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A Within Subjects Analysis of the Validity of the Expectancy Model of Motivation

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ABSTRACT

The validity of the expectancy model of motivation is examined within subjects employing the techniques of cognitive integration theory and functional measurement. Two experiments are reported. In the first experiment levels of expectancy and valence of outcomes associated with hypothetical jobs are experimentally manipulated in a factorial design. Subjects are asked to rate their motivation to accept alternative jobs. The goodness of fit of the expectancy model is analyzed graphically and statistically for each individual subject. Results indicate that, on the average, the multiplicative model of expectancy theory accounts for 83% of the variance in observed responses. The results of the first experiment also permit the derivation of ratio scale values of the levels of expectancy and valence employed for each subject. These derived scale values call into question the validity of scaling levels of expectancy on a continuum of 0.0. to 1.0. In experiment two, the same subjects are asked to rate their level of motivation to accept six actual full-time jobs. Expectancy theory predictions employing ratio scale values of expectancy and valence for each subject exhibit a mean correlation of .47 with observed reponses. Of additional interest, an equal weight model (simple adding of appropriately signed expectancies) employing an arbitrary equal interval coding of levels of expectancy for all subjects correlates with observed responses .50 on average. The implications of these findings for both theory and prediction are explored.

The study of motivation in organizations has been greatly influenced by the work of Vroom (1964). Vroom presented a cognitive model which hypothesizes that the force acting upon an individual to engage in an activity is a function of the sum of the products of the valences of all outcomes and the expectancies that the activity will lead to the attainment of these outcomes. The theory then predicts that the individual will choose to perform that activity having the strongest positive or weakest negative force.

Since its appearance the theory has been revised, modified, and extended by a number of authors. The distinction between first- and second-level outcomes has been clarified by Galbraith and Cummings (1967), Graen (1969), and Lawler (1973). Campbell, Dunnette, Lawler and Weick (1970) have presented a detailed and extended "hybrid" expectancy model which introduces a distinction between task goals and first-level outcomes. The distinction between intrinsic and extrinsic rewards drawn by Galbraith and Cummings (1967) has been further elaborated by House (1971) who distinguishes two different types of intrinsic valence. These suggested revisions have essentially involved the postulation of additional relevant independent variables for the model and the more precise specification and differentiation of different classes of expectancies and valences. However none of the proposed modifications has suggested an alternative to the basic cognitive process of multiplication of appropriate expectancies and valences.

In addition to theoretical development, a considerable body of empirical research has been generated aimed at testing the validity of the model and determining its predictive utility when applied to behavior in organizations. The results of these empirical studies have been mixed. Suttle (1975) cites 61 studies and concludes that 50 provide results generally supportive of expectancy theory, ten provide no support, and one yields mixed results. Mitchell (1974) summarizes 22 studies based upon Vroom's model of motivational force which employ job effort as the criterion variable, and five such studies employing specific job behaviors as the criterion. A total of 34 correlation coefficients are reported in the former category, and 9 in the latter. For both groups the median correlation between expectancy theory predictions and criterion measures is 0.30. Thus, although empirical findings tend to be generally supportive of the expectancy model, the strength of observed relationships is often disappointingly small.

This general pattern of results has recently lead a number of active researchers in the field to suggest that what is needed is not further elaboration of ever more complex models, but rather a careful analysis of the validity of the basic multiplicative model. Mitchell (1974), at the conclusion of his extensive review of research on expectancy theory, presents two quotations as a summary of the current status of the expectancy model. Lawler and Suttle (1973) conclude that "the theory has become so complex that it has exceeded the measures which exist to test it" (p. 502). Behling and Starke (1973b) argue that what is needed at this time is "a shift in research emphasis from extension and refinement to testing of basic interactive relationships" (p. 25). Mitchell himself concludes that "while it is relatively clear that expectancies, instrumentalities, and valences are significantly related to their various criteria, <u>we really know very little about just how the relationship occurs</u>" (p. 1074). (Emphases added).

The research reported here carefully examines the nature of this relationship between expectancies and valences with a view toward the determination of the validity of the model of motivational force put forward by Vroom.

Vroom's Model of Motivation

The proposition containing Vroom's basic statement of the model of motivational force is as follows:

> "The force on a person to perform an act is a monotonically increasing function of the algebraic sum of the products of the valences of all outcomes and the strength of his expectancies that the act will be followed by the attainment of these outcomes." (p. 18)

This proposition can be stated algebraically as follows:

$$F_i = f_i\left[\sum_{j=1}^{n} (E_{ij}\ V_j)\right] \quad (i=n+1,\ldots,m)$$

$$f_i' > 0;\ i \cap j = \phi\ ,\ \phi \text{ is the null set}$$

where

F_i = the force to perform act i

E_{ij} = the strength of the expectancy ($0 \le E_{ij} \le 1$) that act i will be followed by outcome j

V_j = the valence of outcome j

Immediately following the proposition, Vroom states that "It is also assumed that people choose from among alternative acts the one corresponding to the strongest positive (or weakest negative) force" (p. 19).

