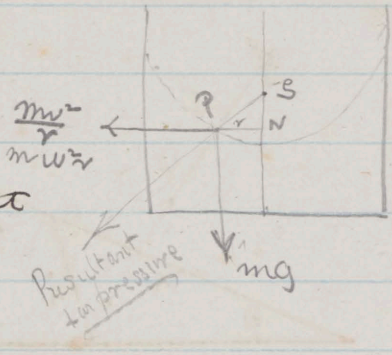
$\frac{21}{3} \cdot x^2 Y \cdot$

23

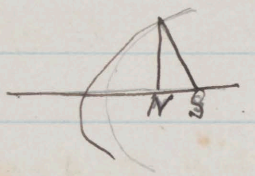
PNG is Δ of forces

$NG : r = mg : m\omega^2 r$

$\therefore NG = \frac{g}{\omega^2}$, a constant



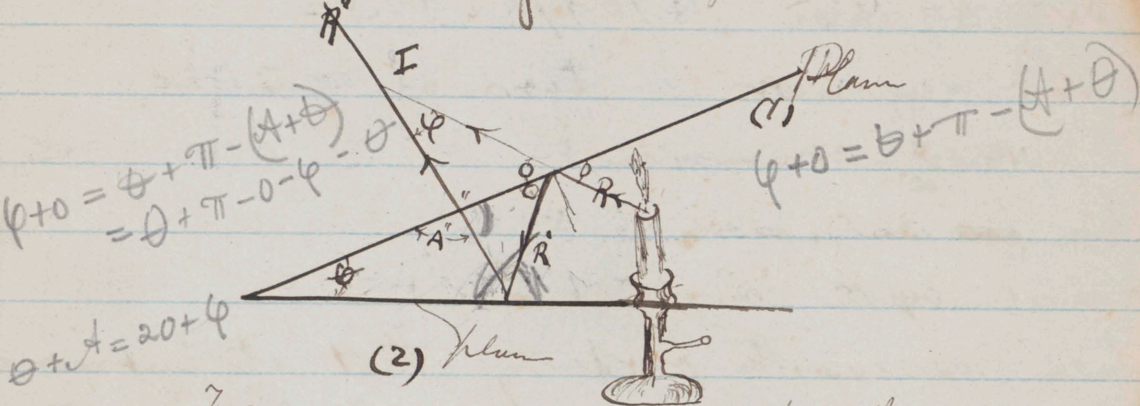
$\therefore$ The ~~free~~ surface is formed by the curve which possesses the property that its subnormal is const. It is a parabola of latus rectum $= \frac{g}{\omega^2}$.



Vol of parabola = $\frac{1}{2}$ vol of hemisphere on same base & of same height.

Vol that escapes = area of base of vessel × height

Principle of Hadley's Sextant.



To prove geometrically that the $\angle \phi = 2\angle\theta$.
Granting that $\angle$ of incidence = $\angle$ of reflectn

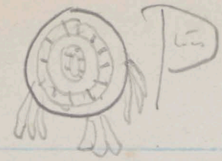
$$\pi - A = \phi + 0 = \theta + \pi - (A+\theta)$$

$$\therefore \theta + 0 = \pi - (A+\theta)$$

$$\therefore \phi + 0 = \phi + \theta + 0 = 2\theta + 0$$

hence $\phi = 2\theta$. Q.E.D.

N.B. R represents the directly incident incident ray on plane (1) which is reflected on plane (2) as R', & from thence again supposed to be reflected on plane (1), as R'' through wh. it passes to meet R. at I

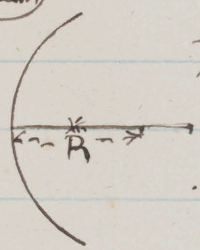


omit 998

Arts 964 = 965 = 967 = 975 = 995 =
Theory of formation of images,
with Concave & convex mirrors :-
As Examples 46. 50. 51

Laws of Refraction –

Example 46 Deschanel

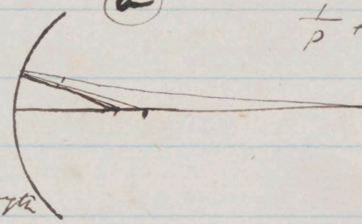


$\frac{1}{p} + \frac{1}{p_1} = \frac{2}{R}$.

p is infinite

$\therefore p' = \frac{R}{2}$ = focal length
= 1 foot.

(a)



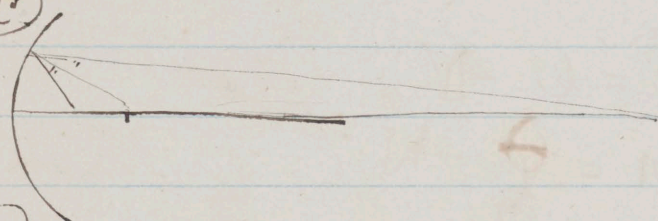
$\frac{1}{p} + \frac{1}{p_1} = \frac{2}{r}$.

$\therefore p' =$

$= \frac{-180''}{15 - 12}$

= 5 feet.

(c)

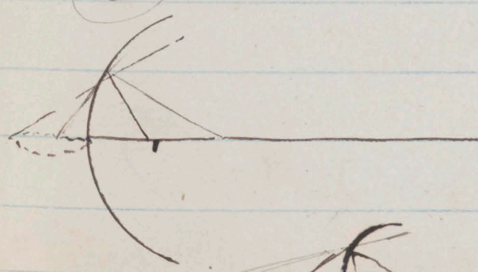


$\frac{1}{p} + \frac{1}{p'} = \frac{2}{r}$

$\frac{1}{10} + \frac{1}{p'} = 1$

$p' = 1\frac{1}{9}$ feet in front.

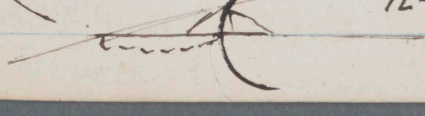
(d)



$\frac{2}{3} - \frac{1}{p'} = \frac{1}{f}, \therefore p' =$

~~$p' = \frac{3}{7}$~~

$\frac{1}{p'} = \frac{4}{3} - 1$



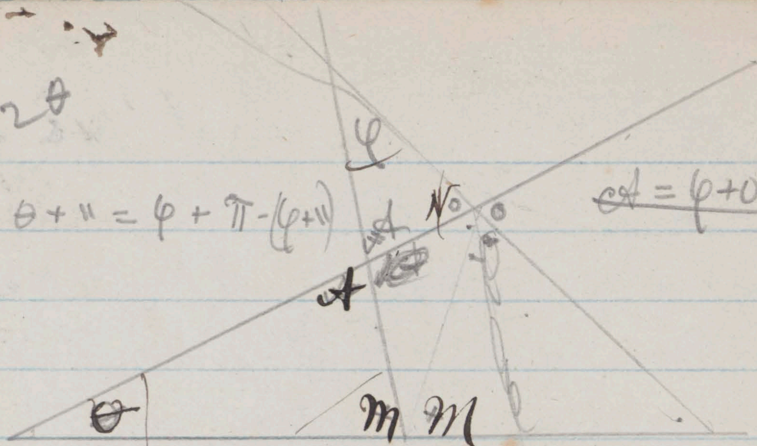
$12 - \frac{1}{p'} = \frac{1}{f} = 1$
p'

$\therefore p' = 3$ feet. behind

$\varphi = 2\theta$

$A = \theta + 11 = \varphi + \pi - (\varphi + 11)$

$\cancel{A = \varphi + 0 = \theta + x}$



$$\pi - A = \varphi + 0 = \theta + x = \theta + \pi - (\theta + A)$$

$$\therefore \quad \theta + 0 = \pi - (A + \theta)$$

$$\therefore \varphi + \cancel{\varphi} = 2\theta \cancel{+\varphi}$$

$$\therefore \varphi = 2\theta$$

Prove that position of P

Draw beam by which the eye would see the image of x.

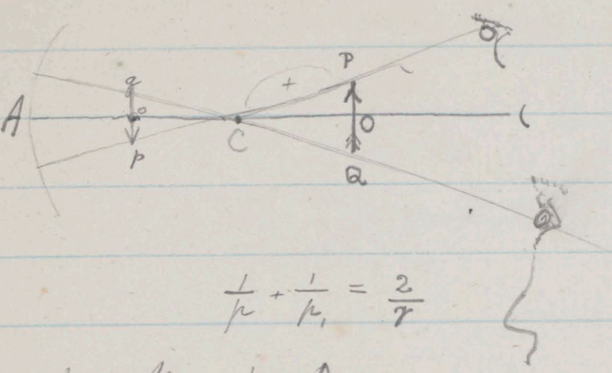
$$\cancel{\varphi} \quad \theta + m = \varphi + N$$

$$\therefore m = \cancel{\theta} + N$$

$$\therefore 2\theta + N = \varphi + \cancel{N}$$

$$\varphi = 2\theta$$

Angle between rays of light = ~~2~~ twice ∠ between planes



+, also is positive when measured in opposite direction to light.

−, & negative otherwise

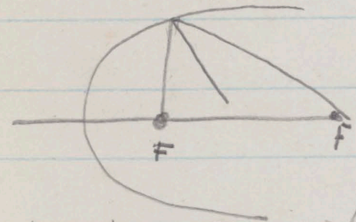
$$\frac{1}{p} + \frac{1}{p_1} = \frac{2}{r}$$

where $p = AO$, $p_1 = Ao$

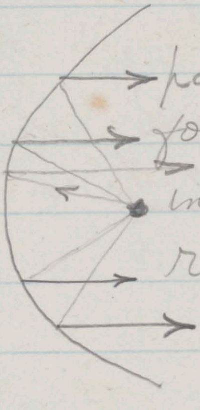
or, when $q = CO$; $q_1 = Co$

$$\frac{1}{q} + \frac{1}{q_1} = \frac{2}{r} \; :$$

Degree of magnitude $= \frac{Co}{CO}$.



of no practical importance when rays are sent from one focus to another.



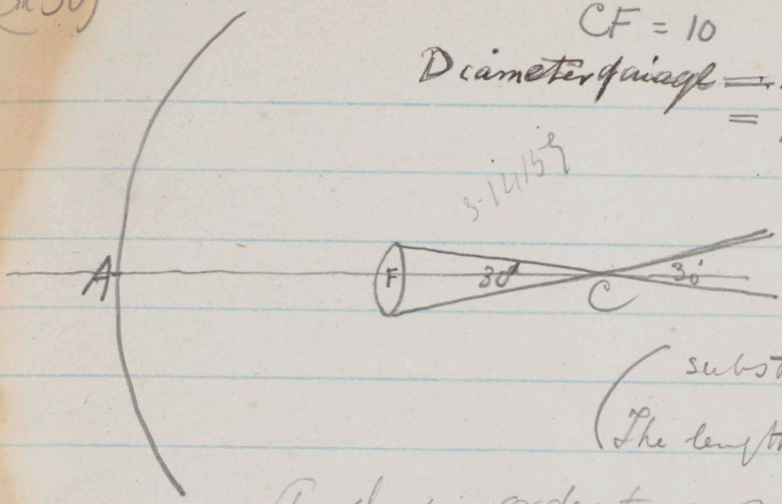
parabolic Section for a lighthouse. in order to send off $\parallel^{le}$ rays.

Michael M. O'Shaughnessy

Michael M Michael M O'Shaughnessy

Joseph Larmor Michael M O'Shaughn

(Ex: 50)



CA = 20

CF = 10

Diameter of image = 10 × Circular measure of 30'

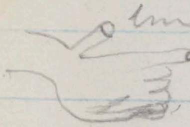
$= 10 \times \frac{\frac{1}{2}\pi}{180}$ metres

$= \frac{314.16}{36}$ c.m.

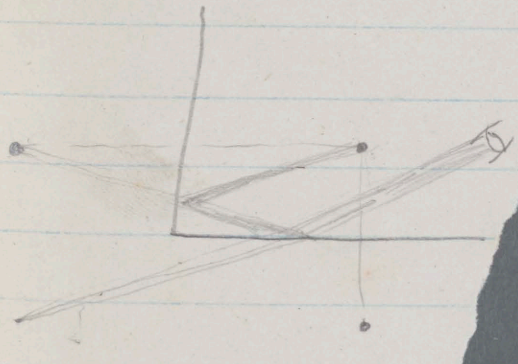
$= 8\frac{2}{3}$ about.

