Productivity (1952)

NO GLA4315

Concepts and Measurement of Production and Productivity

by

Irving H. Siegel

Reproduced as a Working Paper of the National Conference on Productivity, 1952

Bureau of Labor Statistics
U. S. Department of Labor
Washington, D. C.

Concepts and Measurement of Production and Productivity

by

Irving H. Siegel

Reproduced as a Working Paper of the
National Conference on Productivity, 1952

U.S. Bureau of Labor Statistics
U. S. Department of Labor
Washington, D. C.
1952

ACKNOWLEDGMENTS

This study, which has often been interrupted by other tasks, owes its completion largely to the continuing encouragement and guidance of Professor Frederick C. Mills. It was begun in his stimulating price and production seminars in 1940-41. It was also influenced in its early stages by Professor Leo Wolman, who has likewise maintained a friendly interest in its progress. It also reflects an interest developed by my first teacher, Professor (Emeritus) Willford I. King, of New York University.

The present content and emphasis of this study have been shaped by my work experience, in the course of which I have had fruitful associations with Professors Solomon Fabricant and Harold Barger at National Bureau of Economic Research, Mr. W. Duane Evans at U. S. Bureau of Labor Statistics, and Dr. Clement Winston at WPA National Research Project.

I must also record my intellectual debt to the writings of Professors S. S. Kuznets and A. F. Burns and others mentioned in the text.

Finally, I acknowledge with thanks the critical review of Professors A. Bergson, G. J. Stigler, and W. S. Vickrey, as well as the comments of Professors Mills and Wolman.

I. H. S.
May 1951

Foreword

This study, originally prepared as a doctoral dissertation at Columbia University, is reproduced as a working paper of the National Conference on Productivity through the courtesy of the author. It represents the opinions of the author, and not necessarily those of the sponsoring agencies.

The Conference on Productivity is a technical forum for the discussion of productivity concepts and measurements, the current state of knowledge regarding productivity, the adequacy of present research programs, and related questions. It is not an official advisory body, but a common ground for the exchange of opinions and information. The Conference is sponsored by the Bureau of Labor Statistics of the U. S. Department of Labor and the Division of Statistical Standards of the U. S. Bureau of the Budget.

Samuel H. Thompson
(Secretary, National Conference
on Productivity)
Bureau of Labor Statistics
U. S. Department of Labor
Washington, D. C.

March, 1952

TABLE OF CONTENTS

CHAPTER I

SCOPE AND VIEWPOINT OF STUDY

Scope

This study deals with the rationale and techniques of measurement of temporal changes in the "physical volume" of production and in the level of productivity, two algebraically related entities which lie close to the core of economics. Though "real" national product and kindred aggregates are not of prime interest, it has been necessary to make reference to such concepts and measures and to the specialized literature on social accounting. Though the discussion is generally restricted to temporal comparisons, it also applies to some extent to analogous spatial indexes, like international production and productivity comparisons.[1] In the treatment of productivity, attention is centered on the important practical case of output per unit of labor input, but due account is also taken of the theoretically interesting case of output per unit of composite factor input.

[1] Among the recent outstanding writings on international comparisons are: L. Rostas, Comparative Productivity in British and American Industry (Cambridge, Eng.: Cambridge University Press, 1948); E. C. Snow, "The International Comparison of Industrial Output", Journal of the Royal Statistical Society, CVII (Pt. 1, 1944), 1-30; A. J. Brown, Applied Statistics (New York: Rinehart & Co., 1948); S. Kuznets, "National Income and Industrial Structure", Econometrica, XVII (Supplement, July 1949), 205-39; C. Gini, "La comparabilité dans le temps et dans l'espace des évaluations du revenu national", Economie appliquée, II (January-March, 1949), 7-25; H. Staehle, "International Comparison of Real National Incomes: A Note on Methods", Studies in Income and Wealth, XI (New York: National Bureau of Economic Research, 1949), 223-51, with "Comments" by A. Bergson, 252-59; and C. Clark, The Conditions of Economic Progress (London: Macmillan & Co., 1940), supplemented by various 1949 issues of his Review of Economic Progress.

Production and productivity time series, which have been developed systematically in this country since the 1920's, are now used for a variety of purposes at different levels of economic activity -- the nation, the economic sector, the industry, the enterprise, the plant, the department, and the job. Above the enterprise level, they are used, for example, in econometric analysis and model-building; in the projection of related aggregates, like employment; in the appraisal of economic conditions and prospects; in the formulation of government, business, and labor-union policies relating especially to wages, prices, employment, and hours of work; and in the concrete historical study of such abstractions as economic development, growth, and progress. Within the enterprise, individual accomplishment records have been of interest at least since the rise of scientific management before World War I; and "work measurement" programs covering key departmental activities have more recently been introduced by alert management to meet diverse technical, administrative, and planning needs.

There are six chapters in this study. The two following this introductory one are concerned with meaning and with various technical aspects of measurement, and the next two deal with special topics in production and productivity measurement. More specifically, Chapters II and III take up such matters as the definition of production and productivity, alternative choices in the algebraic implementation of these notions, the relation between different measures, and the ulterior significance of indexes in the light of economic theory and other fundamental considerations. Chapter IV discusses the properties of particular gross and net production index formulas and the techniques of adjusting for incompleteness of quantity statistics and of maintaining chronological continuity. Chapter V exhibits some properties of labor and composite-factor productivity measures and considers the partitioning of changes in, say, aggregate man-hours or value added into the "contributions" of productivity and other designated variables. Chapter VI summarizes the findings of the study and considers the outlook for improving and extending production and productivity measurement.

Viewpoint

On the whole, the matters with which we are here concerned have received inadequate attention from index users and makers alike. "Practical" men -- "operators" and administrators -- cherish the illusion that there is a unique concept of "production" or "productivity" which is represented fairly by whatever index is available or constructible. To them, a special interest in technique or in transoperational sense may seem as digressive as stopping to pick daisies in the middle of a battlefield. When the typical statistician or economist has occasion to use production or productivity indexes, he is less careful of the distinction between tweedledum and tweedledee than when he is working in his own Gebiet. Aware of the "usual index-number problems", he proceeds as though acknowledgment of their existence somehow renders them inconsequential. Like others who are less informed, he tends to consider the tools at his disposal sufficiently reliable for his ulterior purposes. Thus, he uncritically accepts the result of deflation of a value index by a price measure as an index of "quantity" expressed in "constant" dollars of the base period, even if the product complex cannot be specified and regardless of the particular formula of the deflator. Exaggerating the degree of conformity of differently constructed measures, he minimizes the importance of determining the index most appropriate to a purpose or context.[2] When interpreting a productivity measure, he commonly

[2] A curious dualism on the matter of index-number differences is exhibited by writers who have had occasion to comment on both practical and theoretical applications of formulas with dissimilar weights. When discussing the usual price or production measures, they emphasize empirical consistency and minimize or ignore the possibility of divergence. But, when they consider

overlooks the danger that it may lie outside the range of the individual productivity relatives. When using a partition formula, he frequently sets up and interprets the particles incorrectly.

But index makers, too, are undiscriminating. They prefer the comforts of convention and the ruts of familiar practice to the search for promising new paths. Without protesting too much, they subordinate purpose and methodology to availability of data. It is remarkable that they have been content for thirty years to construct production indexes without establishing the sense in which the results are to be regarded as numerically significant. A conscious theory is necessary if only because the indicators are not unequivocal indicators of "physical" output; the original quantities lose their pristine clarity when they are converted through weighting to value or labor terms and when the notion of "netness" is introduced. In the absence of any clearcut principle, it is natural

"welfare" or "cost-of-living" comparisons in the so-called "economic theory of index numbers", they tacitly assume the divergence of the Laspeyres, Paasche, and "true" indexes. If formulas and weights do not really matter, then the ingenuity lavished on "economic" index theory has been wasted; if all questions have the same answer, then it does not matter which we are asking.

Examples of dualism may be found, for example, in the works of A. C. Pigou, J. R. Hicks, and P. A. Samuelson. Pigou, the pioneer of "economic" index theory, cautiously states in The Veil of Money (London: Macmillan & Co., 1949), p. 62, that experience shows alternative real income measures (or the price indexes used in their derivation) to be "not often very widely divergent" and then goes on to say: "Like the members of Lord Melbourne's Cabinet, all of them say much the same thing; though in this case none of them precisely knows what it is that he is saying!" In Income: An Introduction to Economics (London: Macmillan & Co., 1946), pp. 12-14, on the other hand, Pigou still seemed perplexed by the problem of choosing among alternatively weighted measures. Hicks, who has also made fundamental contributions to "economic" index theory, generally minimizes the importance of weights in his book written with A. G. Hart The Social Framework of the American Economy (New York: Oxford University Press, 1945), pp. 199-200, but acknowledges that weights for "dissimilar" though close years could yield disparate results. Finally, Samuelson asserts in his Foundations of Economic Analysis (Cambridge: Harvard University Press, 1947) the "indisputable fact that prices do generally rise and fall in about the same proportions" (p. 145) but forthwith proceeds to summarize "economic" index theory, showing the usual inequalities involving the Laspeyres and Paasche formulas (pp. 146-63).

to endow measures with economic significance simply because they are based on materia economica (especially price weights). But it may be argued, with Hayek and Robbins,[3] that aggregates are economically meaningless. At a minimum, distortions of the traditional theory of rational choice are involved in the transoperational interpretation of output measures in terms of "volumes of utilities" or in the allegation of a "downward bias" in such measures due to "quality improvements".

Fortunately, the dangers of verbal algebra, metrical simplism, pedestrian practice, and routine interpretation are rediscovered from time to time. In the past eventful decade, there have been numerous instances in which government, business, and labor officials who would not normally care about technical issues had to take cognizance of statistical crises. During the War, unusual interest was shown in the inadequacy of national income as an indicator of the extent of conversion, in the limited substitutability of ingenuity for information in the Federal Reserve manufacturing indexes, in the logical obstacles to measurement of production and productivity changes in industries undergoing radical alteration in the character of their output, in the difference between accrual and delivery measures of accomplishment of aircraft plants and shipyards, etc. There also was widespread interest in the various expert opinions concerning the probable course of postwar productivity and the implied levels

[3] F. A. v. Hayek, "Scientism and the Study of Society: II", Economica, X (February, 1943), 34-63, especially pp. 39-50; L. Robbins, "Production", Encyclopedia of the Social Sciences, XII (New York, Macmillan & Co., 1934), 462-67, and An Essay on the Significance of Economic Science (2nd ed., London: Macmillan & Co., 1946), pp. 57, 63, 66.

of employment.[4] After the War, it became evident to many for the first time that most industry productivity measures were boobytrapped -- as conscientious technicians had so often warned without success. Labor and management representatives vigorously disagreed on the facts; national productivity conferences were held in Washington in 1946 and 1948 in the hope of clearing the atmosphere;[5] and, in the latter year, a Congressional Committee declared productivity to be one of twelve areas with notable "statistical gaps" and recommended the improvement of underlying production measures and price deflators.[6] One of the features of the historic September 1949 fact-finding report of the President's Steel Industry Board was a digression on the quality of the statistics submitted to it on productivity. A greater appreciation of the need for improved real product and productivity measures for the national economy and for broad economic sectors has stimulated government activities along these lines under the sponsorship of the Council of Economic Advisers and the U. S. Bureau of the Budget.[7] To overcome objections to publication

4 On wartime measurement problems and postwar employment projections, see, for example, the articles by E. B. George on "Gross National Product Projections" in Dun's Review, LXIII (March, May, and June, 1945); F. R. Garfield, "Measurement of Industrial Production since 1939", Journal of the American Statistical Association, XXXIX (December, 1944), 439-54; and I. H. Siegel, "The Concept of Productive Activity", ibid. (June, 1944), 225-28.

5 On the first conference, see U. S. Bureau of Labor Statistics, Summary of Proceedings of Conference on Productivity (Bulletin No. 913, Washington, 1946), especially S. Fabricant's evaluation, pp. 1-8.

6 U. S. Congress, Joint Committee on the Economic Report, Statistical Gaps (Washington: U. S. Government Printing Office, 1948), p. 7.

7 U. S. Congress, Joint Committee on the Economic Report, January 1950 Economic Report of the President (Washington: U. S. Government Printing Office, 1950), pp. 116-17.

raised by interested groups aware of the practical consequences of a few percentage points, the U. S. Bureau of the Census has been obliged to release three differently weighted 1947 indexes, not merely one, for each manufacturing industry.[8] It is also noteworthy that almost the entire discussion at the third national productivity conference (held in January, 1951) concerned technical problems of measurement and interpretation; that all attempts to evaluate the economic implications of newly developed output and productivity statistics ended with a return to technical issues.

The basic idea that informs this study is the interdependence of data, methods, and meaning. The terms "production" and "productivity", like other general terms derived from common speech and enriched by specialized usage, signify families of concepts in the Bridgman sense. There is a spectrum of such concepts corresponding to the alternative sets of operations which might be performed on appropriate data. Thus, each of the possible measures confers a particular meaning on the general term. Ideally, a measure should be selected with reference to purpose or context,[9] but practical choice is severely

[8] Indexes with 1939, 1947, and combined (Edgeworth) weights are shown in U. S. Bureau of the Census, The U. S. Census Index of the Physical Volume of Manufacturing Production, 1939-1947 (processed, Washington, January 18, 1951).

[9] In addition to being endowed with qualities required by the circumstances of measurement, indexes might also be expected to meet certain formal tests satisfied by their constituents. For example, there are the algebraic tests popularized by Professor Fisher and extended by C. Gini, Metron, IV (July, 1924), 3-162, by R. Frisch, Journal of the American Statistical Association, XXV (December, 1930), 397-406, by I. H. Siegel, ibid., XL (December, 1945), 520-23, and by others. Commonsense invariance tests may also be introduced, like those suggested by G. Haberler and E. E. Hagen, "Taxes, Government Expenditures, and National Income", Studies in Income and Wealth, VIII (1946), 5-6. Another type of test -- conformity of the aggregate to the conditions of microeconomic equilibrium -- would seem to rule out the familiar indexes. See the following Econometrica papers, which have carried forward

limited. This limitation is serious because index-number aggregates do not, except in the trivial case of identity of relatives, satisfy the condition which Bridgman calls the "absolute significance of relative magnitudes".[10] Furthermore, the pleasant assumption that the structure of a measure is irrelevant may lead to the kinds of difficulties already illustrated, to what Boulding calls "fallacies of aggregation".[11] The minimum precaution that is indicated is some understanding of the algebraic conditions for one measure to be larger or smaller than an alternative.

But there is still a second kind of meaning to consider -- the trans-operational or ulterior meaning, which depends on the rational model implicit in the very operations of measurement. Thus, an index number suggests a precise comparative evaluation of two aggregates from the standpoint of a mythical "macrotype" imagined as capable of making judgments for a (changing) group over time. Since the relevant behavior of this fictional creature is prescribed by the data, weights, and formula, it is evident at once that the model underlying an index and the theory of choice of rational individuals or "microtypes" are incompatible and of a different order. A production index cannot be explained in terms of "volumes of utilities", even if we assume that utility is a quantity and that there are no problems of interpersonal comparability in each of the periods. If it is agreed that value theory can at best permit only ordinal

the pioneer inquiry of F. W. Dresch: L. R. Klein, XIV (April, 1946), 93-108, and October, 1946, pp. 303-12; K. May, XVII (January, 1947), 51-63; and A. Nataf, XVIII (July, 1948), 232-44.

10 P. W. Bridgman, Dimensional Analysis (New Haven: Yale University Press, 1922), pp. 19-20.

11 K. E. Boulding, A Reconstruction of Economics (New York: John Wiley & Sons, 1950), pp. 186-88.

comparisons, then the only way left open to rationalize the numerical comparisons of the usual index is to "invent" the decision-maker implicit in the details of the index and to judge its plausibility. If this deus ex machina is rejected, if it cannot be regarded as a "probable impossibility" in the Aristotelian sense, then we must concede the arbitrariness of numerical indexes.

In summary, there is obvious need for higher standards among index makers and users. There should be a more general and more consistent awareness of the multiplicity of measures which have a priori plausibility, of the relevance of purpose and context to the choice of a measure, of the nature of the algebraic difference between preferred and derivable or available indexes, and of the conventions which underlie the numerical interpretation of indexes. Only persistent attention to theory and to technique -- and to their linkage -- can lead to the right true end of better solutions to practical problems. The sway of the Baconian idola fori cannot be prevented unless there is a vigilant interest in the nuances of meaning of "production" and "productivity". Definitions are important despite the customary show of impatience with "semantics" and even in the absence of statistical crises produced by dramatic events like labor-management disputes or the mobilization of industry for war. The detection and study of "pathological" cases -- of instances of externality of averages and of extreme divergence of alternatively weighted indexes -- are worthwhile undertakings for they emphasize the dangers which ever lurk in operations on aggregates. Indeed, the progress of measurement requires the recognition of dissimilarity within apparent similarity -- "the obstinate insisting", in James' phrase, "that tweedledum is not tweedledee". Progress cannot be made when vision is curbed by the poor best that has been done; when any authoritative, so-called "general-purpose", or "official" index is supposed to satisfy scientific curiosity about the course of, say, "production" or "productivity" in every connotative sense.

Despite its critical emphasis, the stated ultimate purposes of this study are obviously constructive. In the last chapter, some recommendations are offered for improvement of measures in the light of our results. Although some of the positions taken here are necessarily controversial, many of the concepts and methods employed -- some not mentioned in prior publications and others little known -- should prove of interest to other students of production and productivity measurement. The notion of "subproducts", for example, points the way to compilation of data permitting the construction of indexes which are sensitive to the structure of activity (Chapters II and IV). The notion of the "macrotype" (which reappears in Chapters II and IV) dramatizes the value judgments that underlie numerical comparisons. The "free composition" index (Chapter IV) merits consideration as an alternative to the chain index for a changing product universe. The plural meaning of aggregates (Chapter III) provides the rationale of various processes involving indexes: deflation (Chapter III), the definition of mutually consistent index-number systems (Chapter V), the partitioning of input and productivity changes into "causal" components (Chapter V), etc. Other items which should be of interest to students include: the distinction between measures based on end products and subproducts (Chapters II and IV); the analysis of the difference between alternatively weighted indexes (Chapter III); the statement of the algebraic relationships between gross and net production indexes (Chapter IV) and between "direct" and "quotient" productivity indexes (Chapters III and IV); the demonstration that coverage adjustments could lead to externality (Chapter IV); the reduction of a productivity index to a ratio of "price" indexes (Chapter V); and the demonstration that the quotient of net output and total factor input indexes may rise while all the "gross" productivity relatives for individual products and factors decline (Chapter V).

CHAPTER II

ON THE MEANING OF PRODUCTION AND PRODUCTIVITY MEASURES

In this chapter, we first define production and productivity, then note some of the problems of single-period aggregation and interperiod comparison, and finally consider some problems of interpreting indexes as strictly quantitative ratios. Much of the discussion here relates to matters that would still be important -- perhaps, even more important -- if complete data were available. For choice would then become practicable, and makers and users of measures would have to be more exacting than they now are; and the larger questions of meaning could no longer be neglected.

Definitions

As we have already noted, the terms "production" and "productivity" denote classes of concepts having a common essence. Various genera may be distinguished -- like gross and net production, output per unit of labor input and output per unit of total factor input. Within each genus, there may be lesser categories and, finally, there are the numerous species or concepts corresponding to the different sets of operations that may be performed on pertinent data.

Production Concepts

In its most general sense, production or output is the result[1] of an

[1] To avoid ambiguity, we shall generally limit use of the term "production" to the result of activity and not apply it to the process as well. Thus, the term refers to "output", to "end products" of activity complexes and to "subproducts" (discussed later in this chapter) of elementary activities.

activity or complex of activities intended to convert scarce resources (human and property services, raw and processed materials, etc.) to more satisfying states. These conversion activities include (technological) "transformation" and three kinds of "translation" -- between persons, between places, and between time periods. They thus correspond to the "creation of utilities", of the four varieties commonly enumerated in textbooks. They do not include consumption, the final registration of the latent satisfactions contained in converted resources.

Production is usually measured gross in comparison to productive activities or factor inputs. That is, the contributions of past periods or of more factors than are of particular interest in a given context (e.g., labor) are included. Net concepts are generally preferred, but they are difficult to approximate satisfactorily, as will be noted in more detail later.

In the typical case of social, time-consuming productive activity, problems of principle arise in the definition and measurement of output which are not encountered in the primitive case of producing for one's own consumption. Thus, there is no universal accord on the treatment of "overhead" activities of enterprises (e.g., advertising) and of a nation (i.e., government) intended to secure the conditions for other activities which are more obviously "productive". Many of a nation's overhead activities (e.g., internal and external safety) may, of course, be rationalized as satisfying more or less conscious collective wants, but these are not of the same order as personal wants satisfied through markets. Furthermore, the criteria for determining production at different levels of aggregation are not always consistent. Thus, what satisfies an individual's wants may be frowned upon by society as "illth"; what an establishment produces may entail offsetting costs to the community; what finally emerges from a sequence of activities may not be equivalent from the market viewpoint to the sum of the contributions of the several stages as judged by the factors (or their owners) and their employers. Finally, the time lag between the initiation of productive activity and the availability of the result (especially when the process is roundabout) introduces the dangers of disappointment of expectations, change in the basis of valuation, etc.