The Nature of the Model

(1) The theory is a within-subject choice model. It is designed to predict which of a number of alternative acts a particular individual will choose to perform. As such the appropriate level of analysis for applications of the theory is the single subject. Although Vroom's statement of the model is clearly designed for application to the choices made by a single individual, investigators have consistently "tested" the theory using across-subject designs. If one obtains measures of the force to perform act i from a sample of n individuals, it is impossible to make any prediction regarding the likelihood that any one of these individuals will actually engage in act i . Such predictions require measures of the force currently acting on each individual to perform acts other than act i . The theory could then be used to predict that those individuals for whom the force to perform act i is strongest (relative to the forces acting on that particular individual to perform other actions) would in fact choose to perform act i .

This basic discrepancy between Vroom's original formulation of the theory and subsequent empirical investigations has been pointed out by only four authors (Behling & Starke, 1973a; Mitchell, 1972; Nebeker & Mitchell, 1974; Mitchell, 1974). In his review of the literature Mitchell (1974) was not able to locate a single study which employed a within-subject analysis for the prediction of job effort (utilizing Vroom's model of the determinants of force).[1] An examination of research appearing since the publication of Mitchell's review fails to reveal any studies employing a within-subject design.

(2) The theory consists of a model of cognitive processing. It provides a model of the way in which individuals combine "subjective values" (expectancies and valences) to arrive at an "integrated impression" (force). The basic tenet of the model is that expectancies and valences multiply to determine force. Vroom presents logical and theoretical arguments to support the hypothesized multiplicative cognitive process. None of the revised versions of expectancy theory has seriously proposed an alternative to multiplication as the basic cognitive process involved, although some have correctly pointed out that currently available empirical evidence is inadequate to permit a decision to be made regarding the validity of the multiplying hypothesis (Campbell, Dunnette, Lawler, & Weick, 1970, p. 348).

Empirical investigations however have often compared the goodness of fit of models based upon different combinations of expectancies, valences, and instrumentalities, e.g., simple adding of expectancies or instrumentalities. The examination of the relative goodness of fit of various models to a single set of empirical data can often be suggestive and informative. However, there exist theoretical, algebraic, and statistical grounds for suggesting the exercise of caution in interpreting the results

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[1] Only four studies have employed a within-subject analysis in studying the determinants of valence (Sheard, 1970; Dachler & Mobley, 1973; Holmstrom & Beach, 1973; Nebeker & Mitchell, 1974).

of such curve fitting as evidence in favor of an alternative model of cognitive processing.

(a) Theoretical. Alternative models developed post hoc based upon the results of the statistical analysis of a single set of data are obviously of questionable value. At least sixteen studies have suggested alternative formulations of either the valence or the motivation model, or have employed statistical analyses based upon modifications of the multiplicative relationship (Georgopoulos, Mahoney, & Jones, 1957; Mikes & Hulin, 1968; Graen, 1969; Arvey & Dunnette, 1970; Gavin, 1970; Sobel, 1971; Arvey, 1972; Heneman & Schwab, 1972; Mitchell & Albright, 1972; Dachler & Mobley, 1973; Pritchard & DeLeo, 1973; Pritchard & Sanders, 1973; Reinharth & Wahba, 1973; Schwab & Dyer, 1973; Reinharth & Wahba, 1974; Sheridan, Slocum & Min, 1975). Interpretation of the validity of an alternative model requires attention not only to the relative magnitude of validity coefficients, but also to: (i) the psychological implications of the alternative formulation as a model of cognitive processing, i.e. besides fitting the data does the alternative model make logical, theoretical, or intuitive sense as a model of cognitive processing?, and (ii) the statistical reasons why an alternative paramorphic model may provide a better fit to the data, regardless of the nature of the underlying psychological process (c.f. Dawes and Corrigan, 1974; and Discussion below).

(b) Algebraic. Research results reporting significant main effects for expectancy and/or valence in addition to an interaction effect for their product are sometimes interpreted as evidence favoring a combined adding/multiplying model, e.g., Pritchard & DeLeo, 1973. It is important to keep in mind the fact that if a significant two-way interaction exists, then the observed main effects can be interpreted as an artifact of the scaling of the independent variables and can be removed by an appropriate (and theoretically meaningful) rescaling of expectancy and valence.[2]

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[2] This can be seen more clearly if we remember that any equation of the form:

$$F = \mu + a_1 E + a_2 V + a_3 (E x V)$$

can be rewritten as:

$$F = \mu' + a_4 (E' x V')$$

where

$$E' = E + k_1$$

$$V' = V + k_2$$

and

$$\mu' = \mu - a_1 k_1 - a_2 k_2 + a_3 k_1 k_2$$

$$a_4 = a_3$$

$$k_1 = \frac{a_2}{a_3}$$

$$k_2 = \frac{a_1}{a_3}$$

(c) Statistical. If one has a set of n observations, then an optimally weighted linear model with any (n-1) independent variables can always account for 100% of the variance in the observed responses. If one has more independent variables than observations, then almost any coding of the independent variables and any set of weights will account for a large proportion of the variance in observed responses. Studies which have employed more independent variables than observations (e.g., Sheard, 1970; Holmstrom & Beach, 1973) can thus shed very little light on the question of the underlying model of cognitive processing, regardless of the magnitude of correlations.