(substituting feet.
The length = 1 inch, (about))

And in order to magnifie this you must convert it into a telescope, by the addition of an eyepiece to magnify the image brought near:

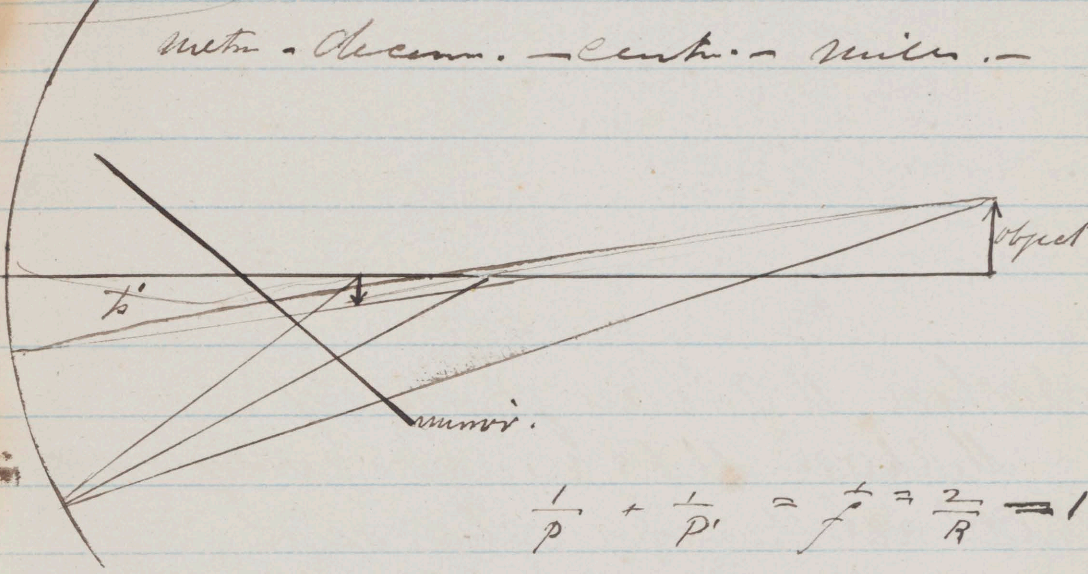


EXAMPLES. 39. 43 = 52 = 54 = 55 = 56 = 57
58 = 60:



Ex. 57

Meter. Decimeter. Centi. Milli.



$$\frac{1}{p}+\frac{1}{p'}=\frac{1}{f}=\frac{2}{R}=1$$

$$\frac{1}{5}+\frac{1}{p'}=1$$

$$\frac{1}{p'}=\frac{4}{5}$$

$\therefore p' = 1\frac{1}{4}$ m.

$$\frac{p}{p'}=\frac{p-f}{f}$$

$$\frac{5}{\frac{5}{4}}=\cancel{\frac{p-f}{f}}$$

$p-f$ = distance of object from principal focus

$$\cancel{\frac{20\times 4}{5}=p-f}$$

$$\cancel{\therefore p-f=16}$$

$$\frac{\text{object}}{\text{image}}=\frac{5}{\frac{5}{4}}$$

Object = 4 times image $\therefore$ latter = $\frac{1}{4}$ dec.

new position of the image

$\frac{1}{4}$ m. from focus:

Michael M O'Shaughnessy
Michael M O'Shaughnessy.
Michael M O'Shaughnessy.
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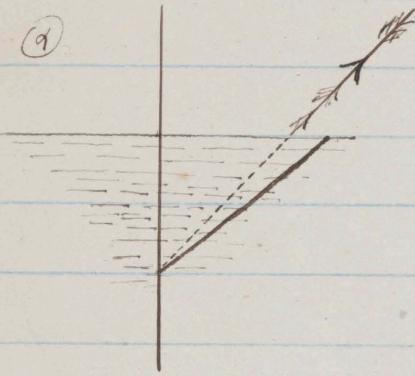
Michael M O'Shaughn
Michael M O'Shaughnessy

Michael M O'Shaughnessy
Michael M O'Shaughnessy
Michael M O'Shaughnessy
M. M

Examples in refraction

Deschanel Page 1142.

52 (α)



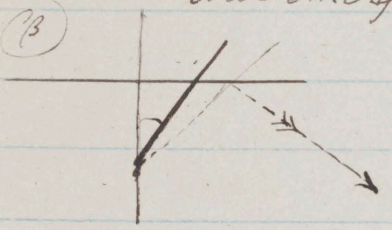
The index of refraction from air to water is $\frac{4}{3}$ & from water to air it must consequently $= \frac{3}{4}$.
$= .75$

And since $\frac{1}{\sqrt{2}} = \sin 45^\circ = .707$

∴ this is less than the sine of the critical ∠

∴ The ray incident on the surface of the water will emerge.

(β)



By a process of reasoning similiar to the preceding we can easily see that the ray incident on the surface of the glass will be reflected ∵ .66 ∠ .707.

Q E D

(54)

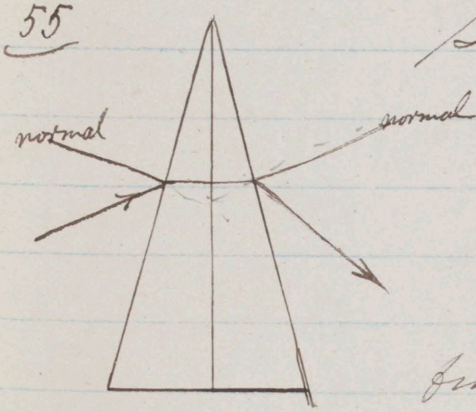
~~The ratio of the sine of ∠ of incidence when a ray passes from air into water~~

When a ray passes from vacuum into any medium, the ratio of the sine of the ∠ of incidence, to the sine of the ∠ of refraction is always > unity, & is called the index of refraction for the medium in question: Thus the index of refraction for glass $= \frac{3}{2}$, & water $\frac{4}{3}$.

54.)

$1 : \frac{4}{3} :: 1.11 : 1.48$ ans. = index from Air into O of Turp.

55

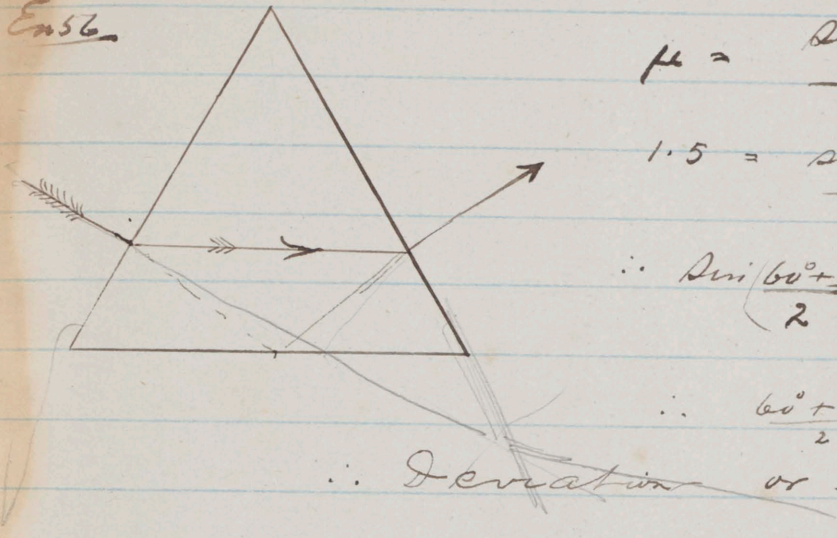


Since μ the ratio of sines of ∠s of incidence & refraction, we have

$$1.6 = \frac{\sin\left(\frac{10^\circ + D}{2}\right)}{\sin \frac{A}{2}} = \frac{\sin\left(\frac{10^\circ + D}{2}\right)}{\sin 5^\circ}$$

by which you can easily find D.

En 56



$$\mu = \frac{\sin\frac{60^\circ + D}{2}}{\sin 30^\circ}$$

$$1.5 = \frac{\sin\frac{60^\circ+D}{2}}{\frac{1}{2}}$$

$$\therefore \sin\left(\frac{60^\circ+D}{2}\right) = \frac{3}{4}$$

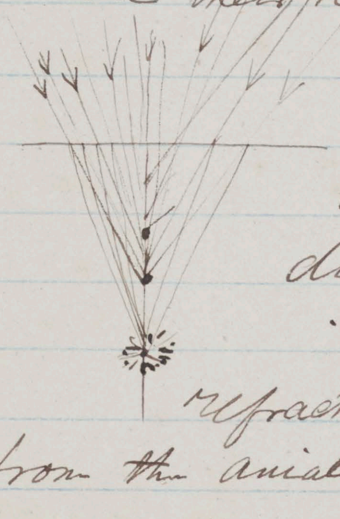
$$\therefore \frac{60^\circ+D}{2} = 60^\circ$$

$\therefore$ Deviation or $D = 60^\circ$

En 57

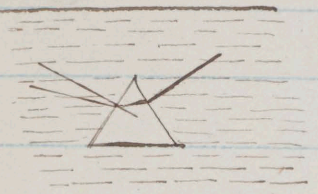
Evidently 3 mm., since index of glass $= \frac{3}{2}$. + objects appear $\frac{2}{3}$ nearer the surface than their/real distance:

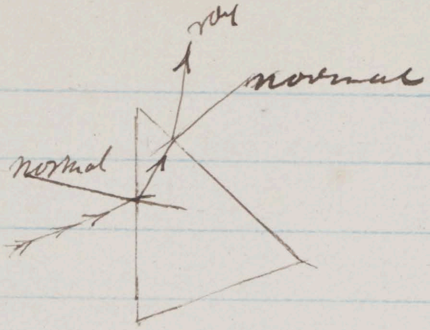
En 58



In air the rays are brought to a focus at a distance of 1 ft from the lens. $\therefore$ owing to the greater refractive power of water they are bent from the axial normal at the surface, so as to meet at a distance of $\frac{4}{3}$ one foot below the surface

Ex 60





Since the ray in passing from a dense medium to a less dense one it must consequently be deflected from the normal at the point of incidence at a > angle than that of incidence, which tends (as in Fig) to send it towards the edge of the prism:

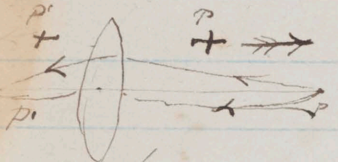
Also when it goes to the other side (forming edge of the prism) it is ~~also~~ further acted on, so as to cause it to move nearer the edge.

LENSES

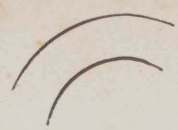
Fot:

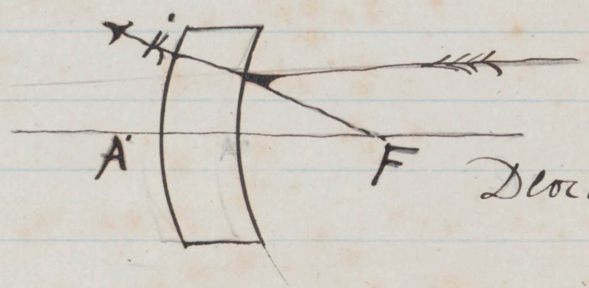
The image will be erect or inverted according as the object and the image lie on the same side or on opposite sides of the optical centre of the lens:

For convex lenses the focal length is to be regarded as positive.



P will be positive when measured from the lens towards the incident light, & p' when measured in the direction of the ~~incident~~ emergent light.



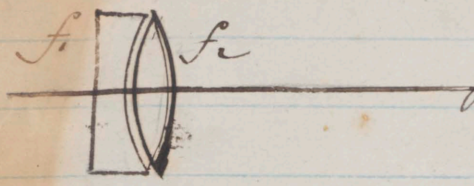


Deviation = $\left(\frac{A'K'}{F}\right)$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

If f is very small so must r_1 & r_2 be & ∴ the lens must be very small: & the way to use a lens like this of very high power is to make it the Objective of a Compound microscope.

I



For first lens $\frac{1}{d_1} + \frac{1}{D} = \frac{1}{f_1}$

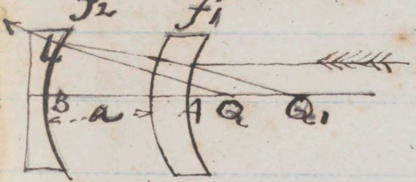
" 2nd " $\frac{1}{d} - \frac{1}{d_1} = \frac{1}{f_2}$

$$\frac{1}{d} + \frac{1}{D} = \frac{1}{f_1} + \frac{1}{f_2}$$

∴ the two lenses are equivalent to a simple lens F, where $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

NB In this case the two lenses are close together.

II (Let a = distance betw. the lenses) A single lens which produces the same deviation in a ray $\parallel^{l}$ to the axis, as the given pair is called the "equivalent lens" of the pair:



[The fig is the typical case when all the lines are positive]

$AQ_1 = f_1 \quad BQ_1 = a + f_1$

$\frac{1}{BQ_2} - \frac{1}{a+f_1} = \frac{1}{f_2}$; The direction $= \angle QB = \frac{LB}{BQ}$

$= LB\left(\frac{1}{f_1+a} + \frac{1}{f_2}\right) = KA \cdot \frac{f_1+a}{f_1}\left(\frac{1}{f_1+a} + \frac{1}{f_2}\right)$

$$\frac{1}{F} = \frac{f_1+a}{f_1}\left(\frac{1}{f_1+a} + \frac{1}{f_2}\right) = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2}$$

(for convex lenses the signs are changed)

∴ The refracted beam passes through the focus.

Example 61 Lenses. Deschanel —

The following relation must stand for both, viz:—

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} + \frac{1}{r_1}\right)$$ where f = focal length of glass lens

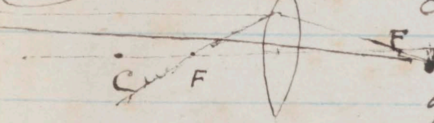
& $$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{r} + \frac{1}{r_1}\right)$$ where f_1 is focal length of Diam

Divide & you get

$$\frac{f_1}{f} = \frac{\mu - 1}{\mu_1 - 1} = \frac{.6}{1.6}$$

$$\therefore f_1 = \frac{6}{16} f = \frac{3}{8} f.$$

(63)



Focal length = 1 foot. ? position of image of a small object at distances of 20, 2, & 1½ feet.