The notions of production and productive activity have been elaborated in formal economic theory, in national income literature, and in the course of development of "physical volume" measures. For our purposes, it is sufficient to note here only two contributions of theory: the determination of the essence of productive activity and the explicit recognition of both gross and net production concepts. The definition of productive activity as the "creation of utilities" is a commonplace going back at least to J. B. Say.[2] But the words "production" and "productive" have long retained a material connotation inherited from classical and popular usage. Marshall insisted that man "only produces utilities", rearranging nature to satisfy needs; but, in deference to tradition, he preferred to let slow time deal with the classical bias of materialism.[3] The second important contribution of theory -- the distinction

[2] On the notions of production and productive activity provided by theory, see Palgrave's Dictionary of Political Economy (2nd ed., London: Macmillan & Co., 1926), III, 213-17; Robbins, "Production," loc. cit.; J. D. Black, Introduction to Production Economics (New York: Henry Holt & Co., 1926); and L. M. Fraser, Economic Thought and Language (London: A. & C. Black, Ltd., 1937), pp. 175-97. Also of interest are the surveys of H. Myint, Theories of Welfare Economics (Cambridge: Harvard University Press, 1948) and K. Marx, Theorien uber den Mehrwert (2 vols., Stuttgart: J. H. W. Dietz, 1905).

[3] A. Marshall, Principles of Economics (8th ed., London: Macmillan & Co., 1920), pp. 63-67. It is noteworthy that, as recently as 1944, the term "production and related workers" was substituted for the term "wage earners" in all U. S. Federal statistical reports.

between gross and net output -- is central to economics. Referring again to Marshall, we observe his use of the term "net product" in the modern sense of wages plus the Ricardian net revenue, as the explicit equivalent on the national level of "national dividend" and "social product".[4] We should also note that several important net concepts have been distinguished on the subjective plane -- involving "surpluses" which individuals and collectivities presumably seek to maximize.[5]

National income literature has enriched the theoretical notions of production and productive activity while providing statistical series for various gross and net aggregates and their components. Out of vigorous discussions of measurement decisions have come important contributions to thought on the character of gross and net output, the nature of government output, the accounting of war production, the distinction between measures of potential welfare and potential (physical) output, and many other matters.[6]

[4] Ibid., pp. 79-81, 511, 827.

[5] Ibid., pp. 829-31, 846-52; F. Y. Edgeworth, Mathematical Psychics (London: C. Kegan Paul & Co., 1881), pp. 56-82; Myint, op. cit., chap. 9; and K. E. Boulding, "The Concept of Economic Surplus", American Economic Review, XXXV (December, 1945), 851-69.

[6] See, for example, the various volumes by S. Kuznets and the volumes in the Studies in Income and Wealth series, published by the National Bureau of Economic Research; C. Shoup, Principles of National Income Analysis (Boston: Houghton Mifflin Co., 1947); S. Kuznets, "National Income: A New Version", Review of Economics and Statistics, XXX (August, 1948), 151-79, and reply by M. Gilbert, G. Jaszi, E. F. Denison, and C. F. Schwartz, pp. 179-97; J. R. Hicks, "The Valuation of the Social Income", Economica, VII (May, 1940), 105-24, follow-up articles by S. Kuznets in the same periodical, XV (February, 1948), 1-16, and May, 1948, pp. 116-31, and a rejoinder by Hicks, August, 1948, pp. 163-72.

Of special interest to us are the operational concepts of gross production and the approximations to net developed for the statistical study of the "physical volume" of output of manufacturing and of other economic sectors.[7] In general, net indexes would be preferred if choices could be made, but gross measures are often used, faute de mieux. As for the interpretation of the constructed series, they have sometimes been regarded as reflecting comparative "volumes of utilities"; but, according to a more persistent tradition, they have also been characterized as approximate indicators of "deflated value added", "net output content", "amount of fabrication", and as analogues of national product in "constant" factor prices.[8] Although there is a distinct preference

[7] In recent years, physical output measures have been prepared by many Federal agencies (Federal Reserve Board, Bureau of the Census, Bureau of Labor Statistics, Bureau of Agricultural Economics, Bureau of Mines, Works Progress Administration, etc.) and by such private organizations as the National Bureau of Economic Research. These refer to practically all economic sectors characterized by directly measurable production -- manufacturing, mining, agriculture, electric and gas utilities, and transportation. Measures of output have been developed by U. S. Bureau of Foreign and Domestic Commerce and by the National Bureau for other economic sectors by means of deflation.

The terms "gross" and "net" are relative and, therefore, are not always used in an unambiguous manner. Thus, a measure that is net from one standpoint may still be gross from another. An estimate of industry output based on the end products of establishments may be net in the sense that duplication in the form of inter-establishment transactions has been eliminated. On the other hand, the measure may still be gross in the sense that the end products have not been adjusted to exclude the contribution made to their value in the same establishments in prior periods and the contribution of other industries.

[8] For comments on operational concepts of production, see, for example, A. F. Burns, Production Trends in the United States since 1870 (New York: National Bureau of Economic Research, 1934) and "The Measurement of the Physical Volume of Production", Quarterly Journal of Economics, XLIV (February, 1930), 242-62; E. E. Lewis, "Some Basic Problems in Index-Number Theory", in Economic Essays in Honor of Wesley Clair Mitchell (New York: Columbia University Press, 1935), pp. 276-92; E. Frickey, "Some Aspects of the Problem of Measuring Historical Changes in the Physical Volume of Production", in Explorations in Economics (New York: McGraw-Hill Book Co., 1936), pp. 477-86; P. G. Hudson,

for pecuniary weights, there is also a growing recognition of the usefulness of other weights for "special-purpose" measures -- e.g., labor weights for output indexes used in productivity computations.[9]

Since output indexes are not unequivocally "physical", doubts concerning their numerical absoluteness as well as their economic significance are to be expected. It may be recalled that Keynes, in his General Theory, rejected the notion of a "volume" of net output as too vague for his "causal analysis" -- and then proceeded to use a wage-weighted employment measure which hardly seems more precise and is subject to the theoretical limitations of other aggregates.[10] Pigou is likewise skeptical, noting that real income and other indexes rest on "extremely shaky foundations". Indeed, "once we abandon the sure ground of

"The Technical Problems and Limitations to the Construction of Indexes of Physical Production", Journal of the American Statistical Association, XXXIV (June, 1939), 239-51; W. Thomas and M. R. Conklin, "Measurement of Production", Federal Reserve Bulletin, XXVI (September, 1940), 912-23; S. Fabricant, The Output of Manufacturing Industries, 1899-1937 (New York: National Bureau of Economic Research, 1940), especially chap. 2 and p. 80, and "Problems in the Measurement of the Physical Volume of Output", Journal of the American Statistical Association, XXXIII (September, 1938), 564-70; H. Barger and H. Landsberg, American Agriculture, 1899-1939 (New York: National Bureau of Economic Research, 1942), pp. 12-14, 325-26; C. F. Carter, W. B. Reddaway, and R. Stone, The Measurement of Production Movements (Cambridge, Eng.: Cambridge University Press, 1948), chap. 1; United Nations Statistical Office, Index Numbers of Industrial Production (Studies in Methods No. 1, New York, 1950); and U. S. Bureau of the Census, The U. S. Census Index of the Physical Volume of Manufacturing Production, 1939-1947, cited earlier.

9 See H. Magdoff, I. H. Siegel, and M. B. Davis, Production, Employment, and Productivity in 59 Manufacturing Industries, 1919-36 (Report No. S-1, W. P. A. National Research Project: Philadelphia, 1939), I, chaps. 1, 2; and U. S. Bureau of Labor Statistics, Productivity and Unit Labor Cost in Selected Manufacturing Industries: 1919-1940 (processed, Washington, February, 1942).

10 J. M. Keynes, General Theory of Employment, Interest, and Money (New York: Harcourt, Brace & Co., 1936), chap. 4.

physical fact we are likely to find ourselves juggling with symbols in an extremely foggy atmosphere", to become involved in difficulties which do not comprise "a fit subject for elementary lectures".[11] Finally, we may cite A. F. Burns' warning, which could well serve as the epigraph to a study like the present one: "Strict logic is a stern master, and if one respected it, one would never construct or use any production index".[12]

Productivity Concepts

Productivity is the ratio between the output and the input associated with given productive activities, both measured in "real" terms. We say "associated with" rather than "corresponding to" since, as has already been observed, output is commonly measured gross while input is generally measured incompletely. If the two aggregates corresponded exactly to the same activities, they might not only be expressed in the same common denominator but also be made numerically equal for one of the periods of a time series. In the fifth chapter, we shall discuss indexes incorporating this identity. Meanwhile, we must note that such indexes are not tautological and stationary.[13]

Our usage of the term "productivity" is not universal and is still uncommon in economic literature.[14] There is no entry for the term in the

11 Income, pp. 14-15.

12 Production Trends, p. 262.

13 The assertion by I. M. D. Little, "The Valuation of the Social Income", Economica, XVI (February, 1949), 20, that productivity has meaning with respect to one resource input but not all combined is not correct if it is intended to apply to indexes.

14 Although the term has generally been used in our sense in National Bureau of Economic Research studies (even in book titles), it is restricted in a 1947 study by G. J. Stigler, Trends in Output and Employment, pp. 42-43, to what we later call the "intrinsic efficiency of labor".

Encyclopedia of the Social Sciences, and only one reference in our sense is listed in the index of A Survey of Contemporary Economics. Our productivity ratio for a single factor and single period resembles the "average productivity" (not marginal) in the static law of variable proportions and the reciprocal of a Walras-Pareto "coefficient of fabrication". If the factor is labor, our ratio through time resembles the Produktivitaet of Marx and Sombart. The Ricardian distinction between "riches" and "value" may also be expressed in the form of a productivity ratio.[15] Marshall's description of "increasing returns" corresponds to a productivity measure in which the denominator refers to "efforts and sacrifices". The "productivity" index which Hicks and other national income students distinguish from a "welfare" index is formally like a productivity indicator in our sense since it compares outputs obtainable from "constant" resources in situations characterized by different productive techniques.[16] Of course, it is possible to argue that a change in techniques implies a change in resources.

Though the typical productivity time series does not imply a functional relationship between output and input, mathematical "production functions" have sometimes been fitted, and these permit the derivation of marginal as well as average productivities for each specified input. Examples of such functions are the applications of the Cobb-Douglas formula to labor and capital through time; R. Stone's regression equations connecting S. Fabricant's output and employment

[15] Marshall (Principles, pp. 813-14) preferred to think that Ricardo was attempting to distinguish between total utility and marginal utility. See G. J. Stigler, "The Development of Utility Theory. I", Journal of Political Economy, LVIII (August, 1950), 311-12.

[16] See Economica articles by Kuznets and Hicks cited in footnote 6; A. C. Pigou, "Comparisons of Real Income", Economica, X (May, 1943), 93-98; and T. Barna, "Note on the Productivity of Labor: Its Concept and Measurement", Bulletin of the Oxford Institute of Statistics, VIII (July, 1946), 206. Although Kuznets notes that Hicks assumes constant resources for the individual producer (Economica, May, 1948, pp. 117-18), he later asserts (p. 122) that Hicks' "productivity" index refers to "total productivity" rather than "yield per unit of resources in the usual sense of the term".

series for all manufacturing; and National Resources Committee's somewhat similar equations for various industries.[17]

The measurement of productivity requires a notion of "real" cost which has significance apart from the corresponding return or output. Marshall, Edgeworth, J. B. Clark, and their classical predecessors were interested in measures reflecting "efforts and sacrifices" or "disutilities",[18] but such notions are now unfashionable. Thus, before Occam's razor was applied to utility itself, cost was eliminated as an independent entity, for "cost in the last analysis is derived from utility".[19] A similar view has led some national income students to deny the validity of a concept like "national cost" analogous to national output.[20] But it ought to be clear, once we leave the discussion of exchange-value determination for a single period (a major preoccupation of modern economics) and proceed to a discussion of economic development, growth, or progress, that resources must be given a status independent of output. In the study of change through

[17] See P. H. Douglas, "Are There Laws of Production?", American Economic Review, XXXVIII (March, 1948), 1-41; R. Stone, "Employment in U. S. Manufacturing", Economic Journal, LIV (July-September, 1944), 246-52; and National Resources Committee, Patterns of Resource Use (Washington: U. S. Government Printing Office, 1938).

[18] Marshall, op. cit., pp. 171-72, 339; F. Y. Edgeworth, Papers Relating to Political Economy (London: Macmillan & Co., 1925), I, 292-94; and J. B. Clark, The Distribution of Wealth (New York: Macmillan & Co., 1899), chap. 24. G. T. Jones' measures of real cost in Increasing Returns (Cambridge, Eng.: Cambridge University Press, 1933) were conceived in the Marshallian tradition.

[19] H. D. Henderson, Supply and Demand (Cambridge: Nisbet & Co., 1920), p. 165. See, also, Kuznets, Economica, May, 1948, p. 123.

[20] See G. Haberler and E. E. Hagen, loc. cit., p. 17; M. A. Copeland and E. M. Martin, "The Correction of Wealth and Income Estimates for Price Changes", Studies in Income and Wealth, II (1938), 103-05, additional remarks on pp. 131-35, and comments by M. Friedman, pp. 125-27; and C. S. Shoup, op. cit., p. 9.

time, the distinction between what factors "do" and what they in some sense "are" is appropriate and necessary. Here, as in macroeconomic analysis, limitation to the categories of static price theory could only frustrate inquiry.

So-called "labor productivity" indexes are of special practical importance. They were apparently introduced by the U. S. Bureau of Labor Statistics in the 1920's.[21] They became better known during the 1930's, when "technological unemployment" was a popular subject of inquiry. During World War II -- a period of labor stringency, other shortages, and controls -- the significance of such measures became even more evident. Since the War, interest in higher labor productivity and its measurement has been manifested in virtually all countries -- those which are "underdeveloped", those living under "planning", those experiencing "dollar shortages", and a United States more aware of its world position. Labor productivity measures are now so widely used for so many purposes that the qualifying adjective is usually understood when it is omitted.

To many, labor productivity indexes are less desirable than measures of output per unit of total input, but conceptual as well as measurement difficulties stand in the way of development of the latter. Not only are there obstacles to the quantification of capital and entrepreneurial services, but there also is no unanimity as to the meaning of "factor cost" or "factor of production".[22]

[21] Simple time comparisons of productivity are, of course, much older. See, for example, E. Atkinson, The Distribution of Products (New York: G. P. Putnam's Sons, 1885), pp. 119-20; and reports prepared by C. D. Wright while first Commissioner of Labor, Industrial Depressions (Washington: U. S. Government Printing Office, 1886), pp. 80-90, and Hand and Machine Labor (2 vols., Washington: U. S. Government Printing Office, 1899).

[22] See S. Kuznets, Economica, May, 1948, p. 123, and Hicks' reply, August, 1948, pp. 167-70; Kuznets, Review of Economics and Statistics, August, 1948, pp. 157-60, and reply by M. Gilbert et al., pp. 189-93. E. Rolph, "The Concept of Transfers in National Income Estimates", Quarterly Journal of Economics, LXII (May, 1948), pp. 327-62, would count all interest, dividends, and taxes as transfers.

Furthermore, there is doubt concerning the relevant aspect of input -- "assembled" or "actual", the "supply" or the "use".[23] Of course, both are relevant; the former, corresponding to a fixed parameter in the short run, is a precondition of productive activity, while the latter is the effective variable.[24] There is also some confusion between technically relevant production factors, whether remunerated or "free", and the traditional income sources, which Professor Fraser would rather call "factors of distribution".[25]

Labor productivity indexes do not reveal changes in the intrinsic efficiency of labor but, rather, the changing effectiveness with which labor is utilized in conjunction with other factors. They do not reflect the specific contribution of labor to output, since they are not measures of marginal productivity. They do not signify that labor alone is responsible for the recorded gains in output per worker or per man-hour. A review of the literature indicates that these points are not so commonly misunderstood as the repeated warnings would suggest. Even the earlier writings made it clear that

[23] See H. S. Davis, The Industrial Study of Economic Progress (Philadelphia: University of Pennsylvania Press, 1947), pp. 23-25.

[24] W. H. Nicholls, Labor Productivity Functions in Meat Packing (Chicago: University of Chicago Press, 1948), pp. 51-52, properly distinguishes three dimensions of labor and of capital -- number of units, duration of use, and intensity of use -- in the formulation of short-run production functions.

[25] Fraser, op. cit., pp. 209-12, 326-27. The distinction between "engineering" and "classical" production functions will emerge more clearly as interest in the former increases. See Econometrica, XVIII (July, 1950), 305-09. In their study of the "Relation of Agricultural Production to Inputs", Review of Economics and Statistics, XXX (May, 1948), 117-26, G. T. Barton and M. R. Cooper include virtually every expense of production as an input, even taxes, but they ignore the "free" technical factors, sunshine and rain.

workers' efforts are not the sole source of rising productivity.[26] Indeed, Marx himself regarded productivity as "dependent on the degree of development in the conditions of production"; and he also distinguished it from intensity of labor effort in explaining "surplus value".[27] Similarly, the better known proposals for linking wages to observed or anticipated productivity rises -- like the AFL's "social wage" demand of the 1920's, the "annual improvement factor" incorporated in some recent wage agreements, and the wage-price policy favored by the Council of Economic Advisers and "Keynesians" -- do not derive from any vulgar theory of imputation but, rather, from "ethical" considerations or from the belief that underconsumption tendencies even threaten economic stability.

It is insufficiently appreciated that labor productivity indexes covering long time periods and broad sectors of the economy are peculiarly appropriate for a society in which man is, in Marshall's words, both "the end and an agent of production". In such a society, current human labor should be rendered "scarce" in comparison to other inputs and to output. The goal should be the simultaneous and steady, in the trend sense, increase in real output and (voluntary) leisure (i.e., in "economic welfare"); or, since work is the complement of leisure, the continuous increase in output relative to current

26 See, for example, "Productivity of Labor in the Cement, Leather, Flour, and Sugar-Refining Industries, 1914-1925", Monthly Labor Review, XXIII (October, 1926), 19; U. S. Bureau of Labor Statistics, Handbook of Labor Statistics, 1924-1926 (Bulletin No. 439, Washington: U. S. Government Printing Office, 1927), p. 527; and L. Wolman, "Labor", Recent Economic Changes in the United States (New York: McGraw-Hill Book Co., 1929), II, 446-47, 461-62.

27 K. Marx, Capital: The Process of Capitalist Production (Chicago: Charles H. Kerr & Co., 1909), p. 569. Cost Behavior and Price Policy (New York: National Bureau of Economic Research, 1943), p. 153, first curiously states that measures of output per man-hour are consistent with the labor theory of value and then curiously suggests that such measures should take account of stored-up labor to be "meaningful" within the framework of the labor theory.

labor expenditure (i.e., the continuous increase in productivity.)[28] In such a society, the use of a labor numeraire or accounting unit for all factor inputs also seems appropriate for the study of long-term progress. Thus, the goal of reducing current labor expenditure for a given volume of output might well be extended to the minimization of the cumulated total labor cost to "generalized" man. The composite labor sum would still make some sense even if the original labor-equivalent of the incorporated or used-up resources were not converted to current labor of a uniform grade.

Classification and Aggregation

Having considered the conventional intension of the terms "production" and "productivity", we now proceed to problems of quantification and to problems of interpretation of the derived measures. For convenience, we shall restrict our discussion to indexes based on weighted aggregates. In practice, such indexes are most commonly sought or used. They include the classic Laspeyres and Paasche measures, which are ratios of weighted aggregates and are also equivalent to weighted arithmetic or harmonic means of relatives; the Edgeworth and Fisher "ideal" measures, which are averages of the Laspeyres and Paasche formulas; and the ratios of reduced aggregates, such as are often preferred for the measurement of changes in net production. The most important of the omitted formulas are geometric means of relatives, which are now infrequently used.[29] In this section, we deal with the construction and interpretation of aggregates. In the third and final section of this chapter, we take up the problems of time comparison.

[28] Of course, current labor productivity can be raised steadily only if adequate provision is made in current output for the replacement and expansion of capital -- only if labor is rendered "scarce" with respect to capital.

[29] For a discussion of the relation between geometric means of relatives and aggregative indexes, see I. H. Siegel, "Index-Number Differences: Geometric Means", Journal of the American Statistical Association, XXXVII (June, 1942), 271-74.

Aggregation within a period involves: the establishment of "internal" comparability, or the grouping of items being measured into classes regarded as homogeneous; and the establishment of "external" comparability of the recognized classes. The classes refer to products, to factor inputs, or to the materials, fuel, etc., which have to be eliminated in the derivation of the reduced aggregates entering into certain net output indexes. The items included in each class are presumably fungible. (Thus, if pulp production is being measured, the "homogeneous" classes groundwood, bleached sulphite, bleached sulphate, etc., may be established. Within each class, every ton is regarded as interchangeable with every other.) When productivity -- or, rather, its reciprocal -- is being measured for a group of products, each class may be considered to have only one member, the average ratio corresponding to the total class output. (Thus, an average unit labor requirement -- man-hours per ton -- will correspond to the entire output of groundwood in tons.)

Following Bridgman, we designate as "primary" the characteristic unit in terms of which all class members are interchangeable (e.g., all tons of groundwood are equivalent). Through "weighting" (e.g., by price per ton), the various "primary" quantities (i.e., total tons of groundwood, total tons of bleached sulphite, etc.) are converted to a common denominator (i.e., dollars of a given period). We may designate the sum of the weighted outputs as a "weighted aggregate", or as a "secondary" quantity. As we shall see in the next chapter, secondary quantities are multiples of specific complex secondary units -- a fact of importance for the interpretation of aggregates and indexes.