(3) The dependent variable predicted by the model is a cognitive construct, the Lewinian concept of "force." The dependent variable is not behavior. This fact obviously presents a problem for any adequate test of the validity of expectancy theory, since the dependent variable of the model is an unobservable. One must therefore employ a criterion measure which is as close in psychological meaning to the concept of force as possible (i.e., least contaminated by other factors), e.g., self-reports of level of motivation or strength of inclination to engage in the activity. Performance scores present difficulties as a valid criterion unless equal ability can be assumed, or unless highly valid and reliable measures of ability are available and can be partialled out of the performance ratings.

Effort may only serve as a valid criterion in a very limited set of circumstances, and even then its use requires an additional assumption or hypothesis. If we have obtained a measure of the force acting upon an individual to engage in one activity, e.g., "going to work in the morning," the theory makes no predictions regarding the likelihood of that activity occurring, nor the effort which the individual might put into that activity. In order to make a prediction, the theory requires that we also measure the force acting upon the individual to engage in other alternative behaviors, e.g., "not going to work in the morning." If the force acting upon the individual to perform one of these alternative actions is greater than the single measure of force which we have obtained, the theory predicts that the individual will choose the alternative action with highest force and hence will put no effort whatsoever into performing the action whose associated force we have measured.

If however we are studying a situation in which there are no alternatives available to the individual, i.e., there is only one set of activities in which he is permitted to engage, we might hypothesize that the effort which an individual would invest in that set of activities is proportional to the force associated with that set of activities for that individual. We would then expect to observe a correlation across individuals between force and level of effort. But it is important to keep in mind that this constitutes an additional untested hypothesis which is not contained within the expectancy model put forward by Vroom (1964), and that we would only expect the hypothesis to be valid in an extremely limited and unrepresentative class of situations. Vroom clearly states that "we view the central problem of motivation as the explanation of choices made by organisms among different voluntary responses" (p. 9). The expectancy model of motivation is designed to explain choices among alternative acts, and does not explicitly address the issue of the magnitude of response following a choice.

(4) The theory does not specifically address:

(a) The measurement or psychophysical scaling issues involved in the way in which objective stimuli are translated into subjective values of expectancy and valence,

(b) the nature of the "response function" between the "integrated impression" (force) and the overt response (behavior). (An assumption is made that the individual will choose to perform that act having the strongest positive or weakest negative force.)

Expectancy, valence, and force are all unobservable cognitive constructs. Obviously some psychological process must exist whereby objective stimulus information is translated by the individual into subjective values of expectancy and valence. Similarly, a psychological process must exist for the transition from the cognitive construct of force into an overt response. The expectancy model says nothing about the nature of these two processes and focuses exclusively upon the process whereby expectancies and valences combine to yield force. The issue has similarly been ignored by investigators. Research designs employing regression-correlation techniques to predict observed responses from physical stimulus values of expectancy and valence confound all three psychological processes in a single model and assume that both the input and the output processes are purely linear (c.f. Birnbaum, 1974).

(5) Since the theory postulates a multiplicative relationship between expectancy and valence, an adequate test of the theory requires that these variables be measured on a ratio scale. Schmidt (1973) has drawn attention to this point. None of the published research to date has employed ratio or even interval scales of expectancy and valence. Schmidt discusses techniques for the derivation of ratio scales developed by Thurstone and Jones (1957) and Krantz and Tversky (1971). These techniques are relatively complex and time consuming. An alternative methodology of functional measurement has been developed by Anderson (1962a,b, 1970, 1971) which permits the joint validation of a theoretical model and derivation of interval (or ratio) scales of the independent variables. This methodology serves as the guide for the design of the experiments reported below.

Hypotheses

The preceding discussion of the nature of the expectancy model and the situations to which its application is appropriate makes clear that an adequate test of the multiplicative hypothesis has not been carried out to date. An adequate test of the validity of the theory requires a design which:

(1) employs the single subject as the unit of analysis,

(2) permits an unambiguous examination of the nature of the model of cognitive information integration which best accounts for the data,

(3) employs a criterion measure as close as possible in psychological meaning to the concept of force,

(4) explicitly addresses the nature of the psychological processes according to which physical stimulus information is translated into subjective expectancies and valences, and by which force is translated into an overt response, and

(5) addresses the issue of the scaling of the independent variables and yields ratio scales of expectancy and valence.

Two experiments are reported which fulfil these methodological prerequisites. The first experiment involves a direct application of Anderson's functional measurement methodology to the examination of the validity of the multiplicative hypothesis of the expectancy model. The expectancy and valence of outcomes associated with each of a pair of two hypothetical full-time jobs are varied in a factorial design. Subjects are asked to judge their motivation to accept one or the other job. The design permits a test of the following two hypotheses:

H1: The force on a person to perform an act is a monotonically increasing function of the algebraic sum of the products of the valences of all outcomes and the strength of his expectancies that the act will be followed by the attainment of these outcomes (Vroom, 1964, p. 18).

H2: When faced with a situation involving a choice between two alternative acts, the strength of an individual's motivation or inclination to perform one act rather than the other is a monotonically increasing function of the difference of the forces associated with the two acts.