I. $$\frac{1}{P} + \frac{1}{p} = \frac{1}{f}$$

$$-\frac{1}{p} = \frac{1}{f} - \frac{1}{P}$$

$$\therefore p = \frac{Pf}{P-f} = \frac{20}{19} = 1\tfrac{1}{19}.$$

II. $$p = \frac{Pf}{P-f} = 2 \text{ feet}.$$

III. $$p = \frac{Pf}{P-F} = \frac{3}{2} \times \frac{2}{1} = 3 \text{ feet}.$$

NB. I took p negative ∵ it was on opposite side of lens from ray of light.

All the images must be real ∵ refraction of light actually occurs.

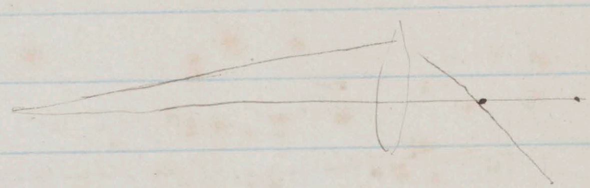
$$\frac{1}{d} + \frac{1}{D} = \frac{1}{f}$$

when $d = D$

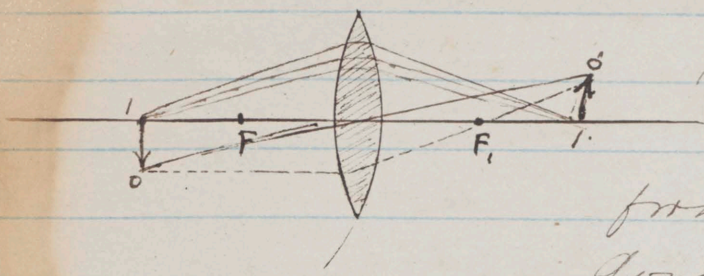
$$\frac{2}{D} = \frac{1}{f}$$

$$D = 2f$$

$\therefore$ Distance between two lenses $= 4f$.



Exbt.

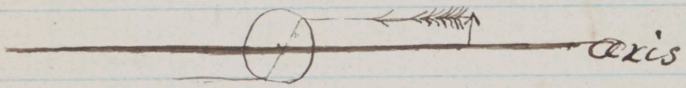


Show that when the distance of an object from a convex lens is double the focal length, the image is at the same distance at the other side

If an object be at any point on the axis, an image of it must be formed at some other point, whether this last image be real, or virtual: and its distance from the lens is regulated by certain constant conditions.

Thus if the rays be all ‖ll or come from infinity the image is formed on the other side of the lens & at its principal focus: As the rays are supposed to come from a point constantly approaching the lens, the image will be formed at its conjugate wh. constantly moves away from principal focus at other side of lens, until finally when the object comes to a distance twice the focal length its image will be formed at a distance on other side exactly =l to the latter.

To find the position of the ~~image~~ formed by a spherical lens:



x distance of the object
y .. .. .. image by 1st refr.
z .. .. 2nd ..
a = radius, μ index of ref.

At first surf. $\rho = a$ $\quad \mu_1 = 1$ $\quad \mu_2 = \mu$

~~at~~ 2nd surf. $\rho = -a$ $\quad \mu_1 = \mu$ $\quad \mu_2 = 1$

$\therefore \frac{1}{\mu_2 q_2} - \frac{1}{\mu_1 q_1} = \left(\frac{1}{\mu_2} - \frac{1}{\mu_1}\right)\frac{1}{\rho}$. substituting for first refraction you get $\frac{1}{\mu y} - \frac{1}{x} = \left(\frac{1}{\mu} - 1\right)\frac{1}{a}$, for second $\frac{1}{z} - \frac{1}{\mu y} = \left(1 - \frac{1}{\mu}\right)\frac{1}{a}$ etc

$$\therefore \frac{1}{z} - \frac{1}{x} = \left(\frac{1}{\mu} - 1\right)\frac{2}{a} = -\left(\frac{\mu - 1}{\mu}\right)\frac{2}{a}$$

If the rays are ||el x is infinite

$$\& \quad z = -\frac{\mu}{\mu - 1} \cdot \frac{a}{2}$$

i.e. principal focus is at a distance from the centre $= \frac{\mu}{(\mu - 1)}\frac{a}{2}$ at opposite side of ~~incident~~ light

$\frac{1}{f} = \frac{\mu_2 - \mu_1}{\mu_2} \cdot \frac{1}{r}$ $\quad f =$

$= \frac{1}{2} \times \frac{3}{2} = \frac{1}{3}$

$\therefore f = 3$

67

The object is 6 feet on one side of a lens and the imaige is one foot on the other side. What is the focal length of the lens.

$$\frac{1}{p} + \frac{1}{p'} = \frac{1}{f}$$

$$\frac{1}{6} + \frac{1}{1} = \text{"}$$

$$\frac{7}{6} = \frac{1}{f} \quad \therefore f = \frac{6}{7}\text{ ft.}$$

69.

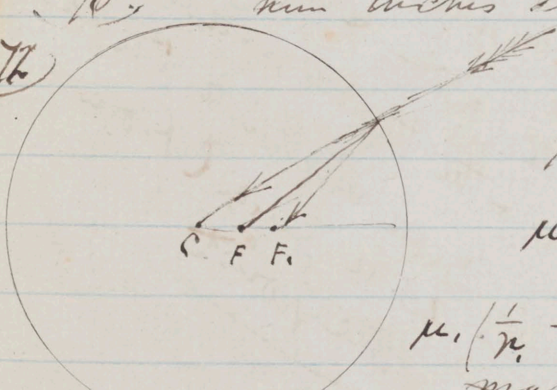
$$\frac{1}{12} - \frac{1}{1} = \frac{1}{f}.$$

$$\frac{-11}{12} = \frac{1}{f}$$

$$\therefore f = \frac{11}{12}.$$

f is negative for a concave lens.

70. nine inches evidently.

71. 

writing down the genl. eqn.

$$\mu_1 \cdot \frac{cz}{r-cz} = \mu_2\left(\frac{cz_1}{r-cz_1}\right) = \cancel{\mu_1 \frac{\mu z_1}{r-\mu_1}} = \cancel{\mu_2 \frac{\mu z}{r-\mu_2}}$$

$$\mu_1\left(\frac{r-p_1}{p_1}\right) = \mu_2\left(\frac{r-p_2}{p_2}\right)$$

$$\mu_1\left(\frac{1}{p_1} - \frac{1}{r}\right) = \mu_2\left(\frac{1}{p_2} - \frac{1}{r}\right)$$

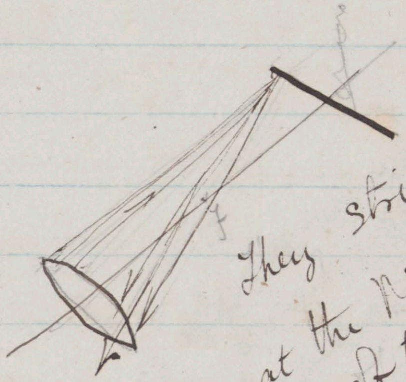
~~making p_1 infinite. $p_2 = f$.~~ $\mu_1 = 1$ & $\mu_2 = \frac{3}{2}$.

$$f = \frac{\mu}{\mu - 1}\,\frac{a}{2}$$

$$-1 = \cancel{\frac{3}{2}\left(\frac{1}{f} - 1\right)} \qquad \therefore \frac{2}{3} f = 2$$

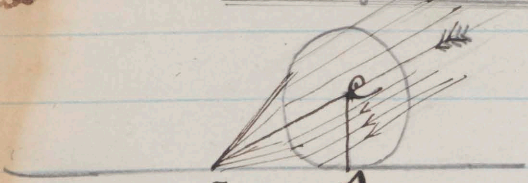
$$\cancel{\frac{1}{3} = \frac{2}{3f} - \frac{2}{3}}$$

81



They strike the mirror as a pencil
of rays at the point & on account
of this they are reflected
back again to the glass
as a pencil from
the point.

May 2nd 1883.



F A

Art 1041

$$CF = \frac{-\mu a}{2(\mu-1)}$$

$$\sin CFA = \frac{CA}{CF} = \frac{2(\mu-1)}{\mu}$$

$$\therefore \sin \text{altitude of Sun} = \frac{2(\mu-1)}{\mu}$$

$$= \frac{2 \cdot \frac{1}{2}}{\frac{3}{2}} = \frac{2}{3}$$

$$\text{magnifying power} = \frac{\text{focal length of object glass}}{\text{focal length of eyepiece}}$$

Astronomical inverts images.
wh. are Real

Can be used for measuring

Tube in Astronomical must be longer. as image is formed in the Telescope

A stop in Astronomical does not diminish the image

Gallilean does not invert
virtual image
& cannot be used for measurements.

A stop in Gallilean diminishes the image, for instance if you Cover $\frac{1}{2}$ object Glass you see only $\frac{1}{2}$ object.

Field of View is measured by the angular radius of the ○ wh. you can see & in a very powerful telescope it is exceedingly small—

Line of collmn from centre of objective to intersection of cross wires.

This peculiar real image of a ⊙ which is formed is the image of Aperture in ~~Astronomical~~ Astronomical Telescope.

[illegible]dered

In the Galilean you will have only a vertical image.

(anc virtual)

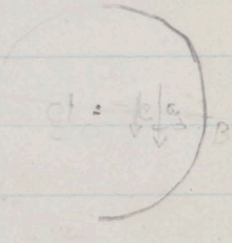
REFLECTING. Telescopes.

[illegible]

[illegible] (for the two types)

En 8?

You take a large sphere of glass that the image might not be refracted:



(1st) an image is formed by [illegible]

$\frac{1}{p} = \frac{1}{p_2} - \frac{2}{a}$ [illegible]

$p = \ldots$ [illegible]

(2nd) The [illegible] from [illegible]

$\therefore p_2 = 2a$

3rd The [illegible] at this [illegible] $\therefore \frac{1}{p_1} = \frac{2}{a} - \frac{1}{2a} = \frac{3}{2a}$

$\therefore$ from a pt. close up to the [illegible]

4 $\therefore$ practically [illegible] is a flat [illegible] the image will be at $\frac{2}{3}$ depth. $p_2 = \frac{4}{3}a$. $\therefore p_1 = \frac{2}{3}a$

Chap XX H II. {First 3 articles; Art 1062 - 63 - 64 - 65}

Ex 73.

By formula in the book

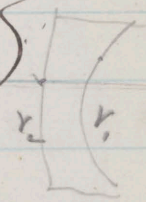
$$f \; \cancel{\text{focal length}} = \cancel{\frac{\mu}{\mu-1}\cdot\frac{a}{2}}$$

$$\cancel{= \frac{25}{15} \times 2}$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} + \frac{1}{r'}\right) \qquad \cancel{= \frac{10}{3}}$$

in this case $= 1.5\left(\frac{1}{4} + \frac{1}{4}\right) = \frac{15}{20} = \frac{3}{4}$ $\cancel{= 3\tfrac{1}{3} \text{ m.m.}}$

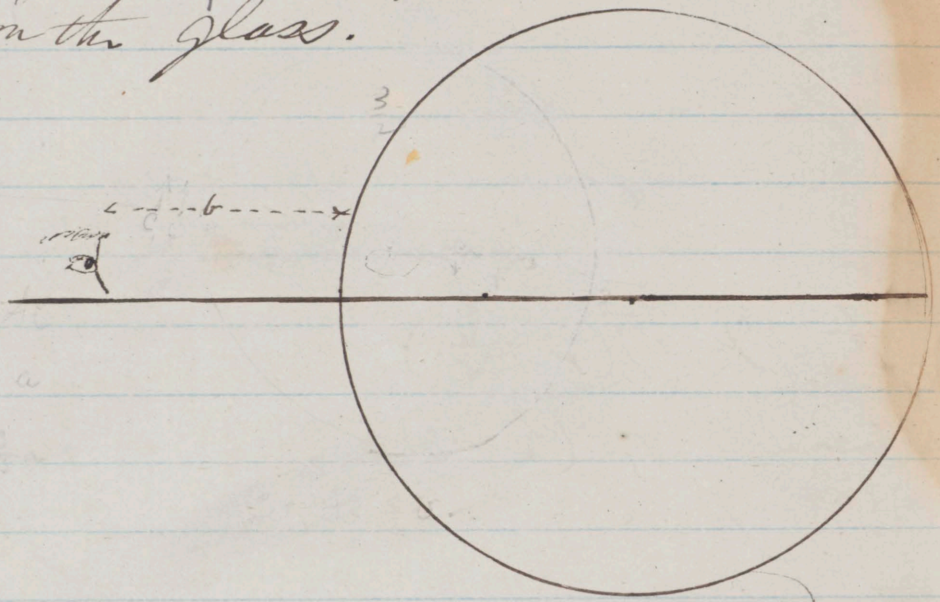
$\therefore f = \frac{4}{3} = 1\tfrac{1}{3}$ m.m.



Ex. 75 axis

In such a case as this one radius is negative

To work out same question as a next page the eye being at a distance (b) from the glass.