Classification and Units

Ideally, the principle of indifference according to which class members are considered equivalent should be determined with reference to the purpose of measurement. In the treatment of output, the selection of classes and units might be approached from either of two general viewpoints. First, the production measure might be intended to reflect the viewpoint of the market -- of the supplier or the industrial or final consumer -- in which case attention should be directed to end products of an activity complex (e.g., of an establishment or industry) expressed in units relevant to supply, demand, or eventual consumption. The resulting measure, usually incorporating price weights, is a familiar type of gross production index. It may give a distorted picture of productivity change since it merely reflects the volume of goods completed during each period -- not the work done, the contribution specifically made to the emerging end products or to goods still in process. The second approach reflects more faithfully the structure of productive activity. For ascertaining the volume of activity in output terms, classification should ideally be based on "subproducts" of the vertical stages through which each end product passes; and each subproduct should be measured in a characteristic unit.[30] The resulting aggregative index would give a much better picture of productivity change than any output measure based on end products. It would also take proper account of changes in the degree of process integration, in the extent of subcontracting, and in departmental rates of operation; and it would permit conception of a hierarchy of consistent productivity measures covering the different levels of aggregation, from the worker up through the plant to the entire economy.

Of course, the subproduct approach is mainly of theoretical interest, since it requires more detailed and more voluminous data than are available. Hence, the approximations to net output measures which are usually discussed or attempted start with data for end products. One of these approximations is

[30] On the notion of "subproducts", see I. H. Siegel, Journal of the American Statistical Association, XXXIX (June, 1944), 219-25, and references to H. S. Jevons and S. Carlson. See, also, E. Rolph, "The Discounted Marginal Productivity", Journal of Political Economy, XLVII (August, 1939), 542-56. The distinction made by Shoup, op. cit., pp. 26, 146-47, between "capitalized" and "expensed" outlays is also relevant.

based on weighted aggregates for end products reduced by weighted aggregates for consumed materials, fuel, etc.[31] A second alternative is based on end-product output (generally in the form of a price-weighted index) aggregated by means of "net" or partial weights referring, say, only to the man-hours or value added in the particular activities of interest. A third possibility, often considered appropriate for the measurement of the unduplicated output of the entire economy or broad economic sectors, involves the combination with full price weights of "finished" products only, those which do not reenter the productive process.[32] In the case of a single industry of simple structure, the establishments of which do not supply each other with any materials, this measure is the same as the gross production index mentioned in the preceding paragraph. It should be noted that these three alternatives need not resemble closely the preferred net index based on subproducts, which involves both an incremental product classification and incremental weights. Furthermore, gaps in the available data prevent their computation, except in a few cases. Thus, manufacturing industry indexes can usually be combined by means of labor or value-added weights, but joint costs in multiproduct establishments generally preclude the unambiguous determination of analogous weights for the individual products of an industry.[33] The extent of intra-industry output duplication

[31] Fabricant, op. cit., pp. 25-26; and R. C. Geary, "The Concept of Net Volume of Output with Special Reference to Irish Data", Journal of the Royal Statistical Society, CVII (Pts. 3-4, 1944), 251-61, especially p. 256.

[32] See W. H. Shaw, Value of Commodity Output since 1869 (New York: National Bureau of Economic Research, 1947).

[33] On the arbitrariness of allocations of joint and general costs, see Cost Behavior and Price Policy, pp. 268-69.

and the magnitude of an industry's consumption of materials, fuel, etc. in "constant" prices are rarely known.[34]

There are instances in which significant end-product classes expressed in acceptable physical units cannot be established. For example, if the end products of an industry are extremely heterogeneous, the number of natural classes may be unmanageable; and, if the output is regrouped into a small number of classes, these new classes may not retain their significance through time. In other words, the qualitative change in the "same" class may seem as important as the change in magnitude. In the case of extreme heterogeneity, the device of "deflation" by a price index cannot yield indirectly a result more satisfactory than is achievable directly. If a suitable product complex cannot be specified at the outset, then the nebulous complex implicit in the derived aggregates can hardly be more suitable. Of course, the deflated result has some meaning, but not the one ascribed to it. A second device -- the substitution of a single proxy series, like consumption of a major material or a crude shipment total -- also fails to meet the real difficulty. A third device -- classification by subproducts rather than end products -- may sometimes prove successful since greater uniformity could result when the productive process is broken down into a sequence of activities. For example, the functionally similar components of two buildings of different size may be aggregated with fewer misgivings than the buildings themselves, especially if the functional components are physically similar and hence differ only in quantity.

[34] Since the aggregate value of output as reported by establishments exceeds considerably (perhaps, by 50 percent) the value of "finished" output of the manufacturing sector regarded as a unit, the value of products is no longer shown for industry combinations in the <u>Census of Manufactures</u>.

When the end product is a "service", a protean combination of elementary activities, the only practicable output unit may be characteristic of input -- e.g., man-years or man-hours. Such a unit would, of course, imply that productivity changes in the actual operations performed are of no interest to the purchaser, that the latter merely "wants what he wants when he wants it". But, if there is interest in the technical effectiveness with which the service is performed, then the service must be redefined in terms of standardized component activities. Thus, the citizenry may be content to pay the militia, policemen, firemen, government clerks, postmen, etc. for man-years of "service", but personnel and budget officers must be concerned with the specific sub-product output of these forces if economy is to be achieved.

Special difficulties arise in time comparisons. As will be noted again later in this chapter, the changes occurring in a progressive society -- in tastes, technology, etc. -- tend to corrode any product classification and may require the reopening of questions which had apparently been settled satisfactorily. Devices intended to maintain statistical continuity often impair continuity of sense. Thus, it is often possible to establish a gross classification which is applicable to the whole time span of interest but which also obscures relevant changes in output composition. It is also difficult to rationalize the results obtained through formal corrections for changes in completeness of reportage of physical quantities and in the assortment of products included in an index. These intended corrections (value adjustment, chaining, etc.) are discussed in Chapter IV.

Agencies which compile and publish production data, like the U. S. Bureau of the Census, follow a variety of criteria in classification, paying particular attention to the conflicting needs of maintaining statistical continuity and of revealing important changes within industries. They also balance the various needs for data and give due consideration to the realities of accounting and their own budget. In the Census of Manufactures, data are usually compiled for relatively broad categories. Though the schedules for 1947 were more detailed than earlier ones, only 6500 "products" were specified; the published quantity figures refer to 6100. In general, the recognized classes relate to end products of establishments; sometimes, they also include items destined for consumption in the same establishment or others of the same industry. The breadth of classification often obscures quality changes of interest. There is also a time lag in the explicit identification of new products. Many items are reported, not in quantity terms, but by value; and shipments have often been reported by manufacturers (indeed, shipments were specifically requested in 1929 and 1947) instead of production. The Census Bureau indexes of manufacturing output for 1939-47, though based on more product series than were ever used before, were derived from quantity data accounting for only 57 percent of the total value of products in 1947; and the industries for which physical data were used comprised only two-thirds of the total value added.[35]

It is even more difficult to devise suitable classes and select appropriate units for the measurement of factor input than output. As has already been suggested, a distinction ought to be made between the input categories and units relevant to a study of welfare or income distribution and the categories and units relevant to a study of the technical relation between output and input. It has also been noted that serious obstacles impede the

[35] See U. S. Bureau of the Census, op. cit., and the "General Explanations" in any volume of the Census of Manufactures: 1947 (Washington: U. S. Government Printing Office, 1950). On criteria apparently used in product classification, see Magdoff, Siegel, and Davis, op. cit., I, 29-35.

development of comprehensive measures corresponding either to a given set of activities (e.g., an industry) or to a given complex of end products. In general, only labor input is reported for a given set of activities; and the recognized classes are broad (e.g., all factory workers combined or all office employees), while the unit refers at best to duration (e.g., man-hours). Greater refinement in the labor classification, like distinction of skills, would permit construction of a greater variety of measures; other units, like "constant" dollars and energy, would also be of interest.

Special difficulties arise in the measurement of factor inputs like entrepreneurial and capital services. There is no characteristic unit for the former; and the latter is measurable, in the absence of serious technical change, in terms of the duration of use of homogeneous equipment classes, but the number of identified classes would be unmanageable.[36] Deflation, as we noted earlier, cannot overcome the basic difficulty though it provides the appearance of a solution.[37] It was also noted earlier that the measurement of non-labor factor inputs in labor units would be tolerable for the study of progress.[38] This view,

[36] Copeland and Martin, loc. cit., suggest the cruder measure of total plant-hours. J. M. Gould, Output and Productivity in the Electric and Gas Utilities: 1899-1942 (New York: National Bureau of Economic Research, 1946), pp. 61-69, uses kilowatts as a measure of electric generating capacity, so that his capital productivity index shows something like the change in capacity utilization. He devised no measure of capital available or capital service in units "independent" of output.

[37] A crude measure of deflated book capital, of a type rejected by Gould for utilities, is used by G. J. Stigler, op. cit., pp. 50-53, in his interesting attempt to measure changes in capital productivity and output per unit of composite factor input in manufacturing.

[38] See F. C. Mills, "Industrial Productivity and Prices", Journal of the American Statistical Association, XXXII (June, 1937), 258-59, and K. E. Boulding, The Economics of Peace (New York: Prentice-Hall, 1945), pp. 76-77. Boulding, incidentally, would like to measure "want satisfactions" per man-hour but settles for the output of goods and services as his numerator.

of course, is unlike that of Dobb and other socialist economists who regard the "failure" of the subjective theory of value to provide a basis for the quantitative expression of capital as a decisive point in favor of the labor theory of value.[39] We do not assert the relevance of a measure of a factor's "quiddity" to the determination of exchange value, so we ought to escape Hayek's censure of "scientism"[40] -- even though we do assert the significance of "real" cost for historical studies and the special significance in this connection of the labor _numeraire_.

Aggregates and Their Meaning

Money and labor are the only more or less usable units for aggregating either output or input. Psychological units -- utility for output and negative utility (disutility) for input -- are ruled out as non-operational, but we shall have more to say about these later. The pecuniary denominator -- e.g., "constant" dollars of a "base" period -- is generally preferred to, and is easier to employ than, the labor denominator.[41] Gross products (which include materials, etc.) may readily be summed in current prices, but conversion of such output to a current labor equivalent would be a real feat. The same is true of

[39] M. Dobb, "On Some Tendencies in Modern Economic Theory", in _Philosophy for the Future_ (New York: Macmillan & Co., 1949), p. 396.

[40] Hayek, _loc. cit._, pp. 40-41.

[41] See Davis, _op. cit._, pp. 20-21, and _Cost Behavior and Price Policy_, p. 154. Two ambitious attempts were made by W. P. A. National Research Project investigators to measure input in labor units: R. K. Adamson and M. E. West, _Productivity and Employment in Selected Industries: Beet Sugar_ (Report No. N-1, 1938) and M. E. West, _Productivity and Employment in Selected Industries: Brick and Tile_ (Report No. N-2, 1939).

factor inputs other than labor. But the problem of expressing all output or input in "constant" price or labor units of a particular period -- a necessary prelude to temporal comparison -- cannot be solved satisfactorily if the different products or factors cannot be measured in appropriate primary units in the first place. Finally, we note that there is no difficulty in principle in aggregating output by means of partial or "net" weights referring either to "direct" money or "direct" labor cost.[42] Such weights do not involve arbitrary allocations of joint or overhead costs.

The significance of index numbers depends on the significance ascribed to aggregates. From the various discussions in the literature, it emerges that pecuniary output and input aggregates have no exact welfare content; that, only under highly restrictive assumptions, can they indicate which of two combinations of output or input is preferred by a collectivity or by its "representative" consumer or producer.[43] The quantitative interpretation of aggregates and indexes is incompatible with the accepted theory of economic value, since the rules of measurement do not correspond to the rules of economic substitution. Price or labor weights cannot be wrenched out of their original

[42] Insofar as possible, "direct" labor weights were used in combining industry products and industry indexes in Magdoff, Siegel, and Davis, op. cit., and V. E. Spencer, Production, Employment, and Productivity in the Mineral Extractive Industries: 1880-1938 (Report No. S-2, W. P. A. National Research Project: Philadelphia, 1940). The same practice is followed by U. S. Bureau of Labor Statistics.

[43] In addition to the cited works of Robbins, Hicks, Pigou, Kuznets, Little, and Samuelson, see R. G. D. Allen, "The Economic Theory of Index Numbers", *Economica*, XVI (August, 1949), 197-203, and O. Lange, "Foundations of Welfare Economics", *Econometrica*, X (July-October, 1942), 215-28. Of general interest is the penetrating study by K. J. Arrow, *Social Choice and Individual Values* (New York: John Wiley & Sons, 1951).

"equilibrium" context and associated with the output or input quantities of other periods without loss of their original meaning. The indifference map of the "representative" consumer and the product substitution map of the "representative" producer cannot be composed of parallel hyperplanes. Marginal rates of substitution would cease to play the role now attributed to them if they really were constant from point to point and from plane to plane. The validity of such maps through time -- together with the fact that the weights have absolute as well as relative constancy -- would, indeed, mean the replacement of "alternative" cost theory by a "real" cost theory. But such maps are not valid -- they do not represent the behavior of any but a mythical creature -- so the implied theory is really one of "conventional" or "arbitrary" cost. Two other facts ought to be noted here. First, there is no unique weighting system inherently superior to all others, so there is no unique measure of output or input change. Second, the behavior implied in a measure of quantity change is inconsistent with the behavior implicit in the correlative measure of change in the weighting concept -- i.e., in price or in the unit labor requirement -- so the two measures cannot be interpreted with reference to one map even in the best of circumstances.

Although output indexes with pecuniary weights are sometimes said to reflect changes in the "volume of utilities" created, aggregates do not represent unique Benthamite sums. Even if utility and disutility were quantities, were measurable on the same scale for each individual, and were convertible between individuals according to known functions, pecuniary output and input aggregates would still imply numerous subjective totals according to the interpersonal distribution of the output and input. This would still be the case if all individuals had the same subjective functions but the interpersonal distribution of benefits and sacrifices were unequal, since marginal utility and marginal disutility are not constants. Apparent definiteness is given to aggregates, at least for a single period, by the transfer of attention away from the individual to "society", a "collective personage"; the identification of equilibrium prices with "social" utility and disutility; and the substitution of the notions of "effective" utility and disutility, computed at the margin, for the true utility totals, which are integrals of marginal utility and disutility. It was in this manner that J. B. Clark sought to connect market and subjective aggregates.[44] But the solution is unsatisfactory. The two "effective" totals are equal, yet the true integrals must be unequal. Indeed, if "society" is rational and endowed with foresight, total utility must exceed total disutility; otherwise, economic activity would not be undertaken in the first place. Clark's Procrustean maneuver amounts to a serious redefinition of the subjective concepts underlying individual behavior. Depersonalization of these concepts deprives them of economic significance, at least to the extent that individuals rather than society make economic decisions. To consider utility with Clark as the quality of inducing "social labor" or with Fraser as the "quality of inducing purchase"[45] is to pour new wine into old bottles without changing labels. In any case, Clark's approach does not suggest the interperiod comparability of subjective totals; and it is even less satisfactory when the discussion is extended to net output aggregates and when output includes goods destined for gradual and intermediate consumption.

The view that pecuniary aggregates have or ought to have more than a vague welfare content inevitably persists. Thus, when a Department of Commerce spokesman claimed merely that the revised gross national product statistics

[44] _Op. cit._, chap. 24.

[45] Fraser, _op. cit._, p. 89

revealed the "structure of the social accounts", it was countered that the measures also have welfare significance.[46] More recently, Samuelson suggested that money aggregates not be compared in ratio form since economists who knew better could not resist the temptation to ascribe cardinal significance to the indexes.[47] Kuznets would like to identify the Hicksian cost-weighted "productivity" and price-weighted "welfare" measures by suitably redefining factor cost.[48] Though Samuelson rejects Kuznets' monism, he does state (and Hicks would agree) that measures of "production possibilities" have no normative significance in themselves but find justification in their illumination of "utility possibilities". The inclusion of such "waste" as war output in measures of national product is rationalized in the same manner.[49] Finally, we note the occasional suggestion (e.g., by Pigou and E. R. Walker) that different price weights ought to be used in physical output and real income measures so that they would more effectively reflect satisfactions.[50]

Time Comparisons

The character of an index is necessarily determined by the choice of classes, units, and weights; and by the techniques employed to make up for gaps

[46] See remarks by J. Rothenberg and E. F. Denison in Studies in Income and Wealth, X (1947), 66-67, 77-78; and M. Gilbert et al., loc. cit., pp. 188-89.

[47] P. A. Samuelson, "Evaluation of Real National Income", Oxford Economic Papers, II (January, 1950), pp. 22-26.

[48] Economica, May, 1948, pp. 122-24.

[49] Oxford Economic Papers, January, 1950, p. 17.

[50] Pigou, Veil of Money, pp. 60-61; and E. R. Walker, From Economic Theory to Policy (Chicago: University of Chicago Press, 1943), pp. 249, 254.

and discontinuities in the data. The validity of such techniques, too often taken for granted, actually depends on the closeness of the results they yield to those which would have been obtained if they were unnecessary -- if complete data were available. We defer discussion of the algebraic conditions they must satisfy to accomplish their purpose and consider here some other aspects of index-number construction and interpretation.

Instability of Reference Frame

The change recorded by any comprehensive measure is most intelligible when it is ascribable to a unique set of causes uniformly acting on all components. Thus, what an index can tell us is best understood when the recognized classes really are homogeneous through time; when the primary unit for each class refers to an attribute of dominant and constant significance, or when the relatives are invariant to a change in units; when all classes indicate the same percentage change (in which event the choice of formula and weights is immaterial); when all plausible weighting systems are equivalent. But such uniformities are not encountered in practice; indeed, as we have already suggested, such uniformities would imply that rational economic behavior is something far different from what it is customarily considered to be. Instability is the rule -- in the statistics and in the situation to which an index relates -- so that interpretation is no simple task. Thus, there are relevant changes in the tastes, income distribution, age distribution, and identity of the population; in institutions, technology, structure of activity, and degree of market imperfection; and in the assortment, quality, and physical specifications of output and in the quality of factors. Such changes suggest, among other things, the impossibility of interpreting a production measure even as an ordinal comparison with reference to a unique map of indifference or substitution curves. Indeed, if a fixed map were assumed while tastes actually adjusted to changes in technology, it could appear that a "representative" consumer illogically preferred the less desirable of two combinations.[51] An index cannot be regarded as an indicator of choice unless a constant criterion is postulated -- unless the measure itself suggests the behavior of a plausible though mythical appraiser contemplating both situations with what Collingwood calls "equal sympathy".[52]

Quality Changes

Of the changes mentioned here, so-called quality changes in output are of particular interest. Failure to take account of these, especially in the case of manufacturing, presumably leads to distorted measures of quantity. A strict distinction cannot be made between quality problems and other measurement difficulties since all ultimately involve the adequacy of classes, units, and weights. Thus, a radical alteration of the internal structure of a broad product class is sometimes regarded as a quality problem. The same is true of a change in the scope of activities included in an industry definition, a change which destroys the relevance of the partial weights. There would be more agreement on an alteration of the properties of a nominal product or an alteration of the significance of a primary unit with the result that the same quantity no longer has the same capacity to satisfy the wants of a consumer with fixed tastes. (The analogous case for capital goods is an alteration of the eventual output obtainable by use of the "same" quantity of equipment with a given complement of other factor inputs.) A quality change sometimes requires an alteration of

51 The remarks of Carter, Reddaway, and Stone, *op. cit.*, pp. 71-72, on British output comparisons for 1935 and 1946 are very pertinent.

52 R. G. Collingwood, *The Idea of History* (London: Oxford University Press, 1946), p. 327. Other remarks of this historiographer on "progress as created by historical thinking", pp. 324-27, are also relevant to our discussion of temporal changes.

the amount of resources used per unit of the same nominal product due to elaboration or curtailment of the process. This situation should be distinguished from a change in resource productivity for a fixed set of activities and from an apparent change due to a relocation of activities -- in both cases without affecting the want-satisfying power of the product. Such quality and pseudo-quality problems could theoretically be handled by means of subproduct measures with classes that could have zero entries.

A commonplace of the literature on production measurement is the assertion that quality changes generally represent "improvements", so that the usual measures have a "downward bias".[53] This idea requires closer scrutiny. It may mean that grossness of classification obscures a shift in assortment of output in favor of goods requiring more factor input, or in favor of goods more highly prized by consumers with given tastes. It may, on the other hand, mean a "true" quality increase (i.e., in a refined product class) from the standpoint of the productive process: the actual output of a new model may be lower than that of the superseded model with given resources. A "true" increase occurs in the case of capital equipment when a unit of the new model is more effectual than a unit of its nominally equivalent predecessor. But conceptual difficulties may arise when attempts are made to identify "true" quality changes in goods made for consumption. Thus, a postulated consumer with fixed tastes in the narrow sense cannot be trusted to compare with "equal sympathy", from the standpoint of one period, the products made at different times in a dynamic society. Indeed, if truly imbued with the outlook of an early period, he may prefer the familiar and regard prospective change as an inconvenience. A consumer with the viewpoint of the later period may likewise be biased, regarding change as progress simply

[53] See Stigler, op. cit., pp. 3, 10-13, and the literature cited in footnote 8.

because it has culminated in the present. There is an alternative quality appraiser whose judgment may be invoked -- a "generalized" consumer equally at home in all periods and whose tastes, being fixed in the broad sense over time, are satisfiable by goods of changing specification. Since he accepts new ways of satisfying his stable categories of wants, he would rule out many of the quality changes often asserted to have occurred.[54] Without some such conception we should probably have to abandon attempts to measure changes in the "physical volume" of the physically changing goods of an advanced industrial society.