In addition to permitting a test of the above two hypotheses, the functional measurement approach also permits the derivation of ratio scale values of the levels of the independent variables employed (in this case, levels of expectancy and valence). In Experiment 2 measures are obtained of subjects' expectancies of obtaining the set of outcomes employed in Experiment 1 in each of a number of real jobs. Subjects are then asked to rate their motivation or inclination to accept one job rather than the other for each possible pair of the jobs. The validity of the following hypothesis is examined:

> H3: Predictions of motivation based upon ratio-scaled values of expectancy and valence for each subject should provide better predictions of observed motivation than any alternative coding of expectancy and valence.

Appendix 1 contains a brief discussion of the functional measurement approach and a detailed outline of the specific techniques involved in the application of the method to the expectancy model.

RESEARCH METHODOLGY

Experiment 1

Subjects judged the strength of their motivation or inclination to accept one of two full-time jobs, JOB X and JOB Y. For each item, the subject was asked to imagine himself in a situation in which he had been offered both jobs and had to make a decision regarding which of the two jobs he was most inclined to accept. The subject was told to assume that:

(1) the two jobs were very similar,

(2) he had equal ability to perform both jobs,

(3) he could expect both jobs to lead to equal outcomes except for the information contained in each item.

Each item then provided the subject with information regarding the probability of a single outcome occurring as a result of choosing JOB X, and the probability of a different outcome occurring as a result of choosing JOB Y. Subjects indicated their inclination to accept one or the other job on a continuous response scale separating the two jobs. Details of the stimulus design, instruments employed, and experimental procedure are discussed below.

Stimulus Design

The design employed was a (5 x 5) x (2 x 3) factorial with five levels of expectancy and five levels of valence associated with JOB X and two levels of expectancy and three levels of valence associated with JOB Y. Verbal probability statements were employed to convey levels of expectancy. The five probability statements associated with JOB X and the two associated with JOB Y are shown in Table 1, along with the subjective probability range of each statement reported by Shanteau (1974, p. 681). The five levels of expectancy for JOB X were chosen to span the range from 0.0 to 1.0 in approximately equal steps. Two intermediate levels were chosen for JOB Y.

Insert Table 1 about here

Levels of valence were operationalized by statements of real job outcomes. The outcomes employed in constructing the items for each subject were selected by the subject from the list of fourteen job outcomes contained in Table 2.

Insert Table 2 about here

In constructing this list an effort was made to include only concrete and specific outcomes to which it would make sense to attach some probability of occurrence or non-occurrence on a job. An outcome was not included if the only uncertainty associated with it could be the amount of the outcome attained, rather than its occurrence or non-occurrence, e.g., "feeling tired at the end of the day." The goal was thus to include outcomes to which it makes sense to attach an expectancy rather than an instrumentality.

Each subject selected from this list the two outcomes which he personally valued most positively, moderately positively, moderately negatively, most negatively and the two to which he was indifferent. The five levels of valence for JOB X were then operationalized for each subject by employing five outcomes which the individual subject had rated as MOST POSITIVE, MODERATELY POSITIVE, INDIFFERENT, MODERATELY NEGATIVE, MOST NEGATIVE. The three levels of valence for JOB Y were operationalized by three different outcomes from the list rated by the subject as MOST POSITIVE, INDIFFERENT, MOST NEGATIVE.

The full factorial (5 x 5) x (2 x 3) design results in a questionnaire consisting of 150 items. Since only six unique combinations of expectancy and valence were employed associated with JOB Y, each combination appears 25 times in the full questionnaire. In order to assure that subjects were always forced to attend closely to and process the stimulus information regarding JOB Y in each item, an additional fourteen dummy items were included in the questionnaire employing different levels of probability and different outcomes for JOB Y from those included in the factorial design. The questionnaire was thus 164 items in length.

Subjects

The subjects were 31 undergraduates, 16 females and 15 males, registered in undergraduate psychology courses. Participation was voluntary. Subjects were run individually for four sessions of 45 minutes to one hour on separate days. Subjects were paid at the rate of $2.50 per session and received the total payment of $10 at the conclusion of the fourth session.

Procedure

Subjects were recruited from four undergraduate psychology courses. The experimenter attended one class in each course and at the conclusion of the lecture briefly described the nature of the research and explained what would be required of subjects. Students interested in participating stayed behind at the conclusion of the presentation and were asked to complete the Job Outcomes Questionnaire. This questionnaire lists the fourteen job outcomes contained in Table 2 and asks the respondent to select the two outcomes which he personally values:

(1) most positively

(2) next most positively

(3) most negatively

(4) next most negatively

(5) neither positively nor negatively

After completing the Job Outcomes Questionnaire each subject was individually scheduled for four one-hour testing sessions. The sessions were scheduled on consecutive days for the majority of subjects. All subjects completed all four sessions within a one-week period.