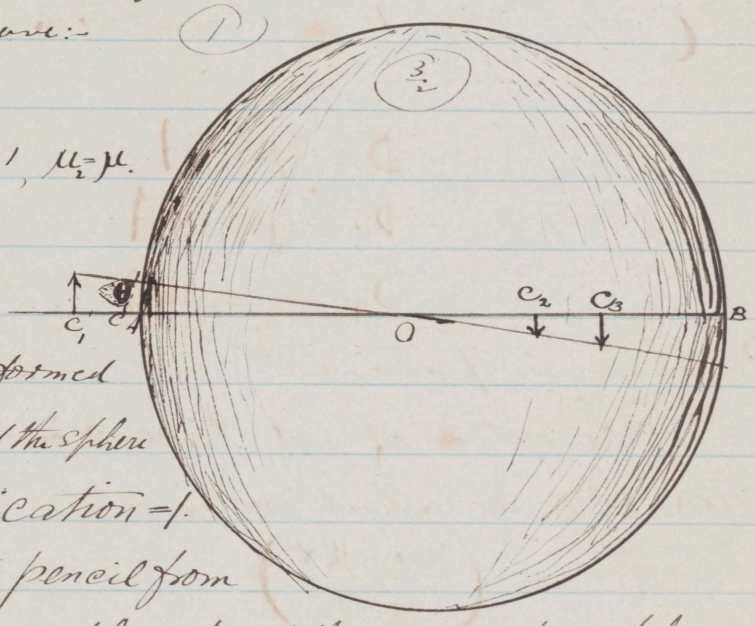
$$\left.\begin{aligned} AC_1 &= \tfrac{3}{4}\,AC. \\ OC_2 &= \tfrac{1}{3}\,a. \\ OC_3 &= \tfrac{3}{5}\,a. \end{aligned}\right\}$$

Example 82 Deschanel.

An eye is placed close to the surface of a large sphere of glass $(\mu = \frac{3}{2})$, which is silvered at the back show that the image which the eye sees of itself is $\frac{3}{5}$th natural size.

(1) From formula we have:-

$$\cancel{\frac{\mu_2}{p_2} - \frac{\mu_1}{p_1} = \frac{\mu_2 - \mu_1}{r}}.$$



in this case, $\mu_1 = 1$, $\mu_2 = \mu$.

$$\therefore \frac{\mu}{p_2} - \frac{1}{p_1} = \frac{\mu - 1}{r}.$$

$$\because p_1 = 0, \ \therefore p_2 = 0$$

and an image is formed at the near surface of the sphere whose magnification $= 1$.

(2). Secondly the pencil from this focus is now reflected at the back of the Sphere.

$$\text{Hence } \frac{1}{p_1} + \frac{1}{p_2} = \frac{1}{f} = \frac{2}{a}; \ \because p_2 = 2a$$

$$\therefore \frac{1}{p_1} = \frac{2}{a} - \frac{1}{2a} = \frac{3}{2a}, \quad \therefore p_1 = \frac{2}{3}a$$

and the magnification in this case $= \frac{1}{3}$.

(3). $\frac{\mu}{p_1} - \frac{1}{p_2} = \frac{\mu - 1}{r}$ { applies to the refraction of a pencil from C_2 in passing out of the glass

$$\therefore \frac{\mu}{AC_2} - \frac{1}{AC_3} = \frac{\mu - 1}{a}$$

& magnification $= \frac{OC_3}{OC_2}$

$$\therefore \frac{\frac{3}{2}}{\frac{4}{3}a} - \frac{1}{AC_3} = \frac{1}{2a}, \quad \therefore AC_3 = \frac{8}{5}a.$$

$$= \frac{\frac{3}{5}a}{\frac{1}{3}} = \frac{9}{5}a.$$

$\therefore$ total magnitude is the product of the 3.

$$= \frac{1}{3} \times \frac{9}{5}a = \frac{3}{5}a.$$

Ramsden's eyepiece
Huygens } in Galb^t. & Haugh Optics

The parts of an object glass make an achromatic combination if

$$\begin{cases} \dfrac{\varpi_1}{f_1} + \dfrac{\varpi_2}{f_2} = 0 \\ \dfrac{1}{f_1} + \dfrac{1}{f_2} = \dfrac{1}{F} \end{cases}$$ which will solve the question

$$D = (\mu - 1)A.$$
$$D_v = (\mu_v - 1)A \qquad v \text{ is violet,}$$
$$D_r = (\mu_r - 1)A \qquad \& \ r \text{ is red.}$$

Dispersive power is
The ratio of ∠ of dispersion to the total deviation of a standard color.

$$\varpi = \left(\frac{\mu_v - \mu_r}{\mu - 1}\right)$$

83

$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}$, where F = focal length of Combination

For first lens

$\frac{1}{f_1}$ is $\because^l$ to $(\mu - 1)$

say $= (\mu - 1)\alpha$

For second lens $\frac{1}{f_2} = (\mu' - 1)\beta$ where α and β are any Constants.

The final focus is to be the same for μ_D as for μ_F.

$\therefore (\mu_D - 1)\alpha - (\mu' - 1)\beta = (\mu_F - 1)\alpha - (\mu'_F - 1)\beta$.

Substituting $.5279\alpha - .6351\beta = .5344\alpha - .6481\beta$

which finds $\frac{\alpha}{\beta}$, & this is the approximate ratio of the focal length.

$\therefore$

$130\beta = \alpha \times 65$

$\therefore \frac{\alpha}{\beta} = 2$.

$\therefore 3 : 2 :: 20 = 13.3$ inches for the Crown

$+ 6.7$ " " "

$f_1 = 2f_2$

$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{20}$

$\therefore \frac{1}{2f_2} = \frac{1}{20}$ $f_2 = 10$

$\beta = 2\alpha$

$(\mu - 1)\alpha - (\mu' - 1).2\alpha = \frac{1}{20}$

$.53\alpha - .642 \times 2\alpha = \frac{1}{20}$

again, the expression for minimum deviation is

$$2i - A.$$

& when the ∠ of the prism is small as in the case of ordinary lenses

$$\frac{i}{r} = \frac{\sin i}{\sin r} = \mu$$

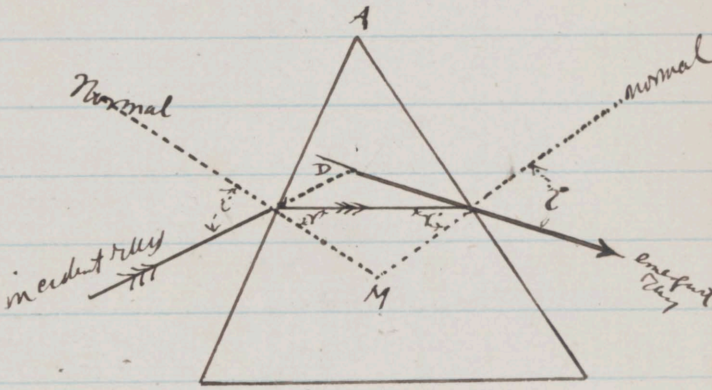
So that twice i, becomes $2\mu r$.

or μA (since $A = 2r$)

and hence the expression for minimum deviation becomes

$$(\mu - 1) A.$$

Required to find an expression for Min^m: deviation when a ray of light passes through a prism of glass whose refractive index is μ.



Let the conditions of the problem be as represented in the Fig.

Then $\sin i = \mu \sin r$.

& $\sin i' = \mu \sin r'$.

The deviation produced at p^t of Incidence $= i - r$

" " " " " Emergence $= i' - r'$

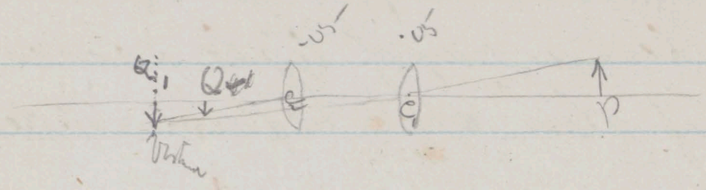
$\therefore$ The Total Deviation $= i + i' - (r + r')$

If we drop a $\perp$ from A on ray. the $\angle$s wh. it makes with the sides are $=^l$ respectively to r and r'

$\therefore$ The $\angle A = r + r'$. and $\therefore$ $\angle D = 2\angle i - \angle A$. $\left\{ \begin{array}{l} \text{when path of ray} \\ \text{is symmetrical} \\ i = i',\ \& \ r = r' \end{array} \right.$

$\therefore$ $i = \dfrac{A+D}{2}$, $\because \dfrac{\sin i}{\sin r} = \mu$

$\therefore \mu = \dfrac{\sin \frac{A+D}{2}}{\sin r} = \dfrac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}$



(1). For the image formed by lens C_1

$$\frac{1}{C_1Q} - \frac{1}{C_1P} = \frac{1}{-\cdot 05}$$

$$\therefore \frac{1}{C_1Q} = \frac{1}{\cdot 1} - \frac{1}{\cdot 05}$$

$$C_1Q_1 = -\cdot 1$$

(2) Before the ~~lens pass~~ light gets to Q_1 it passes thro the 2nd lens.

∴ For the image Q formed by it

$$\frac{1}{C_2Q} - \frac{1}{C_2Q_1} = \frac{1}{-\cdot 05}$$

$$\frac{1}{C_2Q} = \frac{1}{-\cdot 05} - \frac{1}{\cdot 05} = \frac{-2}{\cdot 05}$$

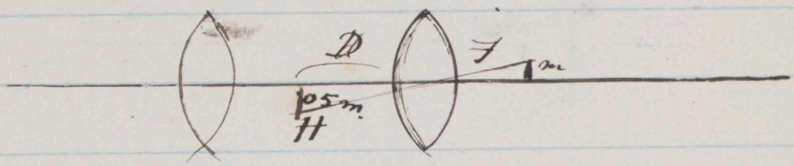
$$\therefore C_2Q = \cdot 025$$

(74)

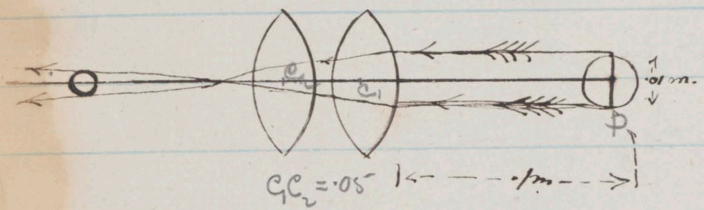
$$\frac{H}{m} = \frac{D}{F} = \frac{d}{d-F}$$

$$\therefore \frac{1}{D} + \frac{1}{d} = \frac{1}{F}$$

$$\frac{1}{D} = \frac{d-F}{dF}$$



(75)



Common focal length ·05 m
Distance apart ·05 m

·05 m Common focal length

$$\frac{1}{p'} = \frac{1}{f} - \frac{1}{p}$$

$$= \frac{p-f}{pf}.$$

p + p' are distances of image + object from the lens

hence $\frac{p}{p'} = \frac{p-f}{f}$

& ab. image $= \frac{f}{p-f}$ AB.

$\therefore$ image $= \frac{·05}{} \times ·01$

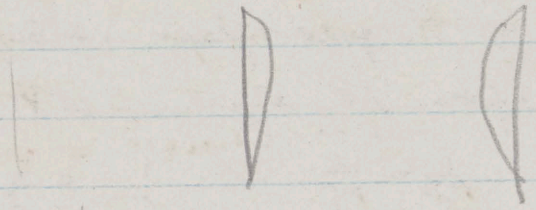
$= ·005$ m.

again $\frac{1}{x} = \frac{1}{f} - \frac{1}{p}$

$$= \frac{1}{·05} - \frac{1}{·1}$$

$$= \frac{·1 - ·05}{·05} = \frac{·05}{·05} = 1\text{ m}$$

rays are diverted so as to meet at Q



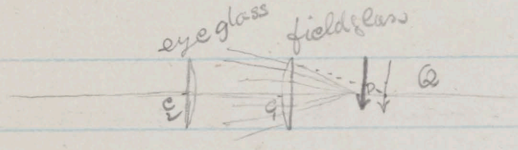
plano convex.
convexo-plane

(1) Corrected for Spherical aberration.
(2) Corrected for Chromatic Aberration

These can be corrected by two lenses at an interval f_1, f_2, and a.

There are two such eyepieces in common use

(1) Ramsdens (employed with crosswires in the focus) or the positive eyepiece. Two $=^c$ lenses at a distance $\frac{2}{3}f$.