Quantitative Interpretation of Indexes

We now address ourselves to a most important question: How can the popular view of an index as a quantitative comparison be rationalized, especially since such a view is not supported by economic considerations? We have already seen that even the interpretation of indexes as ordinal comparisons is permissible only under artificially simple conditions. Historical measures cannot be justified as "economic" comparisons of alternatives from the standpoint of fixed tastes or technology. Output aggregates cannot be interpreted in terms of utility, unless this notion is depersonalized; and output indexes cannot be regarded as ratios of utility "volumes" without further redefinition. The same

[54] To such a consumer, a car of today serves no better the need for private transportation than did the model of yesteryear in its own time, so the later version does not represent some mystic multiple of the earlier one. To him, a change in a product simply keeps it desirable, for his tastes in the narrow sense correspondingly alter. These remarks, of course, refer to output measurement from the end-product standpoint, not the process standpoint. If the purpose were to measure productivity changes in a given set of operations in automobile manufacture, standardization between periods (e.g., on a "sub-product" basis) would be desirable.

In mining productivity measurement, there is a commendable practice of making computations for both ore and recoverable metal -- a practice which recognizes the interest which attaches to measures in terms of processes as well as end products.

may be said concerning efforts to interpret input measures in terms of the disutility experienced by persons in the performance of work or in the use of their property. Besides, the status of disutility is even more dubious than that of utility. All this leaves us with but one answer to our question: An index may be regarded as indicating the precise evaluation and choice made by a mythical personage whose relevant behavior is not "economic" in the ordinary sense but is described by the specific content and structure of the index.

In elaborating our answer, we shall find a few new terms useful. First, we define a "microtype" as an economic man, an abstraction whose relevant personality is described by the familiar equilibrium allocation equations. Next, we introduce two personages of higher order, single-period and multiperiod "macrotypes". The former is a "representative" or generalized individual whose behavior is considered to characterize that of a collectivity for a single period. A multiperiod macrotype is of a still higher order; it is conceived as capable of making a precise comparison of two or more aggregates according to a fixed criterion expressed in the weights. Now, in the quantitative interpretation of an index, we must reject at once the microtype as a personage of insufficient scope to reflect the collective choice of even one period. Utility and disutility, being of the same order, must likewise be rejected; quantifiability has nothing to do with this conclusion. J. B. Clark's socialization of utility, as we have already noted, is tantamount to the introduction of a single-period macrotype whose behavior is *analogous* to that of a microtype. But only the multiperiod macrotype can make interperiod comparisons of a numerical character -- and then only because it is the physical model, the personification, of a formula. In the simple case of a comparison of two output combinations of n end products, we may imagine this mythical creature to have been delegated the task of contemplating both with "equal sympathy" and deciding their relative magnitudes. The individual product series are deemed adequate, whether or not quality has remained constant. The two combinations are like vectors with a common origin in a hyperspace of n dimensions. If the vectors are collinear (in which case all product quantities have changed in the same proportion), then their relative magnitudes are independent of the choice of macrotype. But if they are not collinear, then the result does depend on the macrotype selected. Different principles of indifference may be invoked to render the two vectors collinear; either may be rotated (simultaneously undergoing a change in length) while the other remains fixed, or both may be rotated. The index again is given by the relative lengths of the collinear vectors or of their projections in any plane; the result varies according to the transformation principle used by the macrotype. Another mode of behavior, however, is more commonly built into the macrotype. Instead of collineation of the vectors, the macrotype underlying the ordinary index is asked to accomplish the scalarization of vectors by means of a common system of weights. These weights define the fixed viewpoint from which the macrotype makes a precise appraisal.

The visualization of the physical model underlying measurement has the virtue of clarifying the degree of kinship to the model describing economic choice. It also makes evident the conventional nature of numerical indexes. If all indexes of production or productivity agreed, then there would be no interest in the ulterior justification of any particular one. But, since there is no unanimity, the multiperiod macrotype should be a useful fiction.

A Comment on Ultimate Measurement

Once we note that the categories of man's wants have greater stability through time than the particular product classes satisfying those wants, there is a temptation to proceed further and ask if all output may not after all be summed up, in some ultimate sense, in a single, continuous, comprehensive series.

Is there not a summum bonum to which all output contributes, whatever its specific form? We have already dismissed the subjective, microtypal notion of utility as unsuited to this purpose. If there is an acceptable notion, it must refer to an enduring desideratum of "generalized" man, a multiperiod macrotype. It must be objective, "absolute", and universal -- applicable to all times, places, and cultures. It would evidently be the analogue, on the "return" side, of the ultimate labor measure of composite input.

Perhaps, the only conceivable measure of ultimate output is a Ruskinian one -- the net addition of life-years to a (changing) collectivity made possible by economic activity in its broadest sense (including war).[55] This measure suggests that the extension and perpetuation of life, at least for the collectivity in question, constitute the supreme objective of activity, and that a year of life has constant, additive significance. The contribution of various pursuits (e.g., medical arts) to a life-opportunity sum would be quite different from the contribution to the usual national product total. A "productivity" measure based on a life-opportunity numerator and a composite input denominator expressed in labor would be a most striking indicator of progress. But, needless to add, even Colin Clark would have difficulty estimating the numerator!

[55] See J. A. Hobson, Work and Welfare (New York: Peter Smith, 1948), chap. 1, for an attempt to formulate a vital standard of valuation, "organic welfare", and for a sympathetic estimate of Ruskin's Munera Pulveris.

CHAPTER III

SOME TECHNICAL ASPECTS OF MEASUREMENT

In this chapter, some of the points made in the preceding one are elaborated, and the foundation is laid for the further discussion of production and productivity measurement in the chapters which follow. Among the topics treated here are the interpretation of aggregates and indexes in terms of secondary units; the effects of differences in classification, units, and weights; and the nature of deflation.

On Aggregates

The weighted aggregates of interest here have the form Σxy, where the n positive values of x_i may be the weights and the n positive values of y_i the corresponding quantities to be combined -- or vice versa. The indexes discussed in the next section will be composed of similar aggregates. In the chapters which follow, we shall permit some of the weights to be negative, when we consider certain net production measures; in such instances, the positive weights are associated with gross output quantities and the negative weights with quantities of materials, fuel, etc. to be deducted. Aggregates with only positive weights are relevant to the construction of indexes of gross output, indexes of net output based either on gross quantities and partial (e.g., unit value added or unit labor added) weights or on subproduct quantities, indexes of composite input, "direct" indexes of unit labor requirements and productivity, etc. The weights may refer to the same period as the quantities being combined, as in the Paasche index, or to some fixed "base" period, as in the Laspeyres index; or to some other period, combination of periods, or none at all.

Secondary Units

Two little known facts about aggregates have important bearing on the numerical interpretation of indexes. First, even as the distinctive property of a product or factor class is size with respect to a characteristic primary unit, the weighted aggregate itself is a multiple of an implicit characteristic secondary or composite unit. Second, the aggregate output and aggregate input of a period may both be expressed by the same weighted sum, but this sum represents different multiples of the secondary output unit and of the secondary input unit.

Consider a weighted aggregate, $\Sigma x_o y_o$, where the x_{oi} represent unit value added or the unit labor requirement in the period t_o of the corresponding t_o product quantities, y_{oi}. The dimension of the weighted aggregate is value added or labor consumed (e.g., man-hours). We define a secondary unit of output as $\Sigma x_o \cdot 1$, the value added or labor corresponding to an output complex including one primary unit of each product (i.e., $y_{oi} = 1$). Similarly, a secondary unit of the weighting factor may be defined as $\Sigma 1 \cdot y_o$, the value added or labor quantity corresponding to the hypothetical case in which the output of each product has unit weight (e.g., one dollar of value added or one man-hour of labor). If the weighted sum, however, is to represent "itself" (i.e., value added or man-hours), then we define the composite unit as $\Sigma 1 \cdot 1 = n$. Although the weighted aggregate is constant in its three guises, it represents different multiples of the three secondary units -- i.e., $\Sigma x_o y_o / \Sigma x_o \cdot 1 \neq \Sigma x_o y_o / \Sigma 1 \cdot y_o \neq \Sigma x_o y_o / \Sigma 1.1$. These multiples are clearly "averages" of the product quantities, the weights, and the total value added or labor consumed -- but all the denominators are of the same dimension as the numerators (e.g., all refer to value added or man-hours). To suggest the identity of dimension, as a consequence of which all the multiples are pure numbers (i.e., the units of the numerators and denominators cancel), we retain the superfluous "1" in the denominators.

Let us examine the case in which the same money aggregate corresponds, by definition, to total value added and to total factor rewards -- $\Sigma v_o q_o = \Sigma w_o f_o$, where the v_{oi} and the q_{oi} represent unit value added weights and the corresponding output quantities in t_o, while the w_{oi} and f_{oi} represent rewards per unit of factor input and the corresponding factor quantities. Here, despite the identity of the totals, we have two different multiples, since the secondary output unit differs from the secondary input unit. That is, $\Sigma v_o q_o / \Sigma v_o \cdot 1 \neq \Sigma w_o f_o / \Sigma w_o \cdot 1$.

It should be obvious that a change in classification of products or inputs or a change in primary units alters the number of corresponding secondary units contained in a given weighted aggregate. The reason is that the numerator and denominator are not symmetrically affected. In the numerator, a change in a quantity is compensated by a change in its weight; but, in the denominator, only the weight is modified while the primary unit (hence the secondary unit, too) is simply redefined. Symbolically, we have $\Sigma x_o y_o / \Sigma x_o \cdot 1$ as the number of secondary units contained in the weighted aggregate at the outset. Changing the primary units from x_{oi} to $x_{oi} k_{oi}$, we must correspondingly alter y_{oi} to y_{oi}/k_{oi}. Though the weighted aggregate is invariant to a change in primary units, the number of new secondary units it now represents is $\Sigma x_0y_0/\Sigma x_0k_0 \cdot 1$ ($\neq \Sigma x_0y_0/\Sigma x_0 \cdot 1$). It will be maintained later in this chapter that a technically satisfactory aggregative output index should have the same product classification in both numerator and denominator. That is, both aggregates should be measured in the same implicit secondary unit. If this criterion is extended to the case in which some output entries are zero, we have a rationalization of the "free composition" index, which is suggested as an alternative to the chain measure in Chapter IV.

<u>Effect of Change in Weights</u>

Suppose two different weighting patterns are applied to a given output complex and that the corresponding aggregates are Σxy and $\Sigma x'y$. Under what circumstances would the number of secondary units corresponding to one weighting system exceed, equal, or fall short of the number corresponding to the other weighting system? The two multiples are $Y_x = \Sigma xy/\Sigma x \cdot 1$ and $Y_{x'} = \Sigma x'y/\Sigma x' \cdot 1$. Now, the writer has shown elsewhere[1] that the sign of the

[1] I. H. Siegel, "Further Notes on the Difference between Index-Number Formulas", <u>Journal of the American Statistical Association</u>, XXXVI (December, 1941), 519-24, and "Note on a Common Statistical Inequality", <u>ibid</u>., XXXVIII (June, 1943), 217-22. Other types of expressions are shown in "The Difference between the Paasche and Laspeyres Index-Number Formulas", <u>ibid</u>., XXXVI (September, 1941), 343-50. It should be noted that the same methods of analyzing index-number differences are applicable to differences between secondary-unit multiples.

difference $Y_{x'} - Y_x$ is given by the sign of a double sum composed of ${}_nC_2$ elements,

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} x_i x_j (x'_i/x_i - x'_j/x_j)(y_i - y_j), \quad i \neq j;$$

or by the sign of the von Bortkiewicz weighted correlation coefficient $r_{x:(x'/x)\cdot y}$. That is, $Y_{x'}$ exceeds, equals, or falls short of Y_x according as the x_i-weighted coefficient of correlation between the ratios of the weights, x'_i/x_i, and the output quantities, y_i, is positive, zero, or negative. The double sum snows that, if the Spearman coefficient of rank correlation between the x'_i/x_i and the y_i is $+1$, then $Y_{x'} > Y_x$, necessarily; if -1, then $Y_{x'} < Y_x$, necessarily. These and other critical expressions may be derived by elementary vector or matrix methods, involving the generalized Lagrange identity, or by straightforward, if tedious, algebra.

Suppose that all the quantities are combined with weights in one case and with no (really, equal) weights in another. In terms of secondary units, we have $\Sigma xy/\Sigma x\cdot 1$ for the weighted aggregate and $\Sigma 1\cdot y/\Sigma 1\cdot 1 = \Sigma y/n$ for the unweighted aggregate. Now, the former multiple is greater than, equal to, or less than the latter according as the ordinary Pearsonian correlation coefficient, r_{xy}, is positive, zero, or negative. The critical double sum is

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (x_i - x_j)(y_i - y_j), \quad i \neq j,$$

which shows that the weighted sum must represent a larger multiple when the ranks of the quantities and the weights are identical and a smaller multiple when the ranks are inverse.

On Indexes

In accordance with the introductory remarks to the discussion of aggregates, we shall consider in this section ratios of aggregates of the form $\Sigma xy'/\Sigma xy$, where the weights are positive only.

Indexes and Secondary Units

An aggregative index is technically most satisfactory when the numerator and denominator are both multiples of the same secondary unit, when they are both "absolute" quantities determined with respect to a common scale. That is, the sum $\Sigma x \cdot 1$ should have meaning with reference to both of the aggregates being compared. The product classification, primary units, and weights should remain constant between the two periods, even if some of the quantities are zero in one of these periods. Output and input indexes are technically less satisfactory if the product or input composition is permitted to change, if the two weighted aggregates are deemed comparable simply because they are expressed in a common denominator like dollars or man-hours. Further comment will be made on these matters in the treatment of deflation below and of "free-composition" aggregative indexes (the "frame" of which remains rigid through time even though there are some null quantities) in the next chapter.

It has already been observed that the same aggregate may be regarded as a multiple of different secondary units, according to its meaning, and that these multiples are like "averages", except for dimension. It may now be shown that the numerator and denominator are replaceable by the products of multiples corresponding to "averages" of the weights and of the quantities being combined. Thus, we may write

$$\frac{\Sigma xy'}{\Sigma xy} = \frac{X \cdot Y'}{X \cdot Y} = \frac{\dfrac{\Sigma xy}{\Sigma 1 \cdot y} \; \dfrac{\Sigma xy'}{\Sigma x \cdot 1}}{\dfrac{\Sigma xy}{\Sigma 1 \cdot y} \; \dfrac{\Sigma xy}{\Sigma x \cdot 1}}$$

Indexes and Deflation

Every aggregative index may be written as the quotient of two others. Thus, an index of the Laspeyres type, $\Sigma x_0 y_1 / \Sigma x_0 y_0$, may be rewritten as a quotient, $(\Sigma x_0 y_1 / \Sigma x_1 y_1) \,/\, (\Sigma x_0 y_0 / \Sigma x_1 y_1) = (\Sigma x_1 y_1 / \Sigma x_0 y_0) \,/\, (\Sigma x_1 y_1 / \Sigma x_0 y_1)$. If the original index is a labor-weighted measure of output, the quotient is the ratio between the total employment index and the Paasche unit labor requirement index. If the original index is a price-weighted measure of output, the quotient is the ratio of the gross value index to the Paasche index of price. If the original index were of the Paasche type, the numerator would remain the same, but the deflator would become a Laspeyres measure.

The Laspeyres output index may also be written as

$$\Sigma \left[(x_1 / X_{10:1}) \, y_1 \right] / \Sigma x_0 y_0 \;=\; \Sigma x_1 y_1 / \Sigma \left[(x_0 X_{10:1}) \, y_0 \right],$$

where $X_{10:1}$ is the Paasche deflator. Similarly, the Paasche output index is equivalent to

$$\Sigma \left[(x_1 / X_{10:0}) \, y_1 \right] / \Sigma x_0 y_0 \;=\; \Sigma x_1 y_1 / \Sigma \left[(x_0 X_{10:0}) \, y_0 \right],$$

where $X_{10:0}$ is the Laspeyres deflator. Thus, if the original indexes are weighted by prices, they may be interpreted as indexes of <u>gross value</u> in which the <u>absolute</u> prices of either t_1 or t_0 are <u>adjusted</u> by a flat percentage to the level of one of the periods while the <u>relative</u> prices in both periods are <u>unaffected</u>. The uniform "effect" of a change in "price level" is supposedly "removed". If the original indexes are output-weighted measures of change in unit labor requirements, they may similarly be interpreted as labor input measures adjusted by a flat percentage to the "constant" output level of either t_0 or t_1.

Let the formula $\Sigma qr' / \Sigma qr$ represent a unit labor requirement index with output weights. Its reciprocal, $\Sigma qr / \Sigma qr'$, is a productivity index of the type developed by the W. P. A. National Research Project and preferred by the U. S.

Bureau of Labor Statistics.[2] If the unit labor requirement index is of the Paasche type, the productivity index may be written more specifically as $\Sigma q_1 r_0/\Sigma q_1 r_1$. Now, this ratio may be regarded as an elliptical form. The numerator may be considered to represent output in t_1, the secondary unit in this case being $\Sigma 1 \cdot r_0$; and the denominator may be regarded as signifying labor input in t_1, the secondary unit being $\Sigma 1 \cdot 1$. We may then imagine that this ratio is divided by another, which describes output per unit of labor input in t_0 and has the value unity. This divisor must be $\Sigma q_0 r_0/\Sigma q_0 r_0 = 1$; the secondary unit of its numerator is also $\Sigma 1 \cdot r_0$, but that of its denominator is $\Sigma 1 \cdot 1$. Introducing this divisor and rearranging terms, we get $(\Sigma q_1 r_0/\Sigma q_0 r_0)/(\Sigma q_1 r_1/\Sigma q_0 r_0)$, the ratio of the Laspeyres output index to the labor input index. If we start with the Laspeyres productivity index, $\Sigma q_0 r_0/\Sigma q_0 r_1$, and interpret it as an elliptical form, the numerator and denominator must refer to t_0. Consequently, the ratio of output to input in t_1 must be unity, of the form $\Sigma q_1 r_1/\Sigma q_1 r_1$; and, in the original index, $\Sigma q_0 r_0$ must represent t_0 labor input while $\Sigma q_0 r_1$ signifies t_0 output. Thus, we have

$$\frac{\Sigma q_0 r_0}{\Sigma q_0 r_1} = 1 \Big/ \frac{\Sigma q_0 r_1}{\Sigma q_0 r_0} = \frac{\Sigma q_1 r_1}{\Sigma q_1 r_1} \Big/ \frac{\Sigma q_0 r_1}{\Sigma q_0 r_0} = \frac{\Sigma q_1 r_1}{\Sigma q_0 r_1} \Big/ \frac{\Sigma q_1 r_1}{\Sigma q_0 r_0},$$

where the numerator is the Paasche labor-weighted output index and the denominator is the labor input index.

It is frequently overlooked that indexes derived by deflation ought ideally to meet the standards applied to indexes which are computable directly. The measure that is sought must be possible in the first place; and the intermediate indexes used to approximate it must be appropriate in form and scope. If an output index cannot be visualized because product classes sufficiently

[2] See Magdoff, Siegel, and Davis, op. cit., I, chap. 1; and W. D. Evans and I. H. Siegel, "The Meaning of Productivity Indexes", Journal of the American Statistical Association, XXXVII (March, 1942), 103-11.

homogeneous through time cannot be established, then it cannot be derived by deflation; whether or not the deflator is based on a sample of homogeneous products is irrelevant. If the symbols in the numerator and denominator of the desired index cannot be made to refer to the same product classes, primary units, and weights, then deflation is to no avail and yields an output measure with a different meaning from the one that is sought. Thus, estimates of "real" national product derived by deflation must remain obscure so long as this concept cannot be conceived as a "heap" of goods and services in "physical" terms -- so long as the output of government and the various service industries cannot be quantified in characteristic primary units, so long as homogeneous product classes cannot be established through time for all industries, etc. Furthermore, there is no assurance that satisfactory estimates of net product are obtained when gross price measures are used as deflators. Finally, attempts like those made by the Federal Reserve Board to derive output indexes for machinery and other manufacturing industries with heterogeneous end products by the application of spurious productivity indexes to man-hours indexes can only lead to nebulous results. If an output index cannot be specified directly, then its correlative productivity index cannot be conceived either. Clarity cannot be achieved by substituting one vague notion for another, by simply ignoring a problem which cannot be solved, by resort to mere verbal algebra.

There is a universal tendency to mislabel an index derived by deflation of a value index by a price measure as an output index expressed in "constant t_0 dollars". Apart from the difficulties mentioned above, such a result cannot be derived unless the deflator is of the Paasche type. But, more likely than not, the deflator is of the Laspeyres type with base t_0, in which case the result is a Paasche index expressed in changing t_1 dollars; or of the Laspeyres type with base other than t_0, in which case even the Paasche index is not derived; or of the chain type, which is itself ambiguous. It is also a mistake to assume that a mere shift in the time base of a deflator or of the derived index changes the weight base, whatever the index formula. In the simple case of deflation by an index with fixed weights of period t_c ($c \neq 1, 0$), the result (if the two indexes are comparable in scope, classification, and units) is

$$(\Sigma x_1y_1/\Sigma x_0y_0) \,/\, (\Sigma x_1y_c/\Sigma x_0y_c) = (\Sigma x_1y_1/\Sigma x_1y_c)(\Sigma x_0y_c \,/\, \Sigma x_0y_0),$$

which does not meet the circular test and hence does not necessarily reduce to an internal mean of the quantity relatives, y_{1i}/y_{0i}.