A computer program was written which generated an individualized set of questionnaires for each subject based upon responses to the Job Outcomes Questionnaire. One of the outcomes from each of the above five categories were employed as the five levels of valence for JOB X. The remaining MOST POSITIVE, MOST NEGATIVE, and MOST INDIFFERENT outcomes served as the three levels of valence for JOB Y. The appropriate outcomes selected by each subject constituted the input data to the program. The program then generated four copies of the Job Preference Questionnaire for each subject, one for use in each testing session. Each copy of the questionnaire contained the 150 individualized items of the (5 x 5) x (2 x 3) factorial design employing only those job outcomes selected by the subject, plus the fourteen dummy items. Each copy of the questionnaire contained the subject's name and the session number on the first print-out page. The questionnaire employed on the first day of testing contained detailed instructions to the subject regarding the nature of the choice situation and the type of judgement to be made in response to each item. Also included were a sample item and five practice items.

Figure 1 contains a sample item from the Job Preference Questionnaire. Each item contains information regarding the probability of JOB X leading to one outcome, and the probability of JOB Y leading to a different outcome. The probability statement and the job outcome were underlined. Each item was followed by a response scale 15 centimeters long labeled JOB X at one extreme and JOB Y at the opposite extreme. The midpoint of the response scale was indicated with a slash. The subject was asked to respond to each item by placing a vertical mark on the line separating JOB X and JOB Y at the point best indicating his inclination toward taking one or the other job. Subjects were told to try to make use of the entire length of the continuum by placing their response at the exact point on the line best indicating their inclination toward taking one or the other job. Subjects were told to consider each item as describing a totally new and different situation, and to make their judgement for each item based only upon the information in that item.

Insert Figure 1 about here

The first four items of each questionnaire were extreme valued items employed to anchor the scales. The remaining 160 items were printed in random order by the computer program. Three items were printed per page.

Responses were scored by measuring the distance from the left-most extreme of the response scale to the point indicated by the subject to the nearest millimeter. Responses could thus take on values from 0 to 150, with a score of 75 indicating no preference for one job over the other.

Upon arrival for the first day's testing subjects were presented with their individualized Job Preference Questionnaire for Session 1 and told to read through the instructions and respond to the practice items. At the conclusion of the instructions subjects were asked to explain to the experimenter in their own words the nature of the judgments to be made in responding to each item. Subjects then completed the full 164 item questionnaire. In days two through four subjects explained the task in their own words to the experimenter prior to responding to the complete set of 164 randomly ordered items. The majority of subjects required approximately 40 minutes to complete the questionnaire in each session.

Data Analysis

The model based upon Hypotheses 1 and 2 for the choices made by subjects in responding to the Job Preference Questionnaire is as follows:

$R_{xy} = F_x - F_y$

R_{xy} = resultant force

F_x = force associated with JOB X

F_y = force associated with JOB Y

and

$F_x = E_x \times V_x$

$F_y = E_y \times V_y$

E_x = expectancy of obtaining the outcome associated with JOB X

V_x = valence of the outcome associated with JOB X

E_y = expectancy of obtaining the outcome associated with JOB Y

V_y = valence of the outcome associated with JOB Y

The graphical and statistical tests of the goodness of fit of this model outlined in Appendix 1 were carried out, and the methods described there for the derivation of ratio scales of the independent variables were applied.

Experiment 2

Measures were obtained of subjects' expectancies of obtaining each of five outcomes in six full-time jobs. Subjects were then presented with each of the fifteen possible pairs of the six jobs and asked to judge their motivation or inclination toward accepting one or the other job.

Instruments

Two questionnaires were employed in Experiment 2, an Expectancy Assessment Questionnaire and a Job Choice Questionnaire. Both questionnaires were generated by a computer program.

The Expectancy Assessment Questionnaire was designed to obtain measures of subjects' expectancies of obtaining the five job outcomes which had been associated with JOB X in Experiment 1 (VERY POSITIVE, MODERATELY POSITIVE, INDIFFERENT, MODERATELY NEGATIVE, VERY NEGATIVE) in each of the following six full-time jobs.

(1) Newspaper reporter

(2) Bank teller

(3) Management trainee in a large industrial corporation

(4) Auto assembly line worker

(5) Insurance salesman

(6) Librarian

The title of each job was listed in turn at the top of each page of the questionnaire. The job title was then followed by a list of the five job outcomes which had been associated with JOB X in Experiment 1 for that subject. Below each job outcome the five probability statements associated with JOB X in Experiment 1 were listed across the page (No chance, Somewhat unlikely, Not quite even chance, Fairly likely, Sure thing). Subjects were instructed to respond to the items by circling the probability statement which best indicated the degree to which they would expect to attain the given outcome in the given job. The instructions to subjects stressed that they were to respond to the items in terms of their own personal beliefs regarding the likelihood of the outcomes occurring in each of the jobs.

The Job Choice Questionnaire presented subjects with the fifteen possible pairs of the six jobs. Each item consisted of a response scale 15 centimeters long with the title of one job at the left extreme of the continuum and the title of a second job at the right extreme. For each item subjects were instructed to imagine themselves in a situation in which they were forced to choose between the two full-time jobs. Subjects indicated their inclination toward accepting one or the other job by placing a vertical line on the continuum separating the two jobs at the exact point best indicating their inclination toward taking one or the other job.

The fifteen items were randomly ordered for each subject. Each job title appeared a total of five times in the fifteen items. The items were constructed such that each job title appeared at least twice at both the left and right extremes of the continuum.