The image formed by the eyeglass is at a distance

∴ That formed by the field glass is at Q, when $C_1Q = f$

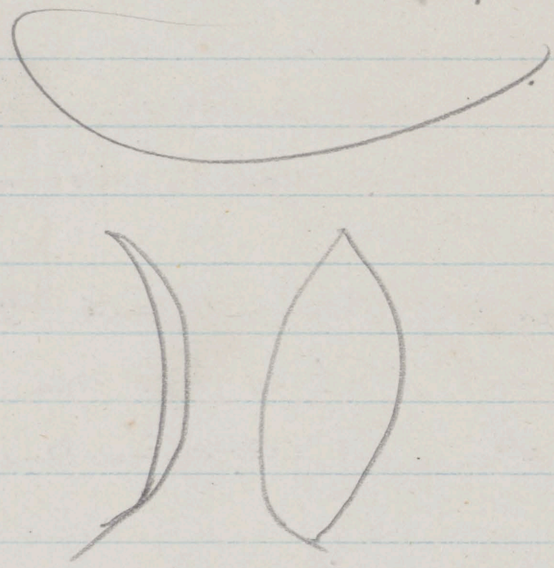
∴ The image of the focus of the objective is at P,

where $\frac{1}{C_1Q} - \frac{1}{CP} = \frac{1}{-f}$ ∴ $\frac{1}{CP} = \frac{3}{f} + \frac{1}{f} = \frac{4}{f}$

$$\therefore CP = \frac{f}{4}$$

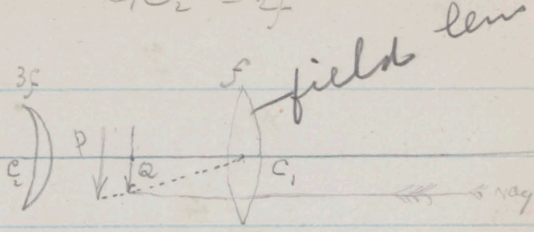
This corrects Spherical Aberration

Chromatic abberations



Concavo-convex

$C_1C_2 = 2f$



C_1C_2

Huygens, or the negative eyepiece.

$$\frac{1}{C_1Q} - \frac{1}{C_1P} = \frac{1}{-3f}$$

$$\therefore \quad \frac{1}{-f} - \frac{1}{C_1P} = \frac{1}{-3f}$$

$$\therefore \quad \frac{1}{C_1P} = -\frac{2}{3f}$$

$$C_1P = -\frac{3f}{2}$$

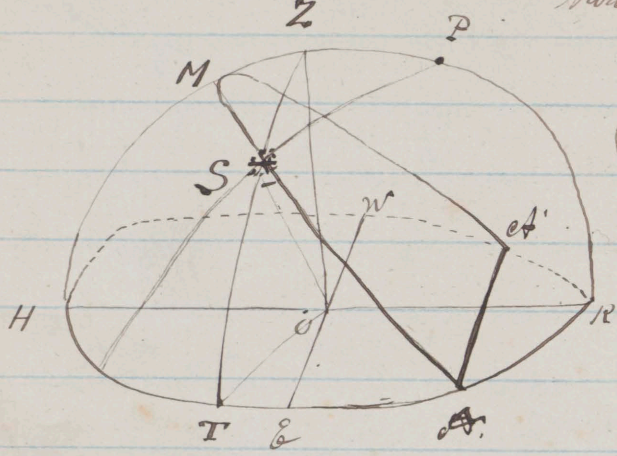
The images are between the two lenses of the eyepiece. And you cant use Crosswires

The ray wh is most bent by Glass at right is least bent by left one

This is an achromatic lens ∴

Michael M O'Shaughnessy

Monday 11 March 1884



∠SPZ is called hour ∠

SP. North Polar distance

determine position

You can find position of star S, whose Diurnal path is ⊙ ASMA' by means of two systems of co-ordinates.

Position will be known when you know ∠MZS between meridian and vertical. & the arc TS. which measures ∠TOS between star & horizon.

The ∠MZS or equivalent HT is called the azimuth (and is usually reckoned from South toward East & West when north pole is above the horizon & from north in other case).

The ∠TOS is called altitude of star, & Complement ZOS, Zenith Distance which constantly diminishes until star culminates, i.e. attains its greatest altitude in the meridian.

ASTRONOMY

(77

May 883

All the stars, the sun & planets appear to revolve round the polar star which comparatively describes a small ⊙.

The earth goes round the sun in a year: 366¼ Sideral {measured from time a certain star rises until it again rises.

365¼ Solar {measured from Sun rise to Sun rise.

Astronomers use sideral day as measure of their time being the most constant day as it varies very little.

The time of the earths rotation being sidereal is uniform.

We want some co-ordinates to determine positions of stars.

If you suppose all the stars projected on a sphere you have the requisite condition.

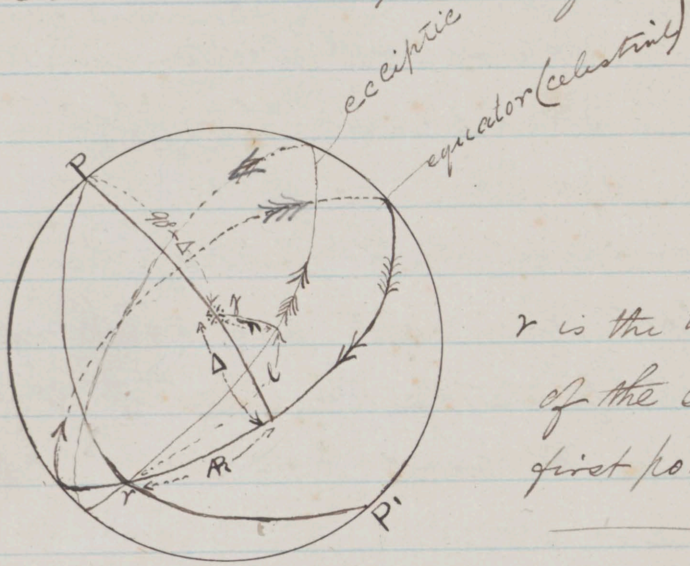
The celestial equator is 90° from the Polar Star.

The Sidereal day is the interval between two successive transits of first point of Aries.

The Declination of a Star is its distance from the equator measured ~~from~~ By the arc of great ⊙ which passes through star & pole.

The declination is north or South according to the side of the ~~star~~ celestial equator on which the star is situated AR

Polar Distance = Complement of declination



♈ is the intersection of the equator & first point Aries

These two co-ords evidently determine the star $\left\{ \begin{array}{l} \Delta = \text{declination} \\ AR = \text{right ascension} \end{array} \right.$

The AR of a star = ∠ made by its Declination ⊙ with that of some other determinate point in the celestial sphere and is measured by the arc of the equator intercepted between them.

♈ first point of Aries is where selected point crosses the meridian of the observer

Practically

You can find the position of γ by measuring backward along the equator, the AR of some heavenly star. with its Declination.

$\omega = 23\frac{1}{2}°$

Another method by latitude & longitude is used for eclipses of the Sun and moon:- given l & λ you can find γ. (along ecliptic)

Diurnal motion

may be represented by the whole celestial sphere rotating round PP' in a sideral day.

$366\frac{1}{4}$ sideral days = $365\frac{1}{4}$ solar days.

one sideral day = $\overset{\text{hours}}{23} = 56'$

The stars rise in the East & set in the West ∴ their motion is represented by direction of two arrows in figure:

The sun moves round the ecliptic in the course of a year & its motion is represented by from West to East arrows in the ecliptic of Fig.

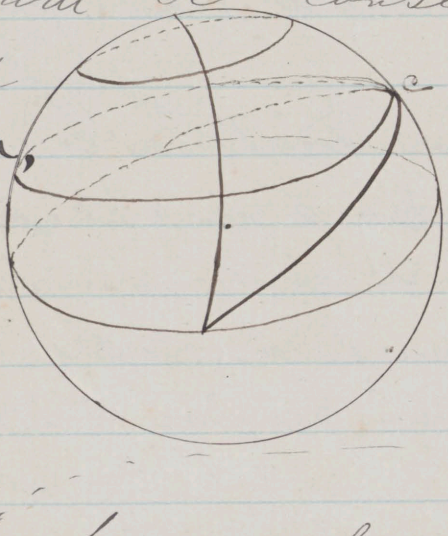
Transit Instrument

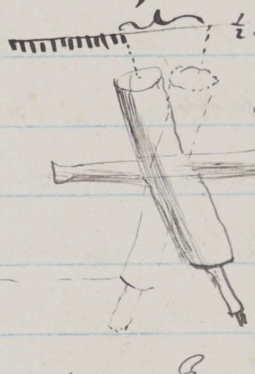
The line of collimation is the straight line joining the centre of the object glass, with the intersection of the cross wires.

1. The line of collimation must be perpendicular to the geometrical axis about which the instrument turns.
2. This geometrical axis must be exactly horizontal
3. It must point accurately East & West.

If these adjustments are imperfect there will be consequent Errors called respectively

Collimation, Level & deviation errors.





½ the gives the correction. Verify?

Corrects. Collimation Error

Thursday.
Thursday
Thursday Thursday

The ecliptic is the plane passing nearly through the sun and the moon and any of the planets that are visible: the moon is sometimes some degrees from it but for rough purposes it answers very well.

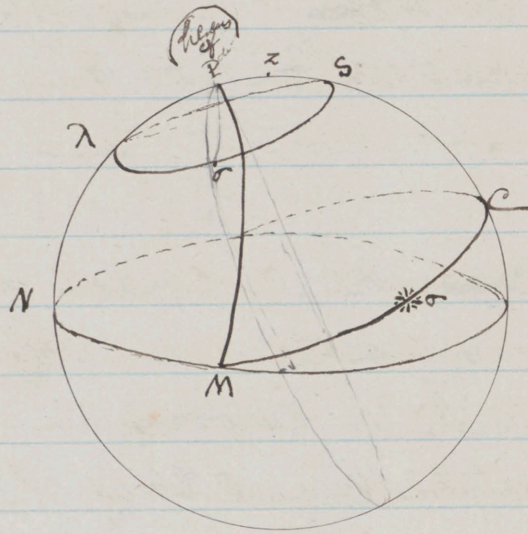
Any star near the pole never sets
Take a star farther down & it sets.

PN = altitude of the pole.
wh = altitude of plane of observation.
=

$P\sigma = 90° - \Delta$ = codeclination = complement of declination
If $90° - \Delta$ > latitude of place :– the star will be a circum polar one and it will never set.

∴ q pt of proj of star is at.

NPS is meridian ○ containing N + S, & pole
when a star crosses the meridian it culminates, as it attains its greatest altitude there.



when $90° - \Delta = P\sigma >$ latitude of place
the the Star is circumpolar

δ ?? =

$PM = 90^\circ - \delta$ (small) = distance from the pole

$PN = \lambda$

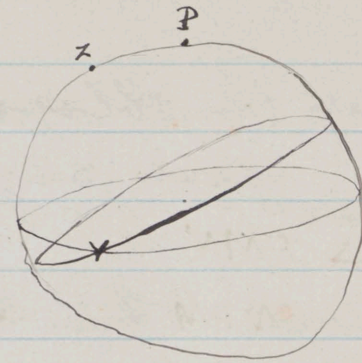
$\therefore$ in the right ∠ᵈ $\triangle$ PMN $\qquad \cos NPM = \dfrac{\tan PN}{\tan PM}$

$\cos NPM = \tan\lambda \tan\delta$

(α) $\quad \cos \angle PM = -\tan\lambda \tan\delta$

The Star makes a complete revolution round P in a siderial day. During that time $\therefore$ it is above the horizon $\dfrac{2\angle PM}{360^\circ}$ of a siderial day

Hence take any almanac read its declinatn & you get time it is above the horizon. At the Spring Equinox the Sun is at Aries 21st March. After that it goes along the ecliptic moving further north of the equator & it culminates at the Summer Solstice and goes down again, when you get Autumnal equinox, & Winter Solstice.



the correction of their Compass

Correction of the Compass

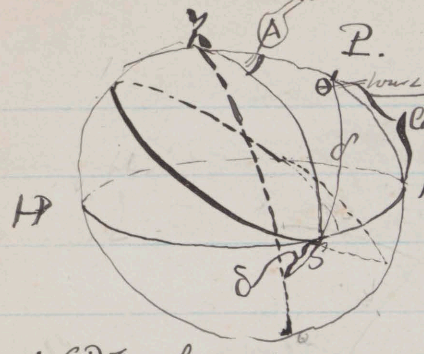
Hence it is the obliquity of the ecliptic which regulates the seasons.

If you apply (α) to the sun you will get length of a day at different seasons and at different latitudes not including the twilight — You will have to get the declination in the nautical or some other Almanac.

To find ∠ between north point of the horizon and the sun at sunset (NM).

$$\cos NM = \frac{\cos PM}{\cos NP} = \frac{\sin \delta}{\cos \lambda}$$

This will give the ∠ between the sun and the North Pole, and by means of it Captains (of vessels made of Iron) ~~regulate~~ calculate ~~their~~ ~~Longitude~~, as the Iron deflects the needle of the Compass from its proper position.



$\angle SPZ$ = hour ∠.

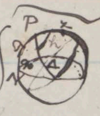
To find Azimuth of Sun at sunrise or at sunset.

In Δ SPR. rt ∠d at R. SR = ∠ SZP = azimuth

$\cos SP = \cos SR \cdot \cos PR.$

or $\sin\delta = \cos A \cdot \cos\varphi.$

$\therefore \underline{\cos A} = \frac{\sin\delta}{\cos\varphi}$



$-\cos h = \frac{\tan \lambda}{\tan \delta}$

if P an altern...
& h.