<u>The Relation between Alternative Indexes</u>

The algebraic difference between aggregative indexes with alternative weighting schemes is a "classical" problem which has interested many economists and statisticians. Although such indexes often are in sufficient agreement to make some observers regard the problem as unimportant, there also are striking instances of divergence; and, in such instances, there is no comfort in knowing that a greater degree of consilience is normal. It may be shown that the difference between the Paasche and Laspeyres indexes is given by the sign of the von Bortkiewicz x_0y_0-weighted coefficient of correlation between the relatives x_{1i}/x_{0i} and y_{1i}/y_{0i}. That is, the Paasche index is greater or less than the Laspeyres according as this coefficient is positive or negative. Incidentally, this coefficient also makes it clear that if, say, a Paasche price-weighted output index exceeds the Laspeyres, then the corresponding Paasche output-weighted price index must also exceed the corresponding Laspeyres. The double-sum development, to which reference has already been made, shows that the Paasche index necessarily exceeds the Laspeyres if the Spearman rank coefficient of correlation between the two sets of relatives has the value +1 and necessarily falls below the Laspeyres if this coefficient has the value -1. Many other expressions have been derived in the discussion of index-number differences. In general, it may be shown that, if one index has x'_i weights and the other has x_i weights, then the former exceeds, equals, or falls below the latter according as $r_{xy_0:(x'/x)(y_1/y_0)}$ is positive, zero, or negative.

Of special interest is the relation between the "direct" labor productivity index -- say, the Paasche measure, $\Sigma q_1 r_0 / \Sigma q_1 r_1$ -- and the usual sort of approximation to which it corresponds, $(\Sigma p_0 q_1 / \Sigma p_0 q_0)/(\Sigma q_1 r_1 / \Sigma q_0 r_0)$. Here, the r_{1i} and r_{0i} represent unit labor requirements of t_1 and t_0, respectively, and the q_{1i} and q_{0i} refer to the corresponding output quantities, while the p_{0i} are price weights. We assume, as seldom actually happens, that the output and labor input indexes of the approximation are identical in scope; and that the direct productivity index also covers the same product classes (as though unit labor requirements could be meaningfully determined in a multiproduct establishment). If the p_{0i} and r_{0i} were proportional, the productivity index and its approximation would be equal, but such proportionality cannot be taken for granted. In general, the approximation may be shown to be greater than, equal to, or less than the direct Paasche index according as the weighted correlation coefficient $r_{q_0 r_0:(p_0/r_0)(q_1/q_0)}$ is positive, zero, or negative. In some circumstances, the approximation may even fall outside the range of the individual productivity relatives.[3]

Similar remarks may be made about the relationship between the direct Laspeyres productivity index and an approximation involving a Paasche price-weighted output index. In this case, the approximation exceeds or falls below the direct index according as $r_{r_1 q_0:(p_1/r_1)(q_1/q_0)}$ is positive or negative.

3 An example of externality, referring to cigars and cigarettes manufacture, is cited by Magdoff, Siegel, and Davis, op. cit., I, 7.

It is also instructive to write the productivity approximation in the form of a product of two indexes, one representing the direct productivity measure and the other showing the shift in the structure of labor-weighted output. Thus, the approximation involving a Laspeyres output index may be written as

$$\frac{\sum q_1 r_0}{\sum q_1 r_1} \cdot \frac{\sum \frac{p_0}{r_0} \cdot \frac{q_1 r_0}{\sum q_1 r_0}}{\sum \frac{p_0}{r_0} \cdot \frac{q_0 r_0}{\sum q_0 r_0}} \; .$$

It is also possible to write this approximation as the product of the direct Laspeyres productivity index and a different index of the shift in structure of labor-weighted output. If the approximation is based on a Paasche production index, its relationship to the direct productivity measure is clearly indicated when it is put in the form

$$\frac{\sum q_0 r_0}{\sum q_0 r_1} \cdot \frac{\sum \frac{p_1}{r_1} \cdot \frac{q_1 r_1}{\sum q_1 r_1}}{\sum \frac{p_1}{r_1} \cdot \frac{q_0 r_1}{\sum q_0 r_1}} \; .$$

Again, it is possible to write an equivalent expression in which the direct productivity index is of the Paasche type.

CHAPTER IV

GROSS AND NET PRODUCTION INDEXES AND ADJUSTMENTS FOR COVERAGE AND CONTINUITY

In the first half of this chapter, we present algebraic formulations of gross and net output measures and observe their relationships. We then proceed to a discussion of techniques intended to overcome difficulties created by the incompleteness of coverage of quantity statistics and by discontinuities of product series. Particular attention is directed to the Mills-Fabricant and similar coverage adjustments, to chain indexes, and to "free composition" aggregative indexes.

Gross and Net Production Indexes

As in the last chapter, we consider aggregative indexes of the form $\Sigma xy'/\Sigma xy$, but we shall allow these symbols to have a broader meaning. To facilitate discussion, we shall designate by G the gross production index, the Laspeyres form of which is $\Sigma p_0 q_1/\Sigma p_0 q_0$, where the weights, p_{0i}, refer to unit prices and the product quantities, q_{1i} and q_{0i}, refer to end products of an establishment or industry. When some of these product quantities equal zero in t_0 or t_1, we shall call the measure a "free composition" aggregative index. If we follow the practice of Hicks, Tintner, and other expositors of the economic theory of production, we may let the expression for G also stand for a net output measure, N, in which some of the quantities represent consumed materials, fuel, etc. and have negative weights. Instead of regarding the consumed items as "negative products", however, we shall follow Fabricant[1] and write $N = (\Sigma p_0 q_1 - \Sigma P_0 Q_1)/(\Sigma p_0 q_0 - \Sigma P_0 Q_0)$ for the Laspeyres form, where the small letters refer to the quantities and prices of end products and the capitals refer to consumed materials, fuel, etc. In this expression, the same item may appear both in the form of q_{1i} (or q_{oi}) and of Q_{1i} (or Q_{oi}). The upper limit of the summation for products differs from the limit for materials, etc., since several materials, etc. enter each product and any one (e.g., coal) may be common to several products. We shall use the symbol M to designate the index of materials, etc. weighted by the corresponding prices, P_{1i} or P_{oi}. Another net product index of interest is based on end products and partial weights. When it incorporates unit value added weights, we shall call this measure V; in its Laspeyres form, it may be written as $\Sigma v_0 q_1 / \Sigma v_0 q_0$. The corresponding output index with labor weights was considered in the last chapter and will receive further attention in the next. Some remarks will also be made here on two other net product indexes which are analogous to N and V but are based on subproducts, rather than end products, and hence tend to reflect the contribution to output on a true accrual basis.

Relationships between Gross and Net Output Indexes

The algebraic conditions which determine the relative magnitudes of a gross output index and various types of net measures are of interest since the former, being easier to compute, is often substituted for the latter, which are generally preferred. It should also be useful to know that some formulas yield results which necessarily lie within the range of certain relatives while other formulas may yield external means of such relatives.

[1] The Output of Manufacturing Industries, chap. 2. In the U. N. report, Index Numbers of Industrial Production, pp. 21-22, the same formula is attributed to Geary, who actually presented a variant measure -- and four years after Fabricant.

First, let us consider the relationship between a gross industry index, G, and the net index, N, for which it is generally substituted. Fabricant has already observed that these two measures are equal if G and M are equal.[2] Geary's algebra suggests that $G < N$ if $G > M$.[3] It is easy to generalize these findings and say that N exceeds, equals, or falls below G according as G exceeds, equals, or falls below M.

Rewriting the formulas for N and G, we reach additional conclusions of interest. If N, G, and M are of Laspeyres form, we find that

$$N = (\Sigma p_o q_o G - \Sigma P_o Q_o M) / (\Sigma p_o q_o - \Sigma P_o Q_o)$$

and

$$G = [(\Sigma p_o q_o - \Sigma P_o Q_o)N + \Sigma P_o Q_o M] / [(\Sigma p_o q_o - \Sigma P_o Q_o) + \Sigma P_o Q_o].$$

The denominator of the second expression, for G, is equal, of course, to $\Sigma p_o q_o$, but we retain the expanded form because it shows that G may be interpreted as a weighted internal mean of N and M -- provided that net output in t_o exceeds zero ($\Sigma p_o q_o > \Sigma P_o Q_o$), as is usually the case. Hence, inequalities other than $N \gtreqless G \gtreqless M$ are ruled out.

Turning to the new expression given above for N, we observe that it could be external to G and M, or even lie outside the range of the relatives q_{1i}/q_{oi} and Q_{1i}/Q_{oi}. Indeed, the expression for N is analogous to that derived in analytical geometry for the x or y coordinate of an external point dividing a line segment in a given ratio, $(\Sigma p_o q_o - \Sigma P_o Q_o) : \Sigma P_o Q_o$. It is also identical with the formula derived in physics for the new center of gravity of a body of known original mass from which a part of known mass has been excised. The

[2] *Ibid.*, pp. 27-29.

[3] *Loc. cit.*, p. 257.

new center (N) often lies outside the line segment connecting the original center (G) and the center of the excised part (M), which has "negative mass".[4]

Although gross output measures must be used in lieu of net for individual manufacturing industries, it is possible to restore some "netness" to industry combinations by the use of partial weights. Let us consider the difference between a composite of gross industry indexes of the Laspeyres type, G_i, with corresponding value of product weights, T'_{oi}, and a composite with industry value added weights, T_{oi}. By methods mentioned in Chapter III, it may be shown that the former exceeds, equals, or falls below the latter according as the weighted correlation coefficient, $r_{T:(T'/T)\cdot G}$, is positive, zero, or negative. If the ranks of the gross indexes and of the ratios of gross value to value added in t_o correspond perfectly, then the composite index with gross value weights exceeds the index with net weights. If the ranks are perfectly inverse, then the composite with gross value weights is necessarily smaller.

What is the nature of the difference between a net output index for a combination of industries based on N_j and the composite measure based on G_j and value added weights, which has just been discussed? (We use the industry subscript j simply to prevent confusion in the comments which follow, where the subscript i is reserved for intra-industry output and consumption.) Again using Laspeyres formulas, we write the difference

$$\frac{\Sigma(\Sigma p_o q_1 - \Sigma P_o Q_1)}{\Sigma(\Sigma p_o q_o - \Sigma P_o Q_o)} - \frac{\Sigma[\Sigma v_o q_o(\Sigma p_o q_1/\Sigma p_o q_o)]}{\Sigma(\Sigma v_o q_o)},$$

where Σ outside parentheses denotes summation over industries and Σ inside parentheses denotes summation within an industry. The sign of this difference obviously depends on the numerators only, since the denominators are both equal

[4] On the mentioned mathematical and physical analogies, see, for example, C. Smith, An Elementary Treatise on Conic Sections (New York: Macmillan & Co., 1892), pp. 4-5, and E. R. Hedrick and O. D. Kellogg, Applications of the Calculus to Mechanics (Boston: Ginn & Co., 1909), pp. 35-36.

by definition. A further simplification may be introduced by conversion of the sums for consumed materials, etc. to a product basis. Thus, we write $\Sigma P_oQ_o = \Sigma(SP_oQ_o)$ and $\Sigma P_oQ_o = \Sigma(SP_oQ_o)$, where the $(SP_oQ_1)_i$ and $(SP_oQ_o)_i$ are sums which correspond exactly in scope to the q_{1i} and q_{oi}, respectively. Since the v_{oi} may also be rewritten in terms of the p_{oi} and $(SP_oQ_o)_i$, we conclude that the sign of the difference depends on

$$\Sigma\left[\Sigma q_1\left(p_o - \frac{SP_oQ_1}{q_1}\right)\right] - \Sigma\left[\Sigma q_o\left(p_o - \frac{SP_oQ_o}{q_o}\right)\frac{\Sigma p_oq_1}{\Sigma p_oq_o}\right]$$

$$= \Sigma\left[\Sigma (SP_oQ_o)\left(G - \frac{SP_oQ_1}{SP_oQ_o}\right)\right]$$

$$= \Sigma\left[\Sigma P_oQ_o(G - M)\right].$$

Thus, if $G_j = M_j$, the two composite measures are equal, as is obvious also from the earlier discussion. More generally, the composite based on N_j exceeds (falls below) the approximation based on the G_j if the G_j exceed (fall below) the corresponding M_j.

If N and the net output approximation with unit value added weights, V, could actually be computed for individual industries, significant divergences would sometimes be observed. For the former is not necessarily an internal mean of the quantity relatives, q_{1i} / q_{oi}, while V would typically be an internal mean (since value added weights are seldom predominantly negative). If we observe that N is factorable into V and some sort of price-margin index, so that in the Laspeyres case we have

$$N = \frac{\Sigma q_1\left(p_o - \dfrac{SP_oQ_1}{q_1}\right)}{\Sigma q_1\left(p_o - \dfrac{SP_oQ_o}{q_o}\right)} \cdot V,$$

it is evident that, even when all the q_{1i} / q_{oi} equal unity, N and V may still diverge since changes occur in the "coefficients of fabrication" (consumption of materials, etc. per unit of output) from t_o to t_1.

What has been said about the relationship between N and V may also be said about the corresponding subproduct indexes, which we may designate N' and V'. The output quantities of the latter two refer, not to end products, but to the fairly homogeneous results of component, repetitively performed, activities. The other symbols must also be reinterpreted so that the formulas show the output of each period on an accrual basis. It is obvious that a subproduct index based on "vertical" components of end products need not be an internal mean of end-product relatives, whether the formula N' or V' is used. But V and V' are related through the fact that both lead to the same net value index when multiplied by corresponding price-margin indexes; that is, the ratio of Laspeyres (or Paasche) indexes, V:V', equals the reciprocal of the ratio of the corresponding Paasche (or Laspeyres) price-margin indexes. A similar identity holds for N:N'. Finally, the ratio N':V' may be analyzed as on the preceding page; the indicated price-margin index, suitably reinterpreted, is the ratio of two others corresponding to V' and N'.

Alternative Expressions for Output Indexes

A number of variant expressions shed additional light on the character of gross and net output measures. In the last chapter, it was noted that aggregative indexes may also be written as weighted arithmetic or harmonic means of relatives, and that they may also be interpreted in terms of deflation. Some other alternative forms will now be cited.

A gross output index may readily be converted to an average of end-product indexes of the same form, but with partial price weights. Thus, if G is a Laspeyres index with price weights, then it is equivalent to

$$G = \frac{a\,G_m + b\,G_w + \dots}{a + b + \dots},$$

where G_m, G_w, etc. are Laspeyres indexes of end products with material cost weights, labor cost weights, etc.; where a, b, etc. represent total material costs, total wage costs, etc. in t_0; and where $a + b + \dots = \Sigma p_0 q_0$, gross output value. If price is broken into but two components, one of which is unit value added, then G is an average of V and another end-product index with residual weights. It is instructive to compare this average with the expression for G in terms of N and M given earlier in this chapter. The weights a and b associated with N and M are the same as those associated with V and the companion index of end products; the substitution of N for V is paralleled by the substitution of M for the companion end-product index, the internal weights of which refer to cost of materials, etc. per unit of output.

There is an alternative form of the net product index which is of particular interest on the national level. The net output may be divided into two parts, one comprising goods (and services) which are "finished" from the standpoint of the economy taken as a unit and the other representing the net change in inventory of "unfinished" items. Thus, we write for the Laspeyres form of N

$$\frac{\Sigma p_0 q_1 - \Sigma P_0 Q_1}{\Sigma p_0 q_0 - \Sigma P_0 Q_0} = \frac{(\sum_F p_0 q_1 + \sum_U p_0 q_1) - \Sigma P_0 Q_1}{(\sum_F p_0 q_0 + \sum_U p_0 q_0) - \Sigma P_0 Q_0} = \frac{\sum_F p_0 q_1 + (\sum_U p_0 q_1 - \Sigma P_0 Q_1)}{\sum_F p_0 q_0 + (\sum_U p_0 q_0 - \Sigma P_0 Q_0)},$$

where the symbol F denotes services rendered consumers and good completed during the period and U refers to goods in process and not yet available for consumption at the end of the period. The sum for materials, etc. includes the initial inventory of uncompleted goods and of materials, etc. plus any intraperiod acquisitions from outside (e.g., imports) entering the production process during the period. In the discussion of welfare, there is a tendency to regard $\sum_F p_0 q_1 / \sum_F p_0 q_0$, which refers to a "heap" of final goods and services, as an appropriate index of changes in real income. Sometimes, the inclusion of capital goods is also assumed, as though these, too, were "finished". In any case, it must not be concluded that the net inventory change within each of the compared periods is negligible or constantly proportional to the weighted "finished" output. Hence, if all other conditions of measurability were fulfilled, it might still be inappropriate to use an ordinary price deflator in lieu of the true price-margin index. The Paasche form of the latter, which would yield the Laspeyres N, is

$$\frac{T_1/T_0}{N} = \frac{\sum_F p_1 q_1 + (\sum_U p_1 q_1 - \Sigma P_1 Q_1)}{\sum_F p_0 q_1 + (\sum_U p_0 q_1 - \Sigma P_0 Q_1)} = \frac{\Sigma p_1 q_1 - \Sigma P_1 Q_1}{\Sigma p_0 q_1 - \Sigma P_0 Q_1}.$$

One more variant of N might be mentioned. Like G and V, it, too, may be written as a weighted average of indexes of the same form as itself but incorporating partial price weights. In the Laspeyres case, the weights incorporated in the averaged indexes refer to components of the p_{oi} and P_{oi}, and the quantities of output and of materials, etc. are as in the original N. The weights used in averaging these indexes to form N add up to net output of t_o.

Coverage Adjustments

A troublesome point in the construction of production measures for manufacturing industries and industry groups is the unavailability of quantity data for the entire area intended to be covered. The reported quantity series usually relate to established products; the newer or less important ones and custom, contract, and repair work are represented in the value statistics only. Though the reported quantities usually account for a substantial part of the gross value of an industry's output, their share in this total fluctuates or even declines through time.[5] Thus, the established products do not necessarily constitute a "representative" sample for the purpose of index construction. Some sort of adjustment seems desirable; and, when none is made, it is implied, if not assumed, that the measure based on the available quantity data and the analogous index for the omitted products have identical movements.

In adjusting for defective coverage, there is, of course, no substitute for knowledge of each case. Any general "across-the-board" adjustment, sound though it may appear on <u>a priori</u> or probabilistic grounds, will not be satisfactory for all cases and may occasionally result in further distortion, rather than correction, of an unadjusted measure. When the adjustment has little effect, when it could just as well have been omitted, its validity is not likely to be challenged. But, when it significantly alters the magnitude or general course of the original incomplete index, the adjustment ought, perhaps, to reinforce doubts concerning the original measure rather than engender confidence in the revised one.

[5] The value adjustments actually made for individual industries are intended to accomplish two purposes -- the elimination of the output of characteristic products made elsewhere and the representation of unreported products made within an industry.

All conceivable adjustment factors are intended to satisfy this equation:

Adjusted output index = unadjusted output index x adjustment factor.

All other formulations of the adjustment process, which often seem cumbersome, reduce to this self-evident statement. Admissible factors would consequently seem to be limited to ratios in which the numerators refer to some measure comprehending the entire industry or industry group in question and in which the denominators are restricted to the scope of the unadjusted quantity index. In the case of an industry, the only generally available factor is the ratio of the gross value index for the entire industry to the gross value index of the products in the unadjusted measure. In the case of an industry group, a similar ratio based on value added, employment, payrolls, or cost of materials, etc. may be computed. For some industries, it may be possible to use ratios referring to unweighted output, the consumption of a principal raw material, machinery hours, etc. But the most widely used adjustment factors, associated in this country with the names of F. C. Mills and S. Fabricant and in the United Kingdom with the name of E. Devons, are based on value or value added, and these will be discussed here in some detail.[6]

The value adjustment is completely effectual if the adjusted index equals the true expanded measure -- that is, if the true index stands in the same ratio to the unadjusted index as the total value index to the covered value index. The ratio of value indexes, of course, is the adjustment factor. Now, if the unadjusted index is of the Laspeyres (Paasche) form, then the adjusted index equals the expanded Laspeyres (Paasche) measure in the event that the

[6] F. C. Mills, Economic Tendencies in the United States (New York: National Bureau of Economic Research, 1932), pp. 90, 92-93; S. Fabricant, *op. cit.*, pp. 362-63; and E. Devons, "Production Trends in the United Kingdom", The Manchester School, X (January, 1939), 55-61.

Paasche (Laspeyres) price index for the total area to be covered is the same as the Paasche (Laspeyres) price index for the area actually covered. But is such agreement between the price indexes of different scope likely to occur? If there were no reason to believe that unreported products were economically different from reported ones, then the assumption of equivalence might readily be granted. It seems to be agreed that price series show less dispersion through time than the corresponding production series. As A. F. Burns noted, all prices are subject to the influence of common monetary factors, but all production series are not affected by any "single dominant force".[7] In explaining his preference for the value adjustment in his 1940 study of manufacturing output, Fabricant also called attention to the fact that prices move within a narrower range than production quantities. But he also had some misgivings; he adopted the value adjustment for his individual industry indexes because, "in the absence of specific and detailed knowledge", the underlying assumptions seemed "least objectionable".[8] More recently, Fabricant has reiterated his preference for adjusted over unadjusted output indexes, but he warns those interested in deriving productivity measures that "in any particular case, we cannot be certain which index is more accurate, nor that the truth lies between the two".[9] When the U. S. Bureau of the Census presented its 1939-47 measures, it designated the value adjusted Edgeworth indexes as "official" and buttressed its choice with tests of an admittedly "suggestive

[7] Production Trends, pp. 260-61.

[8] Op. cit., p. 364.

[9] "Of Productivity Statistics: An Admonition", Review of Economics and Statistics, XXXI (November, 1949), 310.

rather than conclusive" nature.[10]

Some industry index makers have been reluctant to accept the value adjustment as routine. The adjustment was not made in the 1939 W. P. A. National Research Project study of manufacturing trends though it had been considered. U. S. Bureau of Labor Statistics, revising and extending the W. P. A. indexes in 1942, also rejected the general value adjustment since it "may often fail to accomplish its purpose".[11] Neither agency, however, was satisfied with unadjusted indexes but preferred not to introduce an additional assumption. In any case, they did use some techniques which are based on the same kind of assumption. Indeed, they tolerated the strongest possible form of the assumption in using price weights in lieu of unavailable but preferred unit labor requirement weights.