Responses to the Job Choice Questionnaire were scored by measuring the distance from the left-most extreme of the response continuum to the point indicated by the subject. Responses were measured to the nearest millimeter and could thus take on values from 0 to 150.

Subjects

The same subjects were employed in Experiment 2 as in Experiment 1. Subjects did not receive additional payment for participation in Experiment 2.

Procedure

Subjects were presented with the Expectancy Assessment Questionnaire and the Job Choice Questionnaire after completing the Job Preference Questionnaire in Session 4 of Experiment 1. The two questionnaires were contained within the same computer print-out. The subject's name and the questionnaire title were contained on the first page of each instrument. Subjects were instructed to read the instructions and respond to each questionnaire in turn. The experimenter was available to answer any questions regarding the interpretation of the instructions.

Subjects were able to complete the two questionnaires in five to ten minutes.

At the conclusion of Experiment 2 each subject was paid and thoroughly debriefed regarding the nature of the research and the hypotheses being examined.

Data Analysis

Subjects' responses to the Expectancy Assessment Questionnaire were employed to compute predicted responses to the Job Choice Questionnaire.

The following model was employed in computing a predicted response:

$$R_{jk} = F_j - F_k$$

where

R_{jk} = resultant force to choose job j rather than job k

F_j = force associated with job j

F_k = force associated with job k

and

$$F_j = \sum_{i=1}^{5} E_{ij} V_i$$

$$F_k = \sum_{i=1}^{5} E_{ik} V_i$$

E_{ij} = expectancy of obtaining outcome i on job j

E_{ik} = expectancy of obtaining outcome i on job k

V_i = valence of outcome i.

Two sets of predicted scores were computed. One set was obtained employing ratio scale values for the five levels of expectancy and valence for each subject. These ratio scale values were obtained for each subject from the results of Experiment 1 according to the procedures outlined in Appendix 1. A second set of predicted scores were computed employing the arbitrary codings of expectancy and valence in Table 3 for all subjects.

Insert Table 3 about here

For each subject, three predicted scores were computed using each of the coding schemes:

(1) Theoretical model prediction. Scale values of expectancy were multiplied by scale values of valence and summed to determine predicted force for each job:

$$F_T = \Sigma EV.$$

(2) Equal (unit) weight prediction. Expectancies associated with positively valent outcomes were assigned a weight of +1, those with negatively valent outcomes were assigned -1, and those associated with an indifferent outcome were assigned weight 0. The appropriately signed expectancies were then summed to determine predicted force:

$$F_E = \Sigma E.$$

(3) Optimal linear model prediction. The observed responses were regressed on the expectancy scale values. The beta weights obtained from the least squares solution of the regression equation provide estimes of scale values of valence which would optimally fit the observed data. The model is thus:

$$F_0 = \Sigma bE.$$

For each subject observed responses to the 16 items of the Job Choice Questionnaire were correlated with each of the six sets of predicted scores.

RESULTS

Experiment 1

Graphical Analysis

Figure 2 plots the combined effect of Expectancy X (the expectancy of obtaining the outcome associated with JOB X) and Valence X (the valence of the outcome associated with JOB X) on subjects' responses (resultant force). The data reported in Figure 2 are mean responses for the entire sample.[3] The means plotted for each combination of Expectancy X and Valence X are collapsed over all combinations of Expectancy Y (the expectancy of obtaining the outcome associated with JOB Y) and Valence Y (the valence of the outcome associated with JOB Y). The spacing of levels of Expectancy X along the horizontal axis corresponds to the subjective values associated with each of the probability statements. Levels of Expectancy X are coded in Figure 2 as follows:

1 = No chance
2 = Somewhat unlikely
3 = Not quite even chance
4 = Fairly likely
5 = Sure thing.

Each curve corresponds to a level of Valence X (an outcome) and is coded as follows:

1 = Most positive
2 = Moderately positive
3 = Indifferent
4 = Moderately negative
5 = Most negative.

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[3] Data were first analyzed and plotted for each individual subject. The means of these individual level analyses were then obtained and plotted to provide a summary of responses for the whole sample.

Responses were coded such that 0 (zero) corresponds to maximum motivation to choose JOB X, 150 corresponds to maximum motivation to choose JOB Y, and 75 indicates indifference. Thus, the smaller the response magnitude, the stronger was the inclination to accept JOB X rather than JOB Y, and vice versa.

Insert Figure 2 about here

Figure 3 plots the combined effect of Expectancy Y and Valence Y on observed responses collapsing over all combinations of Expectancy X and Valence X. Levels of Expectancy Y are coded as:

1 = Unlikely

2 = Better than even chance.

Levels of Valence Y are coded as:

1 = Most positive

2 = Indifferent

3 = Most negative.

Insert Figure 3 about here

These plots permit a direct analysis of the validity of the multiplicative model of Hypothesis 1, i.e., that $F_x = E_x \times V_x$ and $F_y = E_y \times V_y$. The pattern of results of Figures 2 and 3 provides strong support for the multiplying model. The curves form a diverging fan and are very nearly linear, as is required for a multiplying process. The curves do not however converge at the left-most extreme corresponding to a probability of "No chance". The curves appear to converge at a point to the right of this value, indicating that subjective values of expectancy may take on negative values (see discussion of subjective values below).