To find Suns Azimuth by observation of his altitude

in Δ $\underline{SPZ}$. $\cos SP = \cos ZP \cdot \cos ZS + \sin ZS \cdot \sin ZP \cos A$

$\sin\delta = \cos z \cdot \sin\varphi + \sin z \cos\varphi \cos A.$

$\therefore \cos A = \frac{\sin\delta - \cos z \cdot \sin\varphi}{\sin z \cos\varphi}$

~~or or~~

or $\sin \frac{A}{2} = \sqrt{\frac{\cos\frac{1}{2}(\varphi+z+\delta)\sin\frac{1}{2}(\varphi+z-\delta)}{\cos\varphi \cdot \sin z}}$

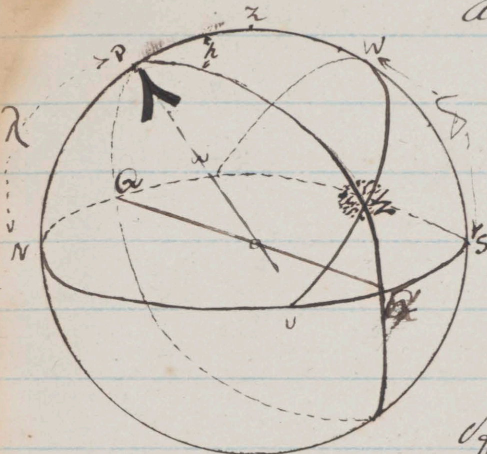
Calculate Azimuth of Sun.
Observe Compass direction of S.

Gives true north.

Sun. Dmit.

Let the bearing of the sun by compass be ... & let his true .. be calculated. The error will be the difference of the two.

~~How~~ to graduate the arc of a Sun Dial:



The sun describes uniformly in a day. The rest of the circle in a night.

It is the shadow of the dial thrown by the sun that makes the hour

The style [bar that throws the shadow] of the dial points towards the pole, i.e. along OP. The plane through the style and the sun contains the shadow. The shadow is the line in which the plane OPΣ cuts the horizontal, i.e. the line in which the meridian through Σ cuts the horizontal, along the line OQ:

NQ = SQ

At ~~x hours~~ ~~of~~ apparent noon the sun crosses the meridian at W, & Q is at S.

Sun does not move uniformly in its orbit, among the Stars.

At x hours after apparent noon

$h = \frac{x}{24} 360^\circ$, to find $Q'S$:—

$$\begin{cases} (P\Sigma = PW = 90^\circ - \delta). \\ PS = 180^\circ - \lambda. \\ \angle PSQ' = 90^\circ. \end{cases}$$

$\therefore \sin SQ' = \sin \lambda . \sin h.$

$\therefore$ at x hours after noon

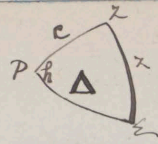
$\sin SQ' = \sin \lambda \sin (15x)^\circ \quad [360^\circ : 24^h :: 15^\circ : 1^h]$

wh. gives the law of the graduation of the dial

The apparent solar day varies throughout the year – A sundial gives you apparent solar time a clock gives you mean solar time. There are two reasons why apparent and mean solar time should not be alike: Even if the sun went uniformly among the stars, the obliquity of the ecliptic would make the days of un=l length, longest at the solstices and shortest at the Equinoxes. In addition to this the sun does not move uniformly.

Positive or Negative

West or East
of meridian



$\sin\frac{h}{2} = \sqrt{\frac{\sin\frac{1}{2}(z+c-\Delta)\ \sin\frac{1}{2}(z \ldots \Delta)}{\sin\Delta\ \sin c}}$

$\sqrt{\frac{\cos\frac{1}{2}(\varphi+\Delta+\alpha)\ \sin\frac{1}{2}(\varphi \ldots \Delta)}{\cos\varphi\ \sin\Delta}}$

$$\left\{\begin{array}{l} Z\sigma = 90^\circ - \alpha \\ P\sigma = 90^\circ - \delta \\ PZ = 90^\circ - \lambda \end{array}\right\}$$

Required: time from altitude of a known Star.

Find h.

$$\therefore \sin\frac{h}{2} = \sqrt{\frac{\cos S.\ \cos(S-\alpha)}{\cos\varphi\ \sin\Delta}} = \text{Ans}$$

The star will culminate in $\frac{h^\circ}{15}$ hours, $\therefore$ if R is the right ascension of the star. First point of Aries ♈ will culminate in $\frac{h+R}{15}$ & this will be the zero of sidereal time at the place.

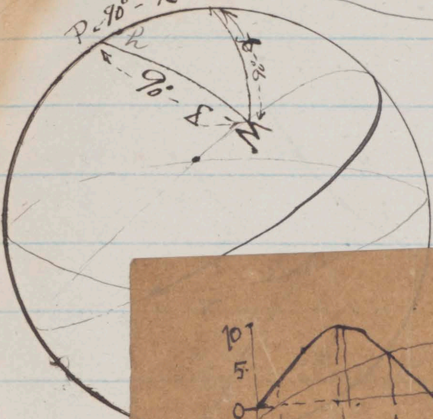
☉ will cross meridian in $\frac{h+R-☉}{15}$ hours and this will be true noon at the place.

$\frac{R-☉}{15}$ hours before true noon, then add equation of time to true solar time, & you get clock time: The equation of time = angle, between declination os of the true and mean suns, when expressed in time = difference between apparent & mean time at any instant

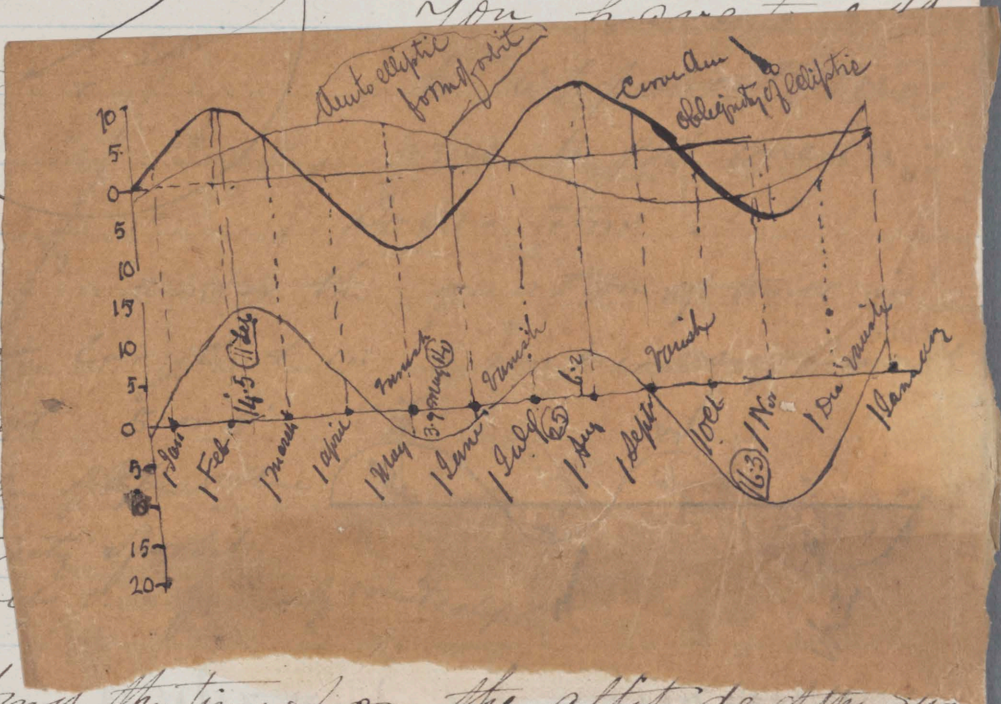
$\cos\omega \sec^2\delta \cdot \frac{dl}{dt}$ ∵ when $l - R$ is a max^m $\frac{dl}{dt} = \frac{dR}{dt}$

$\cos\omega \sec^2\delta = 1$ ∴ $\cos\delta = \sqrt{\cos\omega}$

whence $\sin l - R$ is found $= \tan^2\frac{\omega}{2}$
∴ maxima $l - R = 2^\circ.28' = 10$ seconds nearly



The ("equation of Time") is what you have to add



ven...
abo...
Greatest val...
to ellipti...
Suntrue...

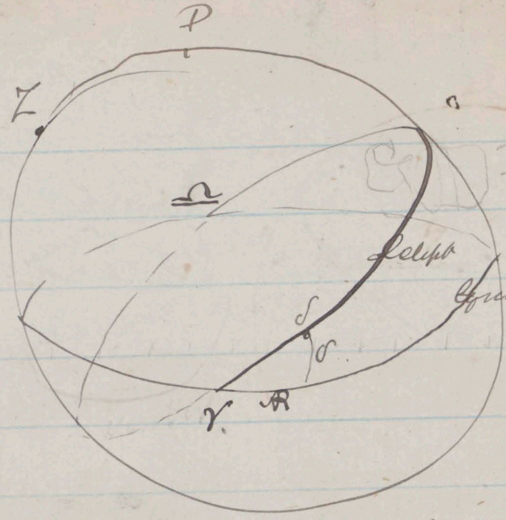
To find the time from the altitude of the sun.

$$\begin{cases} PE = 90^\circ - \delta \\ PZ = 90^\circ - \lambda \\ Z\Sigma = 90^\circ - \alpha \end{cases}$$

The hours $\angle h$ reduced to time at 15° per hour gives you the time after noon. The most convenient formula is, $\tan\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \cdot \sin c}}$ which gives

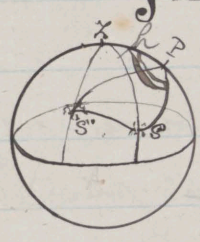
You apparent solar time to which you are to add the "Equation of Time" to get mean solar time.

82



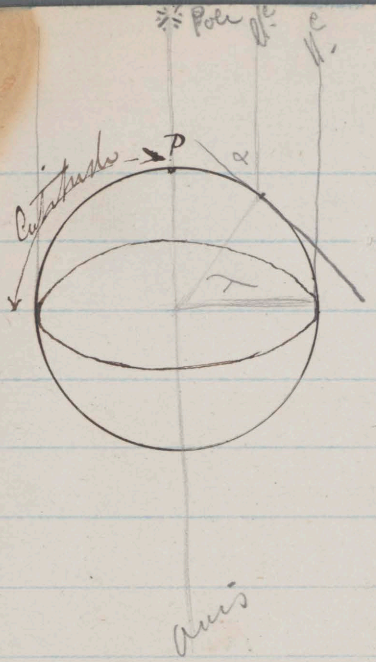
Greatest value of this part.
Term measure R in direction of apparent annual motion of the sun, i.e. opposite to the apparent diurnal motion of stars:

Finding time by observation:
available when Lat is unknown



Determine Lat. & ∠PSZ.
This with Co. lat. PZ. + Zenith dist. ZS. will determine hour ∠

$$\text{Sin hour } \angle = \frac{\text{Sin } PSZ.\ \text{Sin } ZS}{\text{Sin } PZ.}$$



Prove that the latitude of the Pole is =l. its altitude. i.e. in other words to prove that its altitude = 90°.

$\alpha = \lambda$.

Keplers 3 laws of planetary motion.

The planets describe ellipses w. Sun as focus.

Horal gives you $N\sigma$

$N P = \lambda$

$P\sigma = N\sigma - \lambda$

Unmal gives you $P\sigma$ is the compont of the declination

See Ballnorth & Haughton

Pages 49.

79.

Art 3 53

11. 73

14 - 81

15 - 81

Art 56 Chap IX

Imp't ※ 16 =

17 =

18 =

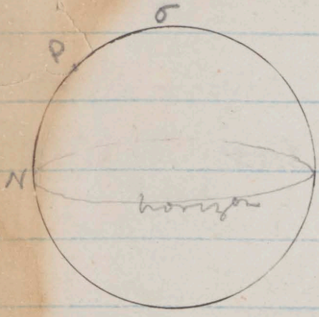
※ Art 1 in Chap IV

5 · phases of moon.

Art 1. Chap V

do not read eclipses

Arts 1·2 VI · 2·3 VII. 2 in Chap VIII



An Astronomical clock points to ~~noon~~ 0 when γ is on the meridian + it is graduated to 24 hours + it gives sidereal time. To find AR of a Star. Observe time when the star crosses the meridian & that time reduced to degrees is its right ascension {at 15° per hour}. Time is 0 when Aries crosses, is x when star crosses. Observation is made by a transit instrument.

A transit instrument measures ARs in conjunction with a clock.

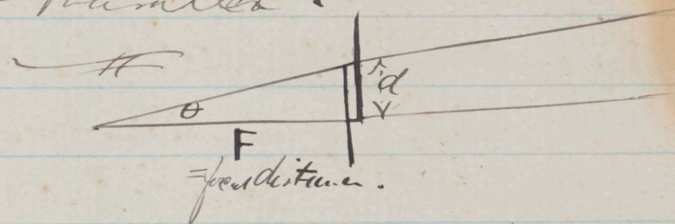
Declination is measured when you measure the altitude when it crosses the meridian, Mural instrument. – Defects which would produce considerable in first

Astronomy.

Azimuth. Correcting Compass by transit Instrument.
Mural fix Lat + Longtde and calculating time
IV. ~~XIII~~. grad.n Sun Dial. XIII. 13.7.8 unimport.
Hamsteads' method for finding 1st point of Aries, & for Obliquity of Ecliptic.

Alberation + Parallax.

Heliometer —

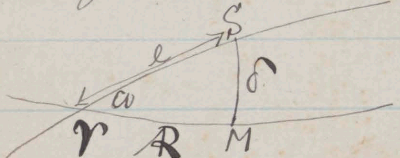


$\theta = \frac{d}{F}$ Used for measuring diameter discs of sun + m...

144-do. 146

See page 91.