A reserved attitude toward the value adjustment still seems warranted. The Census tests do not get to the root of the problem, since they all involve manipulation of quantity series which <u>are</u> available. After all, the price characteristics of the unreported products could well differ from those for available products. Consider, for example, the case of new products, which are not at first reported explicitly by quantity but which are included in the total value of output. The value adjustment, in all likelihood, takes improper account of such products and leads to an understatement; for the prices of new products tend to fall in relation to the prices of established products, so that the price index of the former would be lower than the price index of

10 The U. S. Census Index, pp. 2, 7-9.

11 U. S. Bureau of Labor Statistics, Productivity and Unit Labor Cost in Selected Manufacturing Industries: 1919-40 (processed, Washington, 1942), vii.

the latter over the same interval. Even in the case of new products, as our example below shows, the unadjusted measure could be more correct than the adjusted one. Experience suggests that other kinds of unreported products -- those in the process of displacement, custom goods, goods made on contract, parts, etc. -- may also fail to conform to the price pattern of the standard goods which tend to be reported. Finally, when sufficiently detailed data permit the making of more refined value adjustments than usual, alternative approaches do not always agree; and this fact itself should suggest a cautious acceptance of the adjustments that can usually be made. Thus, the adjusted 1947 index for agricultural machinery on the base 1939 was found by the Census Bureau to be 341, compared to the unadjusted figure of 261. The writer noted that the opportunity to make a detailed coverage adjustment by type of product was overlooked; this more refined method gives the result 301.

To illustrate the dangers which lurk in making coverage adjustments, we offer a hypothetical example which is not too far fetched. An industry makes three products in each of two years, but the prices and quantities are known for only two of the products. The third product is "new", though made in both years; its quantities and prices are not reported but the value of its output is. The price and quantity data for the two established products are shown below, followed by the unknown detailed statistics for the third product:

Product	p_1	p_o	q_1	q_o
A	2	1	3	2
B	3	2	2	1
C (new)	1	3	4	1

The <u>unadjusted</u> Laspeyres industry index, based on the first two products, equals 7/4. The <u>adjusted</u> Laspeyres index -- 7/4 times the ratio of the industry value index (16/7) to the value index for the two known products (12/4) -- equals 4/3, which is even less than the unadjusted measure. But the <u>true</u> Laspeyres index for the three products of the industry is 19/7, more than twice the adjusted measure! The reason for the understatement of the true result is that the output of the new product expanded rapidly while its price fell absolutely and in relation to the prices of the other goods. That is, the price assumption underlying the value adjustment was far from fulfilled. The ratio of the true index to the unadjusted index is 76/49; but the adjustment factor, the ratio of the industry value index to the covered value index, is only 16/21. It should particularly be noted that the adjusted index lies <u>below</u> all of the quantity relatives, while the true index lies within their range. Indeed, even if all the quantity relatives were unity, the adjusted index would still report an output change (a decline) which did not occur.

Thus far, we have assumed that the unadjusted index is of the Laspeyres or Paasche form. It has generally been overlooked that the value adjustment involves no simple assumption concerning price indexes of included and excluded products when the unadjusted measure is, say, of the Edgeworth variety -- the kind used by Fabricant and preferred by the Census Bureau. Perhaps, the Edgeworth can often be assumed with impunity to be close to either the Laspeyres or Paasche formula, since it is a weighted average of these two. But it should be observed that the adjusted index is not equivalent to the true expanded Edgeworth measure even when the Laspeyres price indexes for reported and unreported products are equal and the same is true of the two Paasche price indexes. It is also relevant that the deflation of an index of value of output by the corresponding Edgeworth quantity measure does not yield the Edgeworth price index, but some other weighted average of Paasche and Laspeyres measures.

Now, a few words about the value added adjustment, which has been applied to industry groups and to manufacturing as a whole. This technique may also prove unreliable on occasion. Indeed, the Census Bureau's tests indicate that "at least some of the groups would be appreciably different from any figures now published if it were possible to substitute direct physical measurement for adjustments for the uncovered industries".[12] It further suggests that adjustments based on employment, implying equivalent productivity changes in covered and unrepresented industries, would be more satisfactory. A recent report by the Statistical Office of the United Nations appears to regard this sort of adjustment as promising;[13] and the Bureau of Labor Statistics proposed this alternative some years earlier as more consistent with its preference for labor weights. It may safely be anticipated that some more experience with this alternative will reveal that there still is no substitute for data. In conclusion, we note that Professor Mills used both value added and employment adjustments with the object of obtaining sounder indexes for manufacturing as a whole. This conservative practice seems to go as far as is warranted. There would doubtless be more unanimity concerning value, value added, and employment adjustments if the only purpose of the revised industry and industry group indexes were the subsequent derivation of a general manufacturing series. Indeed, when the Bureau of Labor Statistics rejected routine adjustments in the course of its modification of the W. P. A. indexes, it nevertheless accepted the Fabricant adjusted index for all manufacturing.[14]

12 The U. S. Census Index, p. 13.

13 Index Numbers of Industrial Production, pp. 31-32.

14 Productivity and Unit Labor Cost, p. 1.

Chain and Free Composition Indexes

In addition to the instability of coverage through time, there is the problem of discontinuity of product series due to changes in classification, specifications, and variety of goods made and reported. In such circumstances, it is customary to employ the chain index, which has the authority of Marshall behind it. Another approach, which is virtually unused and deserves consideration, is the construction of a "free composition" aggregative index, a logical extension of the ordinary "fixed composition" measure.

In accordance with the general viewpoint of this study -- that the conventional aspects of measurement should be recognized and faced -- the free composition index appears preferable to the chain measure. It can be interpreted within the same framework as the fixed composition index; it is based on the same assumptions, and any additional ones that are required (like the establishment of synthetic prices for products not made in t_0) have to be introduced consciously. The macrotype that underlies it is visualized as choosing between two output combinations involving the "same" product classes; in one or both of the compared periods, there may be some null quantities, while the fixed composition index has positive entries for each product in both compared periods. The macrotype that underlies the chain index, on the other hand, comes near to being an "impossible impossibility" -- or, at least, one that is not very bright. Unless the chain index is supposed to be a genus apart (in which case, we should learn its recondite meaning), the probable purpose of the ritual of shifting the time base, the weights, and the product classes, and then chaining the links is to derive something like the result that might have been obtained by means of a fixed composition index if the latter were deemed constructible. Actually, the logical extension of the fixed composition index is constructible -- the free composition index -- so the acrobatics of the chain macrotype are superfluous (though they doubtless lead to a different result).

Yet the chain index is a staple while there are few examples of a free composition index. This should not be surprising in view of the general preference, mentioned in Chapter I, for familiar techniques which are not clearly understood. When the easy alternative of mechanically deriving an accepted type of index exists, it would surely be supererogatory to experiment with measures which are clear enough to be disputed. The method of chaining, like the method of deflation, thrives on obscurity, for its implicit assumptions would not go unchallenged if they were evident; and it has had the benefit of authoritative sponsorship, despite its obvious "pseudocontinuity",[15] its well known tendency to "drift", its failure to take full account of a rise due to the introduction of new products and of a fall due to the disappearance of displaced products, etc. Perhaps, the numerous flaws of the Soviet "gross production index" -- which is a crude approximation to a free composition index -- have been so readily discerned by Western observers because the measure is <u>not</u> constructed by the chain method!

We shall now examine the Laspeyres and Paasche free composition expressions for the case of new products and consider also their derivation by deflation. For convenience, we shall assume that we are dealing with the entire (changing) universe rather than with a sample. This assumption, or the equivalent one that the measures are statistically unbiased, has been made throughout this study, but special attention must be directed to the development of "consistent" index estimates in practice when the product assortment

[15] This term is used by G. H. Knibbs, "The Nature of an Unequivocal Price-Index and Quantity-Index: I", <u>Journal of the American Statistical Association</u>, XIX (March, 1924), 55, 60. Other critical remarks on chain indexes may be found in W. I. King, <u>Index Numbers Elucidated</u> (New York: Longmans, Green & Co., 1930), pp. 78-103. Illustrative favorable remarks may be found in A. F. Burns, <u>Quarterly Journal of Economics</u>, February, 1930, pp. 257-61, and B. D. Mudgett, "The Accuracy of Index Numbers", <u>Econometrica</u>, XVIII (July, 1950), 289-90.

is not the same throughout the interval covered. In the actual construction of a chain or free composition measure, it is of utmost importance to distinguish between a product which is new to the universe and one which is simply new to the index.

The Laspeyres free composition measure for the case in which products appear in t_1 which did not exist in t_0 is $(\Sigma p_0 q_1 + \Sigma p'_0 q'_1)/\Sigma p_0 q_0$, where the accented letters refer to the new products. That is, $\Sigma p'_0 q'_0 = 0$. If the q'_{0i} really did not exist in t_0, then the p'_{0i} are hypothetical; we shall say more about these prices later. The Paasche form is $(\Sigma p_1 q_1 + \Sigma p'_1 q'_1)/\Sigma p_1 q_0$, for $\Sigma p'_1 q'_0 = 0$. When products disappear, their output declining to zero by t_1, the numerators of these two indexes, rather than the denominators, are affected. When some products are added while others disappear, the numerators and denominators both include zero entries.

The Paasche free composition index shown in the preceding paragraph is easier to interpret than the Laspeyres. The former is equivalent to the result obtained by deflation of the value index, which expands in scope from t_0 to t_1, by a Laspeyres price index for the products common to both periods. The Laspeyres free composition production index is expressible as the quotient of the expanded value index and the Paasche price index for the t_1 products, $(\Sigma p_1 q_1 + \Sigma p'_1 q'_1)/(\Sigma p_0 q_1 + \Sigma p'_0 q'_1)$. Now, this deflator, like the correlative Laspeyres production measure, contains the prices p'_{0i}, which did not actually exist for products which were not made in t_0. If it could be assumed that $\Sigma p_1 q_1/\Sigma p_0 q_1 = \Sigma p'_1 q'_1/\Sigma p'_0 q'_1$, as in the value adjustment discussed earlier, then the Paasche price index for the products common to both periods is a suitable deflator. But this assumption of "in line" price changes is often economically unsound if the q'_{1i} really refer to new products. For the t_0 prices of products not made in t_0 should be high compared to other t_0 prices as a rule -- indeed, high enough to have precluded production or sale -- while the t_1 prices of the new products typically fall in relation to others.[16] Hence, the "true" deflator, however determined, is normally smaller than the one indicated by standard products and normally yields a higher quantity index.

For genuinely new products, the Laspeyres free composition index not only exceeds the chain index but may also be higher than the chain index with value adjusted links. The free composition index records the full effect of a new product's rise from zero output, while the chain index does not. As for the value adjusted chain index, let us assume for simplicity that the first link is a Laspeyres measure covering the periods t_o and t_1. Let us assume, furthermore, that new products are introduced in t_1. Now, value adjustment of the Laspeyres link leads to

$$\frac{\Sigma p_1 q_1 + \Sigma p'_1 q'_1}{\Sigma p_o q_o} \Big/ \frac{\Sigma p_1 q_1}{\Sigma p_o q_1} .$$

This expression can equal the Laspeyres free composition index only if $\Sigma p_1 q_1 / \Sigma p_o q_1 = \Sigma p'_1 q'_1 / \Sigma p'_o q'_1$. But, as has been argued, the Paasche price index for the products common to both periods normally exceeds the price index of new products introduced in t_1. Hence, the value adjusted link usually rises less than the free composition measure.

Finally, a few odds and ends. First, the free composition approach is preferable to "global" methods often used in international comparisons. Structural differences in the output of the compared economies should be explicitly recognized in the product classification, which may well include

16 See the pertinent remarks by J. R. Hicks, Economica, May, 1940, p. 114. Professor Mills suggests that t_o prices of products introduced in t_1 could in some cases be very low (e.g., byproducts worthless in t_o may become salable in t_1).

items not made in one of the countries and synthetic weights. Second, any principle other than the one suggested here may be used in establishing weights for nonexistent output of one of the periods or countries compared, so long as the results are properly interpreted. This is not to say that choice does not matter; rather, we are forced to make a maximum of sense in the selection of a valuation standard while we still recognize the conventional character of the choice. Third, the free composition approach is applicable, in conjunction with the subproduct approach, to measurement of output from the standpoint of process even during an interval characterized by technological change. Last, the free composition approach is applicable to the completely discontinuous case in which there is reluctance to consider the same nominal product as comparable from period to period because of extreme changes in quality, style, details, etc. The product "frame" would simply have to be broad enough to accommodate the products characteristic of all the periods of the comparison; and the number of synthetic weights would be rather large.

CHAPTER V

PRODUCTIVITY INDEXES AND THE PARTITIONING OF INPUT CHANGES

In this chapter, we first discuss productivity measures relating to composite factor input; then the derivation of mutually consistent indexes relating to labor productivity (or its reciprocal) and other associated concepts, like production; and, finally, the decomposition of the absolute change in factor input into additive components reflecting the "contributions" of productivity (or its reciprocal) and other identified variables.

Composite Factor Productivity Indexes

Although the progress made in the measurement of real net output and composite factor input has been limited, the properties of the implicit net productivity measures are of theoretical interest. As has already been indicated in Chapters II and III, such measures are not tautological despite the equivalence in any period of the output and input aggregates expressed in the money prices of the same period. The remarks which follow are confined to indexes based on money aggregates.

Alternative Expressions

The Laspeyres net productivity index for all factors may be written as

$$\frac{N}{F} = \frac{\Sigma p_0 q_1 - \Sigma P_0 Q_1}{\Sigma p_0 q_0 - \Sigma p_0 q_0} \Big/ \frac{\Sigma w_0 f_1}{\Sigma w_0 f_0} = \frac{\Sigma p_0 q_1 - \Sigma P_0 Q_1}{\Sigma w_0 f_1},$$

where the w_{oi} refer to remuneration per unit of factor input in t_o and the f_{1i} and f_{oi} refer to the corresponding quantities of the various factor inputs in

t_1 and t_o, respectively. The Paasche variant reduces to $\Sigma w_1 f_o / (\Sigma p_1 q_o - \Sigma P_1 Q_o)$.

Another revealing form of the Laspeyres productivity index may be derived at once from the expression shown above and the fact that $\Sigma p_1 q_1 - \Sigma P_1 Q_1 = \Sigma w_1 f_1$:

$$\frac{N}{F} = \frac{\Sigma w_1 f_1}{\Sigma w_o f_1} \Bigg/ \frac{\Sigma p_1 q_1 - \Sigma P_1 Q_1}{\Sigma p_o q_1 - \Sigma P_o Q_1}.$$

According to this result, which may at first seem surprising, the Laspeyres productivity index is equivalent to the quotient of two Paasche _price_ indexes, the numerator being the _factor price_ index and the denominator being the _net product price_ (or the price-margin) index presented in Chapter IV. The Paasche productivity index may similarly be written as the ratio of Laspeyres indexes of factor price and net product price.

Criterion for a Rise or Fall

Under what circumstances does a net productivity index exceed, equal, or fall short of unity? From an inspection of the Laspeyres formulas presented above, it is evident that productivity remains constant in the special case in which all the relatives -- the q_{1i}/q_{oi}, the Q_{1j}/Q_{oj}, and the f_{1k}/f_{ok} -- are equal:

$$\frac{N}{F} = \frac{\Sigma p_o q_o (q_1/q_o) - \Sigma P_o Q_o (Q_1/Q_o)}{\Sigma w_o f_o (f_1/f_o)}.$$

It may be shown that, in general, the net Laspeyres index exceeds, equals, or falls short of unity according as

$$\Sigma S \left[P_o Q_o (q_1/q_o - Q_1/Q_o) + w_o f_o (q_1/q_o - f_1/f_o) \right] \gtreqless 0.$$

In this expression, the subsums $(SP_o Q_o)_i$ and $(Sw_o f_o)_i$ correspond to the q_{1i}/q_{oi} and Σ extends over the product range ($i = 1, 2, \ldots$). The general criterion for the Paasche case is

$$\Sigma S[P_1Q_1(q_o/q_1 - Q_o/Q_1) + w_1f_1(q_o/q_1 - f_o/f_1)] \lesseqgtr 0;$$

that is, productivity rises, remains level, or falls according as this expression is negative, zero, or positive.

Another statement of the criterion may readily be derived in terms of G (gross production index), F, and M (materials, etc. consumption index). Thus, using Laspeyres indexes throughout, we have $N/F \gtreqless 1$ according as

$$G \gtreqless \frac{\Sigma w_of_oF + \Sigma P_oQ_oM}{\Sigma w_of_o + \Sigma P_oQ_o} .$$

Thus, productivity, as measured by the Laspeyres N, rises or falls according as G exceeds or falls below this average of factor input and materials, etc. consumption indexes. A similar expression, with different weights and with the sense of the inequality reversed, may be derived for the case in which all the indexes are of the Paasche type.

In Chapter II, it was observed that service output might for some purposes be measured in units characteristic of input. If such an output indicator were used in the derivation of a net productivity index for a larger group of activities, the criterion for a rise or fall would clearly depend on the measures for the non-service activities. Since the input and output aggregates for the service would be equal in each period, they would be eliminated in the derivation of the criterion.

Possibility of Externality

Both the Laspeyres and Paasche net productivity indexes may lie outside the range of the productivity relatives derived from the q_{1i}/q_{oi} and the corresponding factor relatives. The hypothetical example which follows shows the possibility that the two indexes may move in one direction while the individual productivity relatives move in the other.

Let us assume two products (A and B), each made of the same two kinds of materials (Q and Q') with the same two kinds of factors (f and f'):

	Product A		Product B		*		Product A		Product B	
Quantities	t_1	t_0	t_1	t_0	*	Prices	t_1	t_0	t_1	t_0
q	3	2	4	3	*	p	5	4	3	6
f	4	2	3	2	*	w	1	1	1	3
f'	4	2	5	3	*	w'	1	1	1	1
Q	1	2	2	1	*	P	4	1	1	3
Q'	1	1	1	3	*	P'	3	2	2	2

The Laspeyres net productivity index derived from these data is (25/13) / (22/13)= 25/22, while the Paasche index is (16/1)/(16/9) = 9. Though both of these measures exceed unity, the productivity relatives for A with respect to the two input factors are 3/4 and 3/4, while the corresponding relatives for B are 8/9 and 4/5. Also noteworthy is the disparity between the two net output measures in contrast to the closeness of the two gross output measures, which must be internal means of the individual production relatives; the Laspeyres gross index is 18/13 and the Paasche is 27/19.

The possibility of externality may be removed by the substitution of a measure of net output with unit value added weights -- designated $\bar{V}$ in Chapter IV -- for the measure N used above. If we note that unit value added, v_{oi}, is equivalent to $(Sw_of_o)_i/q_{oi}$ for the same product, then we may write for the Laspeyres case

$$\frac{V}{F} = \frac{\Sigma v_o q_1}{\Sigma v_o q_o} \Big/ \frac{\Sigma w_o f_1}{\Sigma w_o f_o} = \frac{\Sigma v_o q_1}{\Sigma w_o f_1} = \frac{\Sigma q_1 (Sw_o f_o/q_o)}{\Sigma Sw_o f_1}$$
$$= \Sigma S[w_o f_1 (q_1/q_o)/(f_1/f_o)] \;/\Sigma Sw_o f_1 ,$$

which is a weighted internal mean of the productivity relatives for each of factors used in making each of the products. The weights $w_{oj}f_{1j}$ refer to each factor used in making a product; the weights $(Sw_0f_1)_i$ refer to particular products; and Σ again refers to all products combined.

Labor Productivity Indexes

We now return to the discussion of labor productivity, which was begun in Chapter III. It has already been noted that the indirect method of estimating labor productivity change (that is, as the ratio of an output index to a labor input measure) could yield results which lie outside the range of the individual productivity relatives. It has also been observed that a directly defined aggregative productivity index with output weights is necessarily an internal mean of the relatives; and that such an index implies a correlative production index with labor weights. In this section, more is said about the derivation of mutually consistent measures for different entities.

Directly Defined Measures with Production Weights

In recent years, there has been a growth of interest in direct productivity measurement which should lead to the improvement of available industry statistics, to the penetration of now uncovered areas, and to fruitful experimentation with the subproduct approach. Since 1945, the U. S. Bureau of Labor Statistics has been compiling annual series on unit labor requirements (back to 1939) for a few manufacturing industries on the basis of production and man-hour data reported directly by cooperating establishments.[1] This significant innovation has already yielded some indexes with technically desirable features. Since each industry is regarded as a special case, due attention is given to its technological and accounting peculiarities in the development of

[1] See the description given by G. E. Sadler and A. D. Searle, "Measurement of Unit Man-Hour Requirements", in U. S. Bureau of Labor Statistics, Techniques of Preparing Major BLS Statistical Series (Bulletin No. 993, Washington, 1950), pp. 42-49.

its report. Thus, information is compiled, as appropriate, on an end-product, process, or departmental basis. Attempts are made to maintain the same product specifications for all the establishments. The assignment of fixed production weights to the various firms suppresses the effect of interfirm shifts. The industry-wide measures for individual products are combined by means of man-hour weights. Man-hours paid for but not worked are eliminated, and a distinction is made between "direct" and "indirect" factory labor. The detail of the data permits analysis of the relationship between unit labor requirements and such variables as size of firm, degree of capacity utilization, method of production, etc. Information may readily be obtained for interpreting the observed company trends.