Statistical Analysis

The results of the analysis of variance for the full model in terms of variance explained are summarized in Table 4. Analyses were carried out separately for each subject and then averaged. The following results are relevant to the validity of Hypotheses 1 and 2.

(1) The full model (all factors and interactions) can account for 87% of the variance in observed responses (resultant force).

(2) Of the factors predicted by the multiplicative expectancy model:

(a) the two predicted 2-way interactions (E_x x V_x and E_y x V_y) account for 46% of the total variance

(b) the main effects account for 37% of the variance.

The effects predicted by the multiplicative model thus account for 83% of the total variation in resultant force. The predicted effects account for 94% of the total explained variance.

(3) No single effect not predicted by the expectancy model accounts for more than 1.2% of the variance.

Insert Table 4 about here

The critical test of the validity of the multiplicative model is the bilinear analysis outlined in Appendix 1. If the simple multiplying model is valid, the significant 2-way interactions of E_x x V_x and E_y x V_y should be concentrated in the bilinear (linear x linear) component.

Table 5 summarizes the percentage of the 2-way interactions concentrated in the bilinear component for each subject. The E_x x V_x interaction has a median bilinear trend accounting for 90% of this 2-way interaction. The median bilinear trend of the E_y x V_y interaction accounts for 95% of the 2-way interaction. These results constitute strong support for the validity of the multiplicative hypothesis.

Insert Table 5 about here

The graphical and statistical analyses lead to the following conclusions:

(1) Hypothesis 1 is supported. The predicted two-way interactions are significant and are concentrated in the bilinear component. The effects predicted by the multiplying model account for 83% of total variance.

(2) Hypothesis 2 is supported. The two-way interactions not predicted by the model, as well as the higher order interactions, do not account for important components of the overall variation. This indicates that the relationship between F_x (E_x x V_x) and F_y (E_y x V_y) is not multiplicative and hence must be a simple adding/subtracting relationship.

Subjective Values

When the general pattern of results of the graphical and statistical tests of fit provides support for the hypothesized model, the marginal means of the two-way tables employed in the graphical test and the bilinear analysis take on psychological meaning as valid estimates of subjective values of the stimuli. The existence of a clear point of convergence permits the transformation from interval to ratio scaling of the independent variables.

Subjective values of the stimuli were derived for the five levels of expectancy and five levels of valence associated with JOB X. Assignment of specific numerical values on a ratio scale to each of the stimulus levels employed requires recalibration of the interval scales derived from the marginal means. In the case of the levels of expectancy, this was accomplished by assigning the value 0.0 to the point of crossover (the functional zero point for each subject) and the value 1.0 to "Sure thing." The ratio scale values for the remaining levels of expectancy are then assigned in relation to these two points based upon the marginal mean associated with each level.

In addition to the hypothesized multiplicative relationship, the expectancy model also assumes that expectancy takes on values from 0.0 to 1.0 (a unipolar scale), and valence takes on both positive and negative values (a bipolar scale). Such a scaling of the independent variables would predict a fan of curves converging to the left, i.e., converging at the point at which there is no chance of an outcome occurring. The results reported in Figure 2 for the whole sample obviously do not all conform to this pattern. Plots of results for the majority of individual subjects also indicate a crossover effect rather than a convergence to the left. Since this pattern of results was not predicted by the hypothesized underlying scales, a test was required to determine whether clear evidence of a crossover existed, or whether the apparent crossover could be attributed to experimental noise or measurement error. If a true crossover interaction exists, then mean responses to the five levels of valence associated with an expectancy of "No chance" should be significantly different from one another.

In order to rest for such a significant difference a one-way analysis of variance was carried out on the responses of each subject to items in which E_x was coded as "No chance". An F-ratio indicating significantly different responses to the five levels of valence associated with an expectancy of "No chance" was taken as evidence of a true crossover effect. The results for 23 of the 31 subjects exhibited such a significant crossover effect. The value of 0.0 was assigned to the point of crossover for these subjects in deriving the ratio scales of expectancy. For the eight subjects not exhibiting evidence of a significant crossover, the value 0.0 was assigned to the marginal mean associated with expectancy "No chance".

The resulting ratio scale values of the five levels of Expectancy X for each subject are reported in Table 6. Average scale values for the whole sample for each level of Expectancy X are summarized in Table 7.

Insert Table 6 about here

Insert Table 7 about here

To derive ratio scales of the five levels of Valence X for each subject, the crossover point was assigned the value 0.0 and the level of valence with the largest associated marginal mean (i.e., the outcome which the subject had interpreted as most positive in his responses to the Job Preference Questionnaire) was assigned the value +10.0. The resulting subjective values of the five outcomes associated with JOB X for each subject are reported in Table 8.

Insert Table 8 about here

Since the particular outcomes employed were uniquely determined for each subject, and since no single outcome was included for which a common subjective value could be assumed for all subjects, it is impossible to determine the average value of each outcome for the whole sample. The subjective values reported in Table 8 make sense as a ratio scale of the levels of valence for each particular subject. These ratio scale values can thus be employed in computing predicted force scores for each individual subject in Experiment 2. However it is not possible to meaningfully compare or average scale values associated with a given outcome across subjects.