In case of sun whose lat = 0, the connection between R. δ. l., & ω is given by solution of rt. ∠ld Δ.



$\cos l = \cos R \cdot \cos \delta.$
$\sin \delta = \sin \omega \cdot \sin l.$
$\tan \delta = \sin R \cdot \tan \omega.$
$\underline{\tan R} = \underline{\tan l \cdot \cos \omega}$

∴ corresponding small increment of suns R. & longitude, will be in ratio $\cos \omega : \cos l$.

Rankine –
Sims –
Heathers instruments } adjustments of.

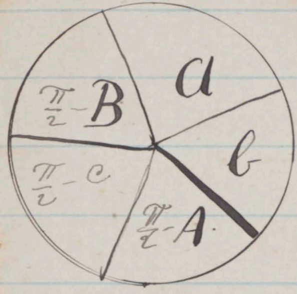
Sims –
Rankine
Williams Geodesy. } Levelling Surveying

~~Rankine~~
Aabbs { S. I. McNeil
Bidder } Measurement of earthwork

Rankines. } Railway Curves

Carpenters Assistant } Practical

sine part = product tans of adjacent parts
= " cosines of opposite "



observation may be harmless in the second. Hence the instruments used to be always separate & separately adjusted. Thus you employ two instruments. These are the principal instruments in an observatory. Δ, + AR. are principal Co-ords.

Effect of Precession on the AR of a star.

The ecliptic is fixed, & ♈ moves backwards along it at 50·2" per annum, ω = const.

For Sun, Express AR in terms of l.

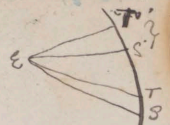
$$\tan AR = \cos\omega \tan l.$$

$$\therefore \sec^2 AR \cdot \frac{dAR}{dt} = \cos\omega \sec^2 l \frac{dl}{dt}.$$

$$\therefore \text{in a year } dAR = \frac{\cos\omega \cos^2 AR}{\cos^2 l} \quad 50 \cdot 2''$$

In a Star Catalogue, the AR, + δ are always put down for a fixed period.

Proof of 2.



Keplers Laws.

sector $SET : S'ET$

$\therefore \angle SET \cdot SE^2 : \angle S'ET \cdot S'E^2$

$\therefore$ ang vel at S. into dis^2 : L^r vel [illegible]

$\therefore$ Sec SET = Sec $S'ET$

Apsides line is changing: { Vbuchen 50''·22 / motion Progr. A.L. 11''·25 — 61''·47

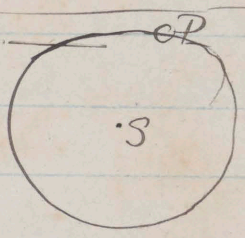
1. Elliptic orbit Sun in focus.
2. Rad. vector sweeps over =^{l} areas in =^{l} times.
3. $T^2 \propto a^3$.

$\therefore$ The planets move under a force directed towards Sun.

$\therefore$ This force $\propto \frac{1}{r^2}$ say $= \frac{\mu S}{r^2}$.

This is the same for all the planets i.e. it is the same law of gravitation for all. μ being called Const. of gravitation.

Ex. – In a $\odot^r$ orbit.



$$T = 2\pi\sqrt{\frac{a^3}{\mu S}}$$

What is value of μ?

Mass of Earth known: att^n of Moon known:– $\therefore$ μ known

radius of orbit = 240 000 miles.

$\therefore$ attraction of Earth $= g \cdot \frac{(\text{rad. earth})^2}{r^2}$ dynamical units

$= g\left(\frac{\cancel{4000}}{240000}\right)^2$ .. ft·sec.

$$g \cdot \left(\frac{2_{+}}{2\omega d}\right)^2 = \frac{v^2}{r} \text{ for the moon}$$

$$\text{ft. sec.} = \text{ft sec.} \qquad v = \frac{2\pi r}{T}$$

$$= \frac{4\pi^2 r}{T^2}$$

~~This~~ This verifies that its is force of Gravity keeps moon in its orbit

Gravitation unit of mass, how great?

$$\left.\begin{array}{r}\text{Attraction of mass} \\ \text{grav}^{\underline{n}}\text{ units}\end{array}\right\} = \frac{M}{r^2}$$

Find E in gravitation units. $\frac{E}{r^2} = g$

but mean density of ♁ $= 5\frac{1}{2}$

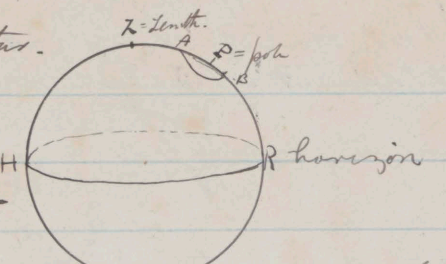
$\therefore$ Can find number of lbs of mass in a Gravitation unit of mass.

I

By means of altitudes of circumpolar stars.

Latitude = $\frac{RA + RB}{2}$

RA = Upper transit. RB. lower one



II To find Lat by means of the meridian altitude of the sun or star or other body whose declin.ⁿ is known.

S = Sun or star in meridian.

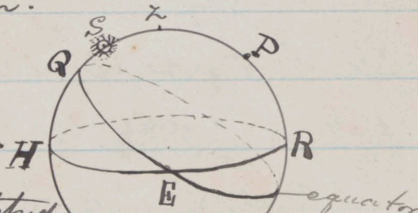
QE = equator.

QS = δ, the known declination.

SH = α, observed meridian altitude.

ZS = $z = 90 - \alpha$, Zenith distance

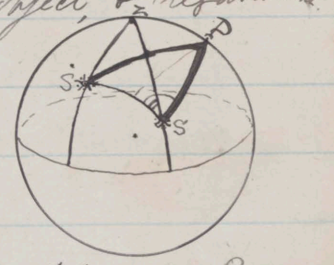
∴ Latitude = ZQ = ZS + SQ = $z + \delta$



z is positive when Zenith is north of the object, & negative when [South]

Similarly North declination +, & south declⁿ −.

III. To find the latitude by simultaneous observations of the altitudes of two known Stars. S S'; Zenith distances z, z'; Polar dist. Δ, Δ'



α = SPS' = Known differ of their AR's

now SS' is determined ∴ $\cos SS' = \cos\Delta \cdot \cos\Delta' + \sin\Delta \sin\Delta' \cdot \cos\alpha$ .... (1)

also ∠ PSS' .. ∴ $\sin PSS' = \frac{\sin\Delta' \cdot \sin\alpha}{\sin SS'}$

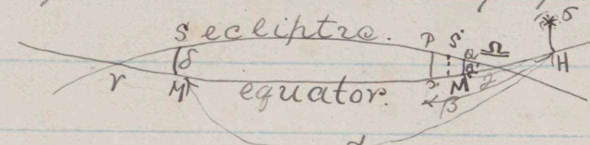
The 3 sides of Δ ZSS' are known ∴ $\sin\frac{1}{2} ZSS' = \sqrt{\frac{\sin\frac{1}{2}(z'+z-SS') \cdot \sin\frac{1}{2}(z'+SS'-z)}{\sin SS' \cdot \sin z}}$

∴ Δ PSZ is known. ∴ $\cos ZP = \sin \text{Latitude} = \cos\Delta \cdot \cos z + \sin\Delta \cdot \sin z \cos PSZ$.

For observations of altitude of sun. is same only. SPS' is an isosceles Δ.

∴ $\sin\frac{1}{2} SS' = \sin\Delta \cdot \sin\frac{1}{2}\alpha$. $\cot PSS' = \cos\Delta \tan\frac{1}{2}\alpha$.

Determination of First point of Aries. ♈ that point of the celestial equator which the Sun's centre occupies when crossing from north to south ...



observe α, β, γ.

Let the declination δ of the sun $= SM$ be observed on some day after the vernal equinox; also α difference between its time of transit and that of a fixed star σ:

Let S be suns place at noon, SM. & σH, the declination os of sun & star: When sun is approaching autumnal pt ♎ let P & Q be two of its positions at noon on successive days. When the declination δ', δ'', are one $>$, & one $<\delta$.

Assume S' as suns position when $PP' = QQ' = S'M' = SM$

$$\therefore \frac{P'M'}{P'Q'} = \frac{\delta' - \delta}{\delta' - \delta''} \qquad \therefore P'M' = (\beta - \gamma) \cdot \frac{\delta' - \delta}{\delta' - \delta''}.$$

$$\therefore MM' = \alpha - \beta + (\beta - \gamma)\left(\frac{\delta' - \delta}{\delta' - \delta''}\right).$$

The suns right asc. AR at S is $♈M = 90° - \frac{1}{2} MM'$.

$$= 90° - \tfrac{1}{2}(\alpha - \beta) - \tfrac{1}{2}(\beta - \gamma)\left(\frac{\delta' - \delta}{\delta' - \delta''}\right).$$

The stars AR $= \alpha + ♈M$.

$$= 90° + \tfrac{1}{2}(\alpha + \beta) - \tfrac{1}{2}\beta - \gamma\left(\frac{\delta' - \delta}{\delta' - \delta''}\right)$$

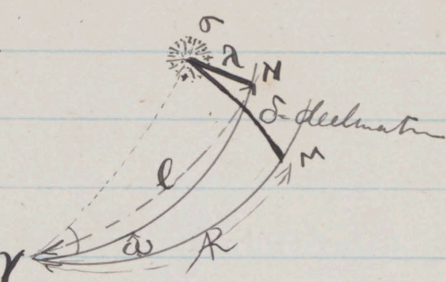
$=$ Answer.

To find obliquity of ecliptic by observation of meridian zenith distance $(z + z')$ of the Sun at the two solstices

$$\left.\begin{array}{l} \lambda - \omega = z \\ \lambda + \omega = z' \end{array}\right\} \therefore \omega = \tfrac{1}{2}(z' - z) \quad \ldots\ldots \quad \lambda = \text{Latitude} = \tfrac{1}{2}(z' + z)$$

Transformation of co.ords.

Let R. + δ given. ∴ ∠ of Δ $\gamma\sigma M$ is given.



$\cos\sigma\gamma = \cos R \cdot \cos\delta.$

$\cot \sigma\gamma M = \sin R \cdot \cot\delta.$

∴ Δ $\sigma\gamma N$ is known,

∴ $\sin\lambda = \sin\gamma\sigma \cdot \sin(\sigma\gamma M - \omega).$ $\tan l = \tan\gamma\sigma \cdot \cos(\sigma\gamma M - \omega)$

Next suppose $l + \lambda$ given. $\begin{cases} \cos\gamma\sigma = \cos l \cdot \cos\lambda. \\ \cot\sigma\gamma N = \sin l \cdot \cot\lambda. \end{cases}$

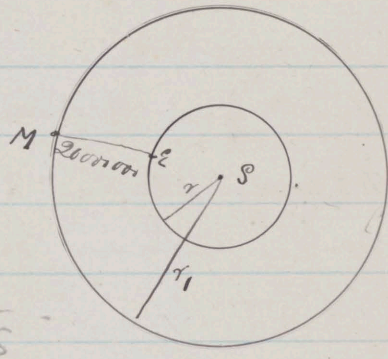
∴ $\sin\delta = \sin\sigma\gamma \cdot \sin(\sigma\gamma N + \omega)$.

∴ $\tan R = \tan\sigma\gamma \cdot \cos(\;\;" \;\;).$

7"

Par. of Mars when in opposition

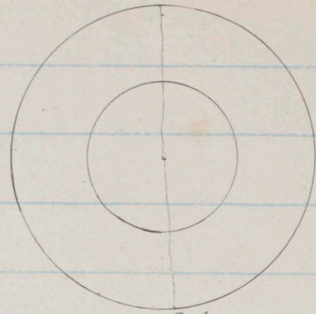
$(r_1 - r)$. $\dfrac{1 \text{ year}}{1 \text{ Martian year}} = \dfrac{r_1^{\frac{3}{2}}}{r^{3/2}}.$



Note 199.

$\frac{1}{M} - \frac{1}{E} = \frac{1}{S}.$

Stars Parr. 254: 257. 258



The maxm ∠ which the radius of the earth's orbit subtends at any star is called the annual parallax of that star.

Reqd. To Determine it:- on the supposition that two neighbouring stars at diff. distance vary in brightness. & that smaller star is so remote as to have no appreciable parallax: ∴ any observed changes in distance between them is due to Parr of nearer star.

The stars must be so situated that they are at the same time in the field of view.

1. Transit of Venus Galbraith & Haughton

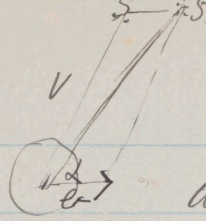
Compression $= \frac{a-b}{a}$ Can be found from the lengths of a degree of latitude in diff^t parts of the Earth.

—

Surface of Earth is a level surface i.e: gravity acts everywhere ⊥ to it because it was once fluid.

Parallax is ∠ subtended at the celestial body by the radius of the earth
Parallax is in altitude only not in Azimuth
To find effect of ∥ax say on declination you must express declination in terms of altitude and azimuth.

Find Moon's ∥ax. i.e. D: deste (S + H^{n}). Differential observation of moon & a near star



α = Earths way.

Aberr of S. $= \frac{v}{V} \sin\alpha$.