It seems that the notion of direct productivity measurement and the derivation of correlative production indexes originated in the study of employment and unemployment problems. At least since the 1880's, when C. D. Wright made his pioneer statistical inquiries into "labor displacement", there has been interest in estimates of the volume of employment consistent with alternative levels of output and productivity. In the 1930's, widespread concern over "technological unemployment" and related issues provided the occasion for estimates of employment opportunities by D. Weintraub, B. Stern, F. C. Mills, and others. The W. P. A. National Research Project, organized during that period for the specific purpose of studying "reemployment opportunities and recent changes in industrial techniques", apparently originated direct productivity indexes but concentrated on their approximation by means of correlative production measures. The formulas were developed as "answers" to "questions" concerning "labor displacement", such as were asked by H. Jerome in his earlier book on mechanization.[2] Once the productivity and production formulas were known, it became evident that they could be written and obtained more simply.

In his book, Professor Jerome, who subsequently continued his investigations at W. P. A. National Research Project, asked two questions which are of interest to us because of the symbolic statements to which they lead. "How much less labor", he asked, "did it require to produce current output than would be required at the productivity rate of the base year?" In our symbols[3] of Chapter III, his answer was $\Sigma q_1(r_1 - r_0)$ for the "displacement". A second question, involving a comparison of the employment consistent with base (t_0) output and current (t_1) productivity and the actual base employment, led to the expression $\Sigma q_0(r_1 - r_0)$. Now, Jerome's purpose did not require conversion of these two answers to relative form, so the simplest derivation of direct measures of productivity or unit labor requirements and correlative production measures with labor weights was overlooked. If the first symbolic statement is referred to current employment, the result is $1 - (\Sigma q_1 r_0 / \Sigma q_1 r_1)$; the index is the Paasche productivity measure. If the divisor were base employment, the result would explicitly include the correlative Laspeyres output index. The quotient of the second expression and base employment is $(\Sigma q_0 r_1 / \Sigma q_0 r_0) - 1$; the index is the Laspeyres measure of change in unit labor requirements, and its reciprocal is the Laspeyres productivity measure.

Another development is based on a partition of the total employment

2 H. Jerome, Mechanization in Industry (New York: National Bureau of Economic Research, 1934), especially pp. 376-77. See, also, Magdoff, Siegel, and Davis, op. cit., I, chap. 1.

3 The writer has added a summation sign to Jerome's expressions to show addition over a group of products.

change into components representing the "contributions" of output, unit labor requirements, and "joint" variations in both. These components, which correspond to terms of a Taylor expansion with zero remainder, resemble Jerome's expressions and also yield the correlative production, productivity, and unit labor requirement indexes when reduced to relative form. The actual components will be shown later in this chapter.

Finally, the mutually consistent indexes may be developed formally by the algebraic implementation of verbal identities.[4] Thus, we start with the verbal equation: man-hours = production x unit man-hour requirements. Since the man-hours index may be written in only one way in terms of production and unit man-hour requirements, we have, for a number of products, the unique expression $\Sigma q_1 r_1 / \Sigma q_0 r_0$. Next, we consider all possible indexes of production and unit man-hour requirements satisfying the identity. There are only two alternatives:

$$\frac{\Sigma q_1 r_1}{\Sigma q_0 r_0} = \frac{\Sigma q_1 r_1}{x} \cdot \frac{x}{\Sigma q_0 r_0}$$

$$= \frac{y}{\Sigma q_0 r_0} \cdot \frac{\Sigma q_1 r_1}{y} .$$

If the first ratio in each of the two equations is to represent a production index, then $x = \Sigma q_0 r_1$, necessarily, and $y = \Sigma q_1 r_0$, necessarily. Substitution of these values for x and y in the second ratio of each equation yields the correlative unit man-hour requirement indexes. The geometric mean of the two possible equations leads to Fisher "ideal" indexes of production and unit man-hour requirements.

4 The discussion which follows is based in part on I. H. Siegel, "The Generalized 'Ideal' Index-Number Formula", Journal of the American Statistical Association, XL (December, 1945), 520-23.

Other Measures

Many other mutually consistent indexes of productivity (or its reciprocal) and other entities may be derived by means of verbal identities. Such directly defined productivity measures and their correlatives are generally more difficult to implement in practice than the indexes just discussed; they require data that are as scarce as, or even scarcer than, say, (allocable) man-hours by product class. Like composite factor productivity indexes, however, these measures also are of theoretical interest.

An instructive example is provided by the possibility of representing a unique payrolls index by two distinct identities, each involving two entities: payrolls = production x unit labor cost = man-hours x average hourly wages. By the method already outlined, consistently weighted Laspeyres, Paasche, and Fisher indexes satisfying these identities may be derived. Now, it is clear from the identities that an indirect productivity (output per man-hour) index may be derived which is equal to the quotient of the correlative indexes of hourly wages and labor cost per unit of output. This result is interesting for two reasons. First, it is the analogue of the result obtained earlier in this chapter for net productivity, N/F, as a ratio of two price indexes. Second, the productivity measure turns out, like V/F, to be an internal average of the individual productivity relatives computed with respect to the distinctively priced man-hour categories. Using the Laspeyres variants, we have as our productivity measure $(\Sigma w_o q_1 / \Sigma w_o q_o) \div (\Sigma e_o m_1 / \Sigma e_o m_o)$, where the w_{oi} refer to labor cost per unit of output, the m_{oj} and m_{1j} to man-hours by type, and the e_{oj} to the corresponding t_o hourly rewards. Now, this productivity measure may be rewritten at once as the quotient of Paasche indexes of hourly earnings and unit labor cost, $(\Sigma e_1 m_1 / \Sigma e_o m_1) \div (\Sigma w_1 q_1 / \Sigma w_o q_1)$. Since the two expressions for t_o payrolls in the original formulation are equal (i.e., $\Sigma w_o q_o = \Sigma e_o m_o$), we may also rewrite the ratio of indexes as an internal average of productivity relatives,

$$\frac{\Sigma w_o q_1}{\Sigma e_o m_1} = \frac{\Sigma q_1 (S e_o m_o / q_o)}{\Sigma S e_o m_1}$$

$$= \Sigma S [e_o m_1 (q_1/q_o) / (m_1/m_o)] / \Sigma S e_o m_1;$$

each $(S e_o m_1)_i$ refers to the t_1 man-hours valued at t_o labor prices, corresponding to a particular product.

The fact that our indirect productivity index reduces, like V/F, to an internal mean of productivity relatives suggests a generalization. The reduction appears possible if the output aggregates include only the relevant product quantities and all are positively weighted; and if the input and output aggregates are of the same scope and dimension, so that they are equal when the incorporated weights and quantities refer to the same period.

The method of identities may readily be extended to the derivation of mutually consistent indexes for three or more multiplicatively related entities. In such instances, a unique aggregative index is factored into others composed of aggregates of the same dimension. The structure of each aggregate is analogous to the structure of the identity; that is, the data required for each correspond conceptually to the entities in the continued index product. Let us consider the third-order verbal identity: payrolls = output per man-hour x man-hours x unit labor cost. Here, the unique expression for the payrolls index is $\Sigma \pi_1 m_1 w_1 / \Sigma \pi_o m_o w_o$, where the π_{1i} and π_{oi} refer to output per man-hour in t_1 and t_o, respectively, and the other symbols refer, as before, to man-hours and unit labor cost. There are six possible equations satisfying this identity, two of which are:

$$\frac{\Sigma \pi_1 m_1 w_1}{\Sigma \pi_o m_o w_o} = \frac{\Sigma \pi_1 m_1 w_1}{x} \cdot \frac{y}{\Sigma \pi_o m_o w_o} \cdot \frac{x}{y}$$

$$= \frac{\Sigma \pi_1 m_1 w_1}{x} \cdot \frac{x}{y'} \cdot \frac{y'}{\Sigma \pi_o m_o w_o} \ .$$

The other four equations are obtained as the aggregates comprising the numerator and denominator are shifted to different positions. In the first equation, it is necessary that $x = \Sigma \pi_o m_1 w_1$ if the first ratio is to be a productivity index; that $y = \Sigma \pi_o m_1 w_o$ if the second ratio is to be a man-hours index. The substitution of these values in the third ratio clearly leads to a unit labor cost index. By a similar argument, we determine $y' = \Sigma \pi_o m_o w_1$ in the second equation. The same reasoning, when also applied to the remaining four equations not shown, yields a total of four distinct indexes for each entity. The geometric mean of all six equations yields the true generalization of the Fisher formula. The indexes so derived for each of the three entities are internal means of the relatives concerned and satisfy the time reversal and (generalized) factor reversal tests.

It should be noted that generalized "ideal" indexes derived for the same entity from alternative verbal identities of the same order or from identities of different order are not identical. Furthermore, any of the distinct indexes derived from one verbalization is not the reciprocal of any of the distinct indexes for the reciprocal entity derived from an alternative verbalization of the same order. For example, none of the four productivity indexes derived from the verbal identity considered in the preceding paragraph is the reciprocal of any of the four aggregative unit man-hour requirement indexes indicated by the alternative identity for payrolls: payrolls = unit man-hour requirements x production x average hourly wages. On the other hand, an index derived from one identity may be equivalent to an index for the same entity derived from a verbalization of a higher order. Thus, the Laspeyres index of output with unit labor cost weights, derived from a second-order identity for payrolls, equals one of the expressions obtained from the third-order identity. That is, $\Sigma q_1 r_o e_o / \Sigma q_o r_o e_o = \Sigma q_1 w_o / \Sigma q_o w_o$, since $r_{oi} e_{oi} = w_{oi}$.

Partitioning of Input Changes

In empirical studies, attempts are sometimes made to decompose the absolute or percentage change in aggregate man-hours, value, or value added between t_o and t_1 into the additive "contributions" of various entities, including production and productivity (or its reciprocal); or to decompose in a similar manner the change in average productivity (or its reciprocal) for a group of products or industries.

Apart from the reservations which may be entertained concerning the economic significance of such statistical post mortems, the technical adequacy of the measures frequently used may also be questioned. A common fault of such partition formulas is the confusion of time bases; all components should logically be computed from either t_o or t_1. Another common flaw is the asymmetry of the component measures; the expression for each of the designated contributory variables should be obtainable from the expression for another by the cyclical substitution of symbols. Furthermore, even symmetrical measures may not make obvious sense when verbalized; they may not unambiguously express the distinctive "contributions" of the variables of interest. Another difficulty, already suggested in the preceding discussion of correlative indexes, is the inconsistency of measures for a variable and its reciprocal when these are derived from alternative partition formulas; thus, the effect attributed in one formula to a productivity change is not the same as that attributed to a change in unit labor requirements in an alternative formula. Finally, two kinds of errors are often committed: the introduction of asymmetry through the absorption of "residual" terms showing the "joint" effects of simultaneous changes in the explicitly recognized variables; and the misinterpretation of such residuals, when they are not absorbed, as the contributions of "all other" variables not explicitly taken into account in the partition formula.

If partition formulas are considered worth using, then it would seem desirable to restrict choice to symmetrical expressions and to avoid interpretation of joint particles by reference to extraneous variables. The writer prefers to preserve the meaning of the expressions in terms of the underlying Taylor expansions and thus would not distribute the joint effects -- even in the two-variable case, where the joint term can be distributed symmetrically. Failure to exhaust the total change when the joint terms are not absorbed or distributed is not a defect but is consistent with the essential mathematical meaning of the partitioning technique.

<u>Two-Variable Case</u>

To illustrate some of the difficulties mentioned above, we shall consider the partition of the change in total man-hours between t_o and t_1 into three components, one representing the "contribution" of the change in unit labor requirements, the second referring to the change in output, and the last to the "joint" change in unit labor requirements and output:

$$\Sigma q_1 r_1 - \Sigma q_o r_o = \Sigma q_o(r_1 - r_o) + \Sigma r_o(q_1 - q_o) + \Sigma(r_1 - r_o)(q_1 - q_o).$$

All of the components are measured from the same time base, t_o, but need not have the same sign. The second-order "joint" effect need not be negligible and it may differ from one of the other two components in sign. Symmetry is destroyed and time bases are confused when this effect is merged with one of the others. Besides, there is no reason to prefer one of the asymmetrical partitions over the other since both,

$$\sum q_1r_1 - \sum q_or_o = \sum q_o(r_1 - r_o) + \sum r_1(q_1 - q_o)$$
$$= \sum q_1(r_1 - r_o) + \sum r_o(q_1 - q_o),$$

are distorted representations. An average of these two has the advantage of restoring symmetry while removing the "joint" particle, but still implies two time bases within each component, t_o and $(t_o + t_1)/2$:

$$\sum q_1r_1 - \sum q_or_o = 1/2 \sum (q_1 + q_o)(r_1 - r_o) + 1/2 \sum (r_1 + r_o)(q_1 - q_o).$$

This sort of "compromise" is favored by Fabricant and was used earlier by the Italian mathematical economist, Amoroso.[5]

Although we have chosen to illustrate the two-variable case in terms of a change in total man-hours, there are other relevant applications. Thus, the change in total value added may be partitioned into components referring to changes in unit value added and output, or to changes in value added per man-hour and in man-hours. Also of interest is the partition of a change in labor-weighted output into the "contributions" of changes in output per man-hour and in man-hours.

It does not appear possible to express a change in average productivity or in its reciprocal as the sum of symmetrical components reflecting changes in output and in man-hours measured from the same time base. There are, however, other partition formulas for average productivity and average unit labor requirements -- and these, curiously, involve the individual productivity ratios and the individual unit labor requirement ratios, respectively, as variables. One particle in the productivity case shows the weighted effect of a change in the percentage of total labor devoted to each product; in the unit labor requirement case, the weighted effect of the change

[5] S. Fabricant, "On the Treatment of Corporate Savings in the Measurement of the National Income", Studies in Income and Wealth, I (1947), 129-30; and L. Amoroso, Lezioni di Economia Matematica (Bologna: N. Zanichelli, 1921), pp. 41-42.

in proportion of total output represented by each product. In general symbols, we have the compromise formulas[6]

$$\Pi_1 - \Pi_o = 1/2\,\Sigma\,[(E_1/\Sigma E_1 + E_o/\Sigma E_o)(O_1/E_1 - O_o/E_o)] + 1/2\,\Sigma\,[(O_1/E_1 + O_o/E_o)\;(E_1/\Sigma E_1 - E_o/\Sigma E_o)]$$

and

$$U_1 - U_o = 1/2\,\Sigma\,[(E_1/O_1 + E_o/O_o)(O_1/\Sigma O_1 - O_o/\Sigma O_o)] + 1/2\,\Sigma\,[(O_1/\Sigma O_1 + O_o/\Sigma O_o)(E_1/O_1 - E_o/O_o)]$$

where Π_1 ($= \Sigma O_1/\Sigma E_1$) and Π_o ($= \Sigma O_o/\Sigma E_o$) refer to average productivity, U_1 ($= \Sigma E_1/\Sigma O_1$) and U_o ($= \Sigma E_o/\Sigma O_o$) to average unit labor requirements, O_{1i} and O_{oi} to (weighted) output of individual products, and E_{1i} and E_{oi} to labor input corresponding to individual products.

The symbols Π and U in general refer to (weighted, if necessary) t_1 or t_o quantities, not indexes. But, if we define the weighted output quantities in t_1 as $O_{1i} = q_{1i}r_{oi}$ and in t_o as $O_{oi} = q_{oi}r_{oi}$, then the change in average productivity becomes the difference between the Paasche productivity <u>index</u> and unity; and the change in average unit labor requirements becomes the difference between the Paasche unit labor requirement <u>index</u> and unity. If we define weighted output quantities in t_1 as $O_{1i} = q_{1i}r_{1i}$ and in t_o as $O_{oi} = q_{oi}r_{1i}$, then $\Pi_1 - \Pi_o$ and $U_1 - U_o$ become differences between unity and the corresponding Laspeyres <u>indexes</u>. These results involve the fact (noted in Chapter III) that the same weighted aggregate has a double meaning, one for

[6] The partition formula used for a change in average productivity by H. Barger and H. Landsberg, <u>American Agriculture, 1899-1939</u>, pp. 258-59 and 264-65, and by H. Barger and S. H. Schurr, <u>The Mining Industries, 1899-1939</u> (New York: National Bureau of Economic Research, 1944), pp. 81-82, is less satisfactory than the expression shown here for $\Pi_1 - \Pi_o$, or the alternative symmetrical expression in which the joint component remains isolated. These writers divide the productivity change into two asymmetrical parts; the first represents the change recorded within component agricultural activities or mining industries and the second, improperly computed as a residual, is said to reveal interregional shifts in the case of agriculture or interindustry shifts in the case of mining.

each identified variable. The effect of combining all output in labor terms here is equivalent to reducing an absolute change in average productivity or unit labor requirements to a percentage change.

Whether or not labor weights are used in the output aggregates, the total change in productivity or unit labor requirements may readily be cast into percentage form. The compromise formulas presented here need merely be divided by Π_o and U_o, respectively.[7]

Higher-Order Partitions

Partition formulas may readily be extended to three or more variables, in which case the number of "joint" particles increases rapidly. Three-variable cases of interest include the decomposition of the total change in payrolls into the parts associated with the entities satisfying the alternative verbalizations considered earlier in this chapter. Another example involves the partitioning of the change in total value added into components referring to value added per man-hour, unit man-hour requirements, and output; or into components referring to unit value added, man-hour productivity, and man-hours. In the three-variable case, there seems to be no "compromise" formula involving three symmetrical additive components.[8]

[7] In *Employment in Manufacturing, 1899-1939* (New York: National Bureau of Economic Research, 1942), pp. 336-37, S. Fabricant uses a compromise expression for $(U_1 - U_o)/U_o$. He assigns Edgeworth price weights to the output quantities.

[8] Of course, symmetrical logarithmic expressions may be developed, like those introduced by J. K. Montgomery, *The Mathematical Problem of the Price Index* (London: P. S. King & Son, 1937). The contribution of each variable is measured by the total difference raised to a distinctive fractional power, the sum of the fractional powers being unity.

Let us consider the three-variable case more closely. By straightforward algebra, we may decompose the total change as follows:

$$\Sigma a_1b_1c_1 - \Sigma a_ob_oc_o = \Sigma a_ob_o(c_1 - c_o) + \Sigma a_oc_o(b_1 - b_o) + \Sigma b_oc_o(a_1 - a_o) + \Sigma a_o(b_1 - b_o)(c_1 - c_o) + \Sigma b_o(c_1 - c_o)(a_1 - a_o) + \Sigma c_o(a_1 - a_o)(b_1 - b_o) + \Sigma(a_1 - a_o)(b_1 - b_o)(c_1 - c_o).$$

Altogether, we obtain seven terms, all measured from the same time base, t_o. The first three represent the "pure" effects of changes in the three explicit variables; the remaining four represent "joint" or "impure" effects, and their share in the total change may be significant. In the two-variable case, it will be recalled, only one of the three terms in the partition formula was "joint".

As was suggested earlier in this chapter, the decomposition is equivalent to a Taylor expansion, with zero remainder, about the t_o aggregate. This fact should warn against the confusion of time bases and the illogical interpretation of the last four terms shown above as the effects of "all other" variables than a_i, b_i, and c_i. Only these explicitly included variables can have effects attributed to them, in this case secondary and tertiary as well as primary. The first three terms shown above (the "pure" effects) are derivable from the first sum, the second three terms from the second sum, and the seventh term from the third sum in the following Taylor development:

$$\Sigma a_1b_1c_1 - \Sigma a_ob_oc_o = \Sigma(\Delta a_iA_i + \Delta b_iB_i + \Delta c_iC_i)f_i + 1/2!\ \Sigma(\Delta a_iA_i + \Delta b_iB_i + \Delta c_iC_i)^2f_i + 1/3!\ \Sigma(\Delta a_iA_i + \Delta b_iB_i + \Delta c_iC_i)^3f_i\ ,$$

where the $f_i = a_ib_ic_i$; the $\Delta a_i = a_{1i} - a_{oi}$, etc.; and the A_i, B_i, and C_i are the partial differential operators $\partial/\partial a_i$, $\partial/\partial b_i$, and $\partial/\partial c_i$ applied to the f_i. Finally, it may be shown, if appropriate substitutions are made in the partition formula, that the change attributed to productivity is not the same as the change attributed to its reciprocal in the alternative three-variable partitions of, say, total value added.

CHAPTER VI

SUMMARY AND OUTLOOK

Our purpose is to restate briefly the argument of the preceding chapters, to recall some of the more significant findings, and to indicate the direction which future work might take in the light of these findings and of the remarks in Chapter I.

Summary

The recent history of production and productivity measurement affords ample evidence that this field has not escaped the usual difficulties associated with the quantitative treatment of mental constructs in the social sciences. Here, too, the rudimentary principles of logic and scientific method are frequently ignored by the trained as well as the untrained as they follow the furrows of convention. The essential fact of the mutual interdependence of data, techniques, and meaning is commonly overlooked. The multiplicity of plausible measures of a general term is insufficiently appreciated, and so is the relevance of purpose and context of measurement to the selection of a particular index. Proceeding on the assumption that such ideas are nevertheless important, we have inquired into the fundamental aspects of measurement and meaning of production and productivity, paying particular attention to the difference between economic valuation and the weighting process, to the difference between verbal algebra and the precise content of a measure, to the circumstances under which indexes satisfying verbal identities also have the desirable property of being internal means of relatives, and to the algebraic conditions for one aggregate or index to exceed or fall below another which may not be available or constructible.

Our inquiry has been restricted to aggregative indexes, which are the most important type in use and which are also expressible as weighted arithmetic or harmonic means of relatives. Most of our illustrations refer to the Laspeyres and Paasche formulas, which underlie the "compromise" measures of Edgeworth and Fisher. The character of an index is determined largely by the classification principle and the "primary" units selected for the products or factors concerned and by the criterion used in establishing the external comparability of these classes. The "same" aggregate represents different multiples of characteristic "secondary" units as the breadth of the classes is altered. Furthermore, the "same" aggregate comprises different multiples of the secondary units corresponding to the variables identified in its symbolic expression and to the continued product of these variables. The relationship between the secondary multiples contained in aggregates with different weights or between differently weighted indexes may readily be analyzed by means of the von Bortkiewicz weighted correlation coefficient and other expressions derivable by elementary vector (or matrix) methods involving the generalized Lagrange inequality.