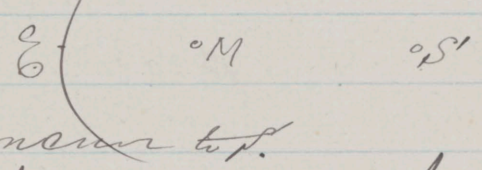
$= 20''.45 \sin\alpha$

∴ page 209. A will be displaced to A'

when $AA' = 20''.45 \sin AO$

Hence find Sun's distance.

298



orbit always concave to S.
but even in the most unfavourable
position the force on the moon, is towards
S.

Read 300 [301 after].

312. Period of Nutation = 19 years.

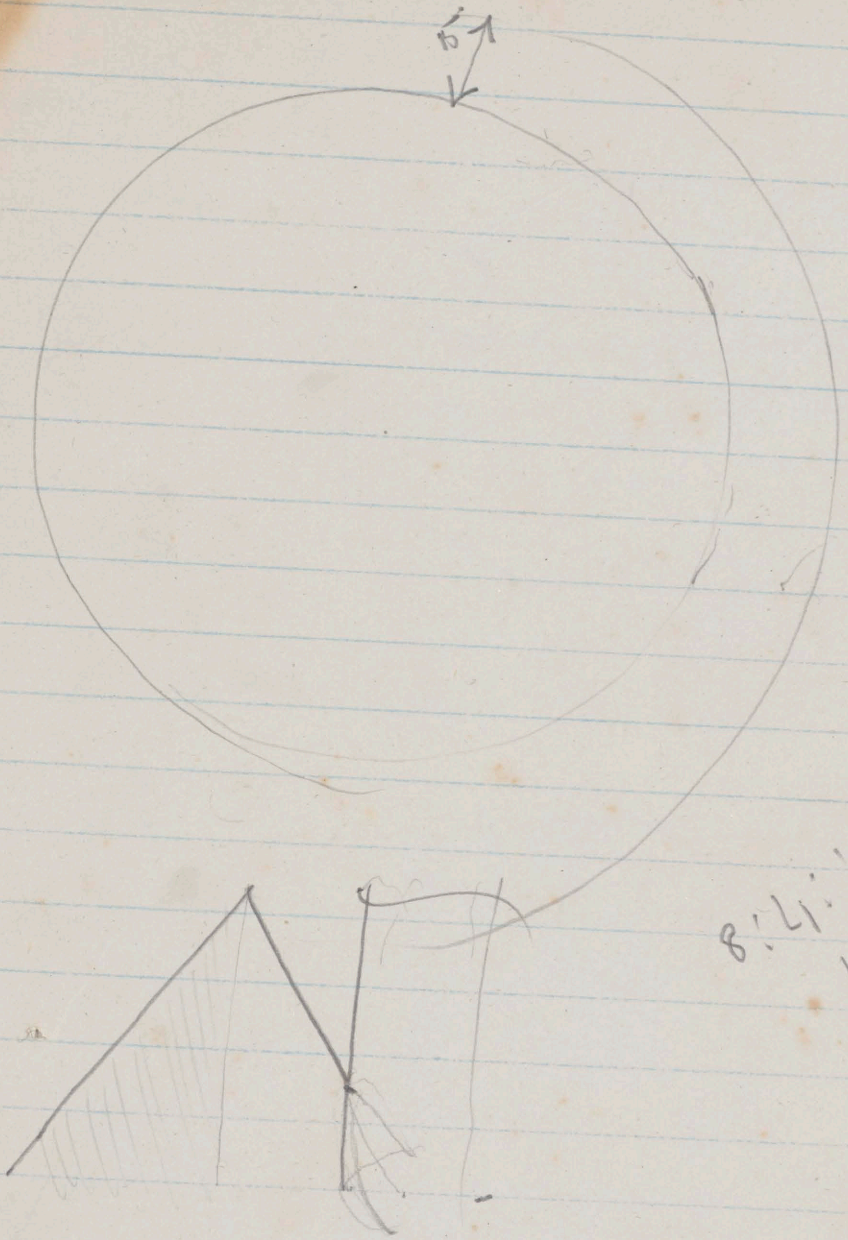
[316].

[329] What is meant by clearing lunars.

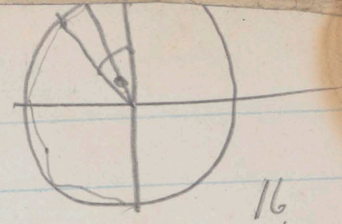
346
347 367
[352] 368
366 369
370.

Finished Astronomy

9 May



8 : 4 :: 13
1½ feet



16
4

12

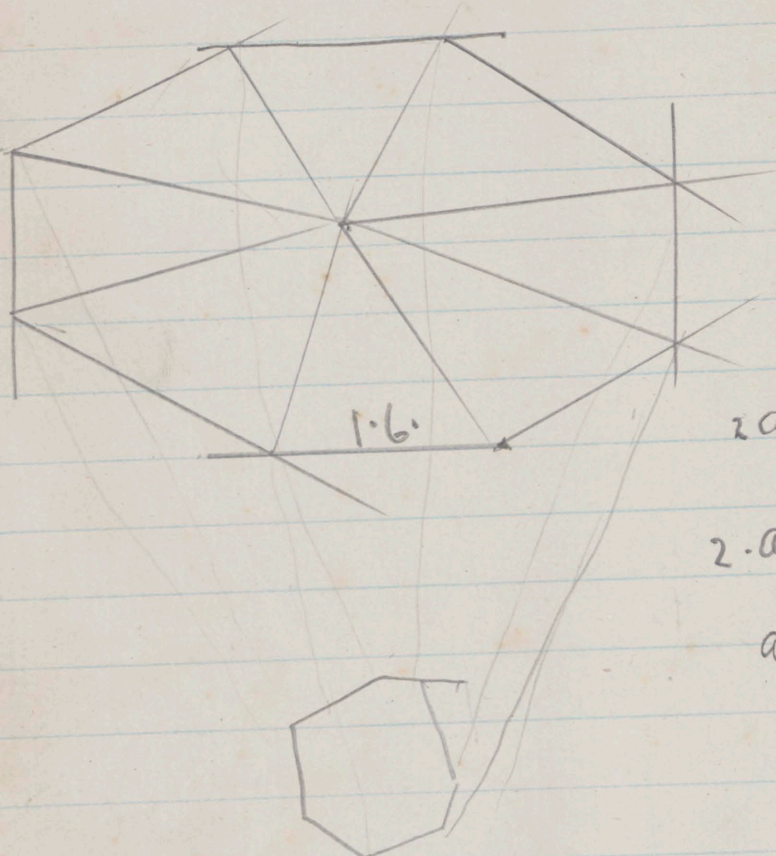
12

$\frac{1}{2}$

180
135°

45°

$65\frac{1}{2}$

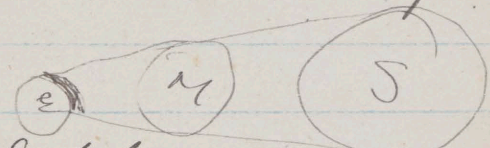


$2\,a \cdot \sin 45^\circ = \frac{3}{2}$

$2 \cdot a \frac{1}{\sqrt{2}} = \frac{3}{2}$

$a = \frac{3}{4\sqrt{2}}$

Halley's method of finding secular change in the length of the day.



Time of eclipses recorded by old astronomers: place where they occurred & when occur at present &c. Thus shortening of day found out.

$V = ft$

$= \frac{3}{\cancel{35}_{7}} \times \cancel{60}^{12}$

$= \frac{36}{7}$

$= \underline{2240 \cdot 5 \cdot 15}$

216
108
1296
185
15

Acceleration

$\frac{\text{Force}}{g \cdot M} = \text{Acceleration}$ Edwd Townsend

$\frac{80-50}{32 \cdot 5 \cdot 2240} = \text{acceln}$ Edwd Townsend

Edwd Townsend

$\text{Acceln} = \frac{3\cancel{0}}{32 \cdot 5 \cdot 224\cancel{0}} = \frac{3}{35840}$

Profs: E Townsends $= \frac{3}{7168\cancel{0}}$ $360\cancel{0}$

Edwd Townsend

Edwd Townsend

$\frac{3\cancel{0} \cdot \cancel{32}\ 4}{5 \cdot \cancel{2240}\ \cancel{28}\ 7} = \frac{3}{\cancel{35 \times 2}\ 7} \times 360\cancel{0}$

$\frac{1080}{7}$

$154\frac{2}{7}$ feet

$V = \frac{f}{t} = \frac{3}{35 \cdot 60}$

$= \frac{1}{700}$

Edward Townsend.

Edward Townsend

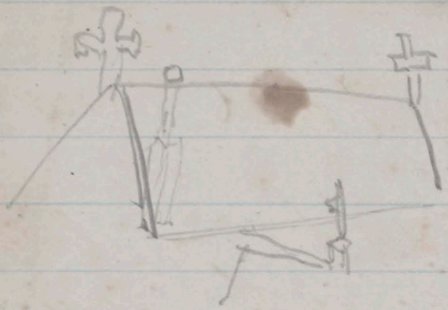
$\frac{1}{2}$ 2240·5

pg 2. g (radius of earth)

dynamical unit

Amount of attraction which the earth exercises on the moon —

attraction of bodies varies inversely as ~~their masses~~ as the square of the distance between them & directly as their masses.



Rathkeale

24th Sept. 1883

Dear Mr. O'Shaughnessy

I am very sorry I cannot send you the required cheque as all your survey was incorrect and we had to send another man to do it and of course Miss Enright would not pay two men to survey her meadowing

Yours etc.

Henry Potter

Gyroscope Theory of [illegible]

Arts 78-79-80-81

How to set transit so that
3 errors would be as
little as possible.

89.

Ask . get altitude
of Pole star. [illegible]
very approximate

Flamsteads Method. [illegible]
[illegible]

[illegible]

Co-ord. changes - Breeze [illegible]
lat. climate.

Correspondence
with H Potter
relative to
Rathkeal
Survey. #

to your last note.
wh. I couldn't avoid

Dear Mr. Potter.

You will I am sure excuse me for not replying earlier if I intended replying at all — as important university matters prevented me from doing so.

To say the least it caused me some surprise but not much uneasiness as whether I got the money or not, I heartily assure you did not ~~[illegible]~~ disturb my equanimity. That All Four Survey was ~~wrong~~ incorrect caused me no small share of amusement and the prospect of seeing the genius who absolutely proved it to be so, will be no small compensation for the time I lost about the affair. If I got before him I might soon let you ascertain and himself understand, his extraordinary acquirements as an arbitrator ~~[illegible]~~ of work performed by me.

103

Subject

Will say no more about the ~~business matter~~ but expect as a matter of courtesy, and politeness you will answer me the following:-

1 ~~Had this man~~ seen a
Did this man see before, after, or during the performance of the survey, any copy of the list I made out.

2 — Did you or anyone in your office Cheque, or Calculate as I requested you ~~I think~~ the survey colums from the acreage I made out.

3- Will you kindly enclose me a copy of the correct list made by this ~~gentleman~~ person whoever he may be, that I may be somewhat of a judge of my abilities to measure any other piece of meadowing.

And Oblige

Yours,

W. H. Humphrey

Are you aware Sir that in each of my collegiate courses I had a distinguished career? Are you aware, may I ask you, ~~did you know~~ the honourable position I was allotted in the "All Ireland Competition" of the Royal University — — — — — — — — —

Where the measurement of a piece of a meadow of information — was as ~~as~~ a drop of water to the ~~mighty~~ ocean.

As a matter of politeness you might answer ~~to~~ the following and my correspondence with you on the matter shall cease.

Had this man a list of my measurements, before, during or after the work.

Did you — as I requested ~~you~~ throw ~~you~~ your eye through ~~down~~ the money calculations as I had only a very limited ~~short~~ time to do them, and ~~so then~~ none to cheque them.

Will you kindly send me a copy of the so-called correct list ~~and latest~~ edition that I may judge of mis ~~[illegible]~~ All unconstrued [illegible]

Mr. Potter Auctioneer Rathkeale.

Dear Mr. Potter, ×

That last note of yours caused me a little surprise but not much uneasiness as whether I got the money or not I assure you it did not much trouble me. × ×

To say that "All your survey was wrong" caused me no small share of amusement; and the ~~first~~ prospect of seeing the genius of an arbitrator who said it was so ~~will be I am~~ would be great compensation for the trouble I got with the affair. If I got before him I might very soon let him, and you ascertain his abilities as a correcting surveyor, by burying his ~~stupid~~ ~~falsehoods in his still more~~ ~~dem~~ented ~~Criticism~~. Statement in the Oblivion which the deserves and ~~to~~ acquirements, and ~~his~~ ~~or~~ bringing him to a position to duly appreciating the justice of his consequence

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W_a W_b be work done by force while the body moves from the fixed pt over dis ab

$$W_b - W_a = \frac{1}{2} m \left(V_b^2 - V_a^2 \right).$$

C = whole work possible at bgs > W by P.

$$W_a = C - P_a$$
$$W_b = C - P_b$$

$$W_b - W_a = P_a - P_b = \frac{1}{2} m V_b^2 - \frac{1}{2} m V_a^2$$

$$\therefore P_a + \frac{1}{2} M V_a^2 = P_b + \frac{1}{2} m V_b^2$$

Finis

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