The plural significance of the "same" aggregate is of fundamental importance for the construction and interpretation of indexes. This fact provides the rationale of deflation for cases in which the recognized classes are homogeneous through time. It also provides the basis for regarding various "direct" productivity indexes as elliptic forms, as measures of output per unit of input in only <u>one</u> period while the productivity ratio for the other period is <u>unity</u>. The restoration of the explicit symbols for the latter ratio and the rearrangement of terms constitute the inverse of deflation; this inverse process shows that a direct labor productivity index is the quotient of two appropriately weighted indexes -- e.g., a labor-weighted output index and an unweighted labor measure, or a labor-weighted index of hourly earnings and an output-weighted index of unit labor cost. In general, the plural significance of an aggregate explains why ratios of aggregative indexes reduce in some cases but not in others to aggregative indexes of the ratios. This fact also provides the key to the construction of systems of mutually consistent indexes which satisfy verbal identities, are internal means of relatives, are expressible in terms of their correlative indexes, and lead to the true generalization of the two-variable "ideal" index. Finally, this fact underlies the formulas for partitioning a change in, say, total input into the sum of the pure "contributions" of the explicit variables and higher-order "joint" effects.

It is important to know that some methods may lead to measures which are not internal means of relatives. Thus, a productivity measure computed as the quotient of a price-weighted gross output index and a labor input index yields the product of a direct output-weighted productivity measure and an index showing the shift in the structure of production. The latter factor may be sufficiently large to make the result external to the productivity relatives, even though the direct productivity measure is an internal mean. Deflation and value adjustment may also lead to externality. Net output indexes based on reduced aggregates and net productivity measures, too, might lie outside the range of end-product relatives -- though the former would generally be true averages of net output relatives (like $[q_{1i} - (SP_oQ_1)_i/p_o] \div [q_{oi} - (SP_oQ_o)_i/p_o]$ in the Laspeyres case) and the latter would generally be true averages of net productivity relatives (like quotients of the indicated net output relatives and the corresponding input relatives $(Sw_of_1)_i/(Sw_of_o)_i$). Since the gross output index is median to the net output index and the index of consumption of materials, etc., it could be a poor substitute for the net index, which it is often intended to represent.[1] Subproduct indexes could also lie outside the range of end-product relatives, but measures based on end-product quantities with "net" weights would generally be internal means.

Although subproduct indexes can even more rarely be constructed than net indexes based on reduced aggregates, the merits of the former should be better known. They would more faithfully reflect the structure of productive activity; give a truer account of the formation of net output; permit the derivation of a hierarchy of consistent output and productivity measures for the various levels of economic organization; remain invariant under changes in the degree of integration of the productive process; escape distortion as the rate of completion varies; sometimes permit satisfactory measurement even though heterogeneity precludes establishment of satisfactory end-product classes; and sometimes permit measurement of production and productivity movements over an interval disturbed by major technology changes.

The "free composition" output index is a preferable alternative to the chain index when new products are introduced or old ones disappear. Indeed, the chain index would seem, at best, to be interpretable as an approximation to some sort of free composition index. The latter is the logical extension of the usual aggregative, or "fixed composition", index; it simply includes zero entries for items not made in any of the compared periods and, when the particular formula requires, it also includes corresponding

[1] This is demonstrated by V. R. Berlinguette for Canada in an unpublished paper presented at the Harvard Meeting of Econometric Society, September 5, 1950.

hypothetical weights. Thus, if the weighting factor is price, the hypothetical weight for a product not yet made might be set at the lowest price which precludes output and sale, the other prices being given. In an underdeveloped country, the prices of nonexistent goods might be fabulous, and the choice of different base periods could lead to substantially dissimilar results. But each choice has its particular meaning, and a later base requiring fewer hypothetical prices is not on this account "better". The free composition index would, more sensitively than the chain index, register the effects of a change in the composition of the universe of products. While the former requires conscious arbitrary decisions, the latter conceals in its structure arbitrary assumptions which may be even less tenable.

The production and productivity indexes used for historical measurement do not have any "economic" import. They are not intended as comparisons of alternative opportunities confronting a "representative" decision-maker with a fixed substitution map; or as ordinal comparisons of "social states" according to a consistent rule applied to the choices of individuals with fixed maps. They are intended as "quantitative" comparisons of some "physical" stuff -- despite the customary intrusion of pecuniary weights, etc. -- and in no deeper sense could they be "surrogative". Thus, a production measure cannot indicate the relative "volumes" of personal or "social" utilities, and an input measure cannot represent the ratio of disutility "totals" corresponding to "efforts and sacrifices". It is possible, however, to interpret historical indexes in a pseudo-economic, transoperational manner which should prove instructive. Thus, any formula may be regarded as describing the relevant behavior of a mythical "macrotype", an appraiser capable of comprehending from a consistant standpoint and of valuing as a numerical ratio the achievements of different periods. This demon may embody a valuation system (weights) associated with a particular period, but he is culturally attached to none; and he is not daunted, for example, by quality conundrums which have not been quantitatively resolved and which have apparently not been considered important enough to preclude a decision to measure in the first place. In the simple case of gross output measurement, our demon may be considered to have the task of numerically comparing two vectors referring to n products (there may be some null quantities). He generally solves this problem by scalarization of the vectors but he could also do so (as the "economic" theory of indexes would suggest) by collineation of the vectors after transforming either or both according to some principle of equivalence or indifference.

For historical studies, the notion of independent "cost" is meaningful even though it is not for value theory. The two aspects of activity may, then, properly be quantified and compared in their characteristic units: what factors "do" may properly be related to what they in some sense "are". When satisfactorily constructed, a labor productivity index indicates the changing effectiveness with which labor is used in conjunction with other factors. This idea has probably not been so widely misunderstood as is often supposed. The specific productivity of labor -- its marginal productivity, other factors held constant -- can be determined only if a mathematical function connecting output to labor and other inputs is established. Since "man" -- a generalized macrotype grown plausible through familiarity -- is both the end and an agent of production, labor productivity measures also have some significance for the determination of "progress" in the sense of "economic welfare". For the same reason, even crude measures of composite input expressed in labor terms could yield significant long-run productivity indicators. Unfortunately, it is impossible to reduce all output to an "ultimate" quantity of some <u>desideratum</u> of generalized man for comparison with composite input in labor terms.

Outlook[2]

It would seem, from the nature of the progress of the past thirty years, that only limited advances in production and productivity measurement may be anticipated. Penetrations into areas not now represented by measures are likely to be occasional, few, and often tentative. Some further improvement, along conventional lines, may be expected in areas already covered. But major advances are improbable without a shift to the subproduct basis of compiling and classifying output and labor data and such a reorientation of Federal and other statistical reporting systems on a grand scale seems very unlikely. True, national trials like depression and war have in the past stimulated interest in production and productivity measurement, but the present world crisis, even if protracted, would probably add only a few series for bottleneck and war-important industries as it dries up some present sources. There is also the danger, in such a crisis, that token measures and "official" indexes will be given a privileged status; and that available "general-purpose" measures will be adapted to all sorts of unwarranted uses. After some disillusionment, there may be a disposition to regard measurement again as an art and to add to the stock of data. The stage will then have been set for a constructive and apparently feasible task: the education of index makers and users to a higher level of sophistication.

A brief review of the history of measurement makes evident the role of catastrophe as a stimulant. The apparent "technological unemployment" consequent upon the post-Civil War industrialization and the labor unrest of

[2] Based in part on two unpublished papers by the writer, "Progress and Problems of Physical Production Measurement" and "A Half Century of American Productivity Measurement", read at the 110th Annual Meeting of the American Statistical Association (1950).

the 1880's provided the background of the first U. S. Labor Commissioner's monumental studies of "labor displacement" and of the relative effectiveness of hand and machine labor. In the surge of statistical activity following World War I, production indexes as we now know them were developed by Stewart, Persons, Day-Thomas, and the Federal Reserve Board. In the 1920's, probably the first productivity indexes were developed by the Bureau of Labor Statistics from data in the Census of Manufactures, which was transferred from a quinquennial to a biennial basis after 1919.[a] These indexes and the Bureau's cross-section studies of industry productivity were motivated by fears of a new technological revolution. The improvement of this agency's employment statistics during the same decade was partly the result of the widespread concern over the sharp 1921 recession. Renewed anxiety over the relation between mechanization and work opportunities in the 1930's led to the comprehensive Bureau of Labor Statistics, Works Progress Administration, and National Bureau of Economic Research studies of production and productivity trends.

But World War II meant the virtual suspension of measurement for many industries, as the manufacturing census was discontinued, as the character of output changed, and as the quality of the Federal Reserve indexes deteriorated. On the other hand, the War also provided the occasion for the introduction of the Department of Commerce concept of gross national product and for the popular use of national "productivity" ratios or indexes for making postwar employment projections. After the War, the Bureau of Labor Statistics inaugurated direct productivity reporting (1945), the Census Bureau developed detailed manufacturing measures for the years 1939 and 1947, and the Department of Commerce issued more refined estimates of "output" for broad economic sectors. As was noted in Chapter I, the inadequacies of the traditional types of production and productivity indexes for collective bargaining purposes aroused considerable public and behind-the-scenes controversy during and after the War. As new war clouds gather, it will become evident that available industry statistics are no better, in view of the demands to be made on them, than the statistics available in 1941. The end-product emphasis of data compilations for industries converting to war production will lead to chronological discontinuities at an early date. Present legislation calls for a new manufacturing convass no sooner than 1953 -- and what happened when the Federal Reserve indexes lost their Census rudder is still fresh in memory.

Our study points to numerous projects which could be undertaken to advance production and productivity measurement. We have already noted that success would often require improbable excursions into the realm of increasing costs. The key to substantial further progress, we repeat, is the compilation of subproduct data. Availability of such data would permit refinement of indexes now available; extension of measurement to areas not now covered (e.g., certain manufacturing industries, construction, trade, finance, government, and personal and professional services); development of a hierarchy of consistent indexes for different levels of aggregation and of consistent indexes of different periodicity; and the closer study of the distribution of productivity gains and of the relevance of productivity to wage payments. Another important advance would result if data permitting construction of net output measures based on reduced aggregates were also made available. It would also seem worthwhile to test the value adjustment by specific inquiries into the comparative price behavior of products reported by value only and those reported by both value and quantity. For example, the movements of prices of complete products and of parts made by the same manufacturer should be traced through time; and case studies should be undertaken to establish the course of prices of new and standard products of particular industries. The reliability of deflation as a means of deriving approximations to directly constructed indexes also deserves attention. Some of the evidence adduced to support the value adjustment really applies to this question instead. Another inquiry should be directed to the comparability of the results obtained by means of the free composition, chain, and value-adjusted chain indexes as the product universe is expanded or contracted.

In addition to the compilation of indexes based on subproduct data, other productivity projects might be listed. Thus, it is desirable to clarify further the concept of factor of production and to continue consideration of the possibility of significantly quantifying capital and entrepreneurship for the purpose of composite productivity measurement. The statistical consequence of classifying labor input by skill, etc., would also be of interest. The relationship between productivity indexes computed directly and those derived as quotients from output measures with pecuniary weights should be tested whenever the former are constructible. More historical studies of individual firms are desirable, and the instrument for such studies is already available. Perhaps, the Bureau of Labor Statistics could finally tell us what happens to productivity, computed both on a subproduct and end-product basis, as full-capacity utilization is approached in job-lot and line-assembly establishments. The results given by formulas for partitioning total input changes should be examined more closely. How similar are the "contributions" measured forward in time and backward; and how similar are the "contributions" of reciprocal variables as estimated from alternative partition formulas? Finally, there has been too little economic analysis of the implications of the vast store of productivity information already available. We ought to know more, for example, about the role of productivity in what Colin Clark calls the "morphology of growth" -- about the connection between productivity and the stage of development, interindustry correlations, and future levels achievable in industrial societies.

Last but not least, our agenda must include the improvement of standards of index makers and users. It is especially regrettable that there is no indigenous theory of just what "physical" production indexes are supposed to measure, that the student must turn to the literature of other fields (like national income accounting and welfare economics) to get a clue. Eventually, the increase in the sophistication of index makers and users should lead to a demand for better data and methods. More immediate objectives would be cultivation of a favorable attitude toward technical inquiry and experimentation -- and of an appreciation of the multiplicity of operational meanings of the terms "production" and "productivity", even though alternative choices may not have such dramatic consequences as Shylock's losing a pound of flesh and Dido's gaining an incredible real estate bargain.

SELECTED REFERENCES

Adamson, R. K., and West, M. E. _Productivity and Employment in Selected Industries: Beet Sugar_. W. P. A. National Research Project Report No. N-1. Philadelphia: October, 1938.

Allen, R. G. D. "The Economic Theory of Index Numbers", _Economica_, XVI (August, 1949), 197-203.

Barger, H., and Landsberg, H. H. _American Agriculture, 1899-1939: A Study of Output, Employment, and Productivity_. New York: National Bureau of Economic Research, 1942.

________, and Schurr, S. H., _The Mining Industries, 1899-1939: A Study of Output, Employment and Productivity_. New York: National Bureau of Economic Research, 1944.

Barna, T. "Note on the Productivity of Labour: Its Concept and Measurement", _Bulletin of the Oxford Institute of Statistics_, VIII (July, 1946), 205-16.

Barton, G. T., and Cooper, M. R. "Relation of Agricultural Production to Inputs", _Review of Economics and Statistics_, XXX (May, 1948), 117-28.

Berlinguette, V. R. "Limitations on Measurements of Physical Production", A paper presented at the Harvard Meeting of the Econometric Society, September 5, 1950.

Black, J. D. _Introduction to Production Economics_. New York: Henry Holt & Co., 1926.

Burns, A. F. "The Measurement of the Physical Volume of Production", _Quarterly Journal of Economics_, XLIV (February, 1930), 242-62.

________. _Production Trends in the United States since 1870_. New York: National Bureau of Economic Research, 1934.

Carter, C. F., Reddaway, W. B., and Stone, R. _The Measurement of Production Movements_. Department of Applied Economics Monograph No. 1. Cambridge, Eng.: Cambridge University Press, 1948.

Copeland, M. A., and Martin, E. M. "The Correction of Wealth and Income Estimates for Price Changes", _Studies in Income and Wealth_, II, 85-119. New York: National Bureau of Economic Research, 1938.

Davis, H. S. _The Industrial Study of Economic Progress_. Philadelphia: University of Pennsylvania Press, 1947.

Day, E. E., and Thomas, W. _The Growth of Manufactures, 1899 to 1923_. Census Monograph No. 8. Washington: U. S. Government Printing Office, 1928.

Evans, W. D. and Siegel, I. H. "The Meaning of Productivity Indexes", Journal of the American Statistical Association, XXXVII (March, 1942), 103-11.

Fabricant, S. The Output of Manufacturing Industries, 1899-1937. New York: National Bureau of Economic Research, 1940.

_______. Employment in Manufacturing, 1899-1939: An Analysis of Its Relation to the Volume of Production. New York: National Bureau of Economic Research, 1942.

_______. "Of Productivity Statistics: An Admonition", Review of Economics and Statistics, XXXI (November, 1949), 309-11.

Fraser, L. M. Economic Thought and Language. London: A. & C. Black, Ltd., 1937.

Frickey, E. "Some Aspects of the Problem of Measuring Historical Changes in the Physical Volume of Production", in Explorations in Economics: Notes and Essays in Honor of F. W. Taussig, pp. 477-86. New York: McGraw-Hill Book Co., 1936.

Garfield, F. R. "Measurement of Industrial Production since 1939", Journal of the American Statistical Association, XXXIX (December, 1944), 439-54.

Geary, R. C. "The Concept of the Net Volume of Output with Special Reference to Irish Data", Journal of the Royal Statistical Society, CVII (Pts. 3-4, 1944), 251-61.

Gilbert, M., Jaszi, G., Denison, E. F., and Schwartz, C. F. "Objectives of National Income Measurement: A Reply to Professor Kuznets", Review of Economics and Statistics, XXX (August, 1948), 179-97.

Gould, J. M. Output and Productivity in the Electric and Gas Utilities, 1899-1942. New York: National Bureau of Economic Research, 1946.

Gourvitch, A. Industrial Change and Employment Opportunity -- A Selected Bibliography. W. P. A. National Research Project Report No. G-5. Philadelphia: July, 1939.

Hayek, F. A. v. "Scientism and the Study of Society: II", Economica, X (February, 1943), 34-63.

Hicks, J. R. "The Valuation of the Social Income", Economica, VII (May, 1940), 105-24.

_______. "On the Valuation of the Social Income -- A Comment on Professor Kuznets' Reflections", Economica, XV (August, 1948), 163-72.

Hudson, P. G. "The Technical Problems and Limitations to the Construction of Indexes of Physical Production", Journal of the American Statistical Association, XXXIV (June, 1939), 239-51.

Keynes, J. M. General Theory of Employment, Interest, and Money. New York: Harcourt, Brace & Co., 1936.

Knibbs, G. H. "The Nature of An Unequivocal Price-Index and Quantity-Index: I", Journal of the American Statistical Association, XIX (March, 1924), 42-60.

Kuznets, S. S. National Product in Wartime. New York: National Bureau of Economic Research, 1945.

________. National Income: A Summary of Findings. New York: National Bureau of Economic Research, 1946.

________. "On the Valuation of the Social Income -- Reflections on Professor Hicks' Article", Economica, XV (February and May, 1948), 1-16, 116-31.

________. "National Income: A New Version", Review of Economics and Statistics, XXX (August, 1948), 151-79.

Magdoff, H., Siegel, I. H., and Davis, M. B. Production, Employment, and Productivity in 59 Manufacturing Industries, 1919-36. W. P. A. National Research Project Report No. S-1. 3 vols. Philadelphia: 1939.

Mills, F. C. Economic Tendencies in the United States: Aspects of Pre-War and Post-War Changes. New York: National Bureau of Economic Research, 1932.

________. "Industrial Productivity and Prices", Journal of the American Statistical Association, XXXII (June, 1937), 247-62.

Pigou, A. C. "Comparisons of Real Income", Economica, X (May, 1943), 93-98.

Robbins, L. "Production", in Encyclopedia of the Social Sciences, XII, 462-67. New York: Macmillan & Co., 1934.

________. An Essay on the Significance of Economic Science. 2nd ed. London: Macmillan & Co., 1946.

Rostas, L. Comparative Productivity in British and American Industry. National Institute of Economic and Social Research Occasional Paper No. 13. Cambridge, Eng.: Cambridge University Press, 1948.

Searle, A. D., and Kaplan, C. J. Major Sources of Productivity Information. Processed. Washington: U. S. Bureau of Labor Statistics, June, 1949.

Sadler, G. E., and Searle, A. D. "Measurement of Unit Man-Hour Requirements", in Techniques of Preparing Major Statistical Series, pp. 42-49. U. S. Bureau of Labor Statistics Bulletin No. 993. Washington: 1950.

Samuelson, P. A. "Evaluation of Real National Income", Oxford Economic Papers, II (January, 1950), 1-29.

Shaw, W. H. Value of Commodity Output since 1869. New York: National Bureau of Economic Research, 1947.

Shoup, C. S. Principles of National Income Analysis. Boston: Houghton Mifflin Co., 1947.

Siegel, I. H. "The Difference between the Paasche and Laspeyres Index-Number Formulas", Journal of the American Statistical Association, XXXVI (September, 1941), 343-50.

________. "Further Notes on the Difference between Index-Number Formulas", Journal of the American Statistical Association, XXXVI (December, 1941), 519-24.

________. "Note on a Common Statistical Inequality", Journal of the American Statistical Association, XXXVIII (June, 1943), 217-22.

________. "The Concept of Productive Activity", Journal of the American Statistical Association, XXXIX (June, 1944), 218-28.

________. "The Generalized 'Ideal' Index-Number Formula", Journal of the American Statistical Association, XL (December, 1945), 520-23.

________. "Progress and Problems of Physical Output Measurement". A paper presented at 110th Annual Meeting of the American Statistical Association, December 27, 1950.

________. "A Half Century of American Productivity Measurement". A paper presented at 110th Annual Meeting of the American Statistical Association, December 28, 1950.

Stigler, G. J. Trends in Output and Employment. New York: National Bureau of Economic Research, 1947.

Thomas, W., and Conklin, M. R. "Measurement of Production", Federal Reserve Bulletin, XXVI (September, 1940), 912-23.

United Nations Statistical Office. Index Numbers of Industrial Production. Studies in Methods No. 1. New York: September 15, 1950.

U. S. Bureau of the Census. The U. S. Census Index of the Physical Volume of Manufacturing Production, 1939-47. Processed. Washington: January 18, 1951.

U. S. Bureau of Labor Statistics. Productivity and Unit Labor Cost in Selected Manufacturing Industries: 1919-1940. Processed. Washington: February, 1942.

_______. Selected References on Productivity. Processed. Washington: October, 1946.

_______. Summary of Proceedings of Conference on Productivity, October 28-29, 1946. Bulletin No. 913. Washington: 1947.

West, M. E. Productivity and Employment in Selected Industries: Brick and Tile. W. P. A. National Research Project Report No. N-2. Philadelphia: February, 1939.

W. P. A. National Research Project. The Work and Publications of the W. P. A. National Research Project on Reemployment Opportunities and Recent Changes in Industrial Techniques. Philadelphia: October, 1939.