Experiment 2

Table 9 reports correlations of predicted and observed scores for the Job Choice Questionnaire of Experiment 2. For each subject two sets of predicted scores were computed, one set employing ratio scale values of expectancy and valence derived in Experiment 1, the second employing the arbitrary scale values in Table 3. Within each coding system, predicted scores were obtained for the theoretical model ($F_T = \Sigma ExV$) summing products of expectancy and valence, for the equal (unit) weight model ($F_E = \Sigma E$) summing appropriately signed expectancies, and for the optimal linear model ($F_0 = \Sigma bE$) regressing observed responses on expectancies.

Insert Table 9 about here

Hypothesis 3 is not supported. Of the six sets of predicted scores obtained, those employing the theoretical model and individualized ratio scale values of expectancy and valence have the lowest mean correlation with observed responses. The predictive validity of the theoretical model employing individualized ratio scale values (R^2 = .22) is far less than that of the optimal linear model employing the same scale values of expectancy (R^2 = .83). The equal (unit) weight model predicts as well as the theoretical model using ratio scale values. Predicted scores of expectancy and valence employing arbitrary equal interval scale values of expectancy and valence exhibit correlations with observed responses which are as large or larger than those obtained from predictions based upon individual ratio scale values of expectancy and valence.

DISCUSSION

The Multiplicative Process

The results of Experiment 1 provide strong support for the hypothesized multiplying process of the expectancy model. The effects predicted by the multiplying model accounted for 83% of the variance in responses to the Job Preference Questionnaire items. In addition, the pattern of results lent support to the hypothesis that when faced with a choice between two alternative actions, the strength of motivation to perform one act rather than the other is a function of the difference of the forces associated with the two acts.

These results are very encouraging. However, it is extremely important to keep in mind the precise nature of the judgment or choice task employed in Experiment 1 and the extent to which these judgments are representative of "real" choices which individuals make in organizational settings.

The Job Preference Questionnaire of Experiment 1 was designed to permit an examination of the way in which individuals combine information regarding the expectancy and valence of outcomes in arriving at judgments regarding their motivation or inclination to accept one job rather than another. The goal was to set up a highly simplified choice situation in which the only information upon which a subject could base a decision was the probability of attaining a single outcome in each of the two alternative jobs. It was assumed that "stripping down" or simplifying the choice situation in this manner would permit an unambiguous examination of the validity of the multiplying process hypothesized by the expectancy model. Such a process of simplification obviously loses a great deal of the richness and complexity of "real-world" choices and decisions. However such simplification is thoroughly justifiable in testing the multiplicative hypothesis of the expectancy model. Theoretical statements of the expectancy model begin with the multiplication of individual expectancies and valences as the essential underlying process, and then build explanations of more complex choices in terms of sums of products of expectancies and valences for all relevant outcomes. Thus, by basing our initial test of the validity of the model upon a highly simplified choice situation we have not "lost" the essence of the model via a reductionist process, but rather have focused directly upon the basic underlying postulate of the theory.

The results of Experiment 1 can therefore be interpreted as strong support of the hypothesized multiplicative process of the expectancy model. In fact, the results are of a sufficient magnitude to suggest that, in addition to constituting a valid <u>model</u> of the choice process, the multiplicative formulation may hold promise as a valid <u>explanation</u> of the underlying psychological processes involved (although such a statement must of course be viewed as speculative given the current status of our knowledge). On the other hand, the results of Experiment 1 alone cannot be interpreted as providing support for the validity of expectancy theory as a model of actual choices made by individuals in organizational settings. There are a number of factors which suggest that such generalization of the results may be premature.

First, the results of Experiment 1 are based upon the judgments of subjects in response to hypothetical situations in the laboratory. It can not be claimed that responding to questionnaire items in the laboratory is the "same" as making real choices among various occupations, jobs, or work behaviors. The implications of the choices for the individual are obviously far different in the two situations. Whether the underlying cognitive processes evoked in the different situations are the same must await an adequate within-subjects analysis of such real-world choices.

Second, Experiment 1 was designed to test only the validity of the multiplicative hypothesis of the expectancy model. The expectancy and valence of only a single outcome associated with each alternative were varied. The results do not allow any conclusions to be drawn regarding the hypothesis that total force is a function of the sum of products of expectancies and valences associated with an action. A test of this hypothesis would require a more complex design in which the expectancies and valences of more than one outcome associated with each alternative were varied.

Finally, the need to create a judgment situation permitting an unambiguous interpretation of the way in which expectancies and valences are combined by a subject resulted in the construction of items which are unlikely to be highly representative of real choice situations. Subjects were told to assume that the two hypothetical jobs are identical in all respects, except for the information contained in each item. Subjects were then told the probability of attaining one outcome in JOB X and the probability of attaining a different outcome in JOB Y. The items were constructed to permit a test of the multiplicative hypothesis, not to be maximally representative of real choice situations. This lack of representativeness is another reason for caution in generalizing the results.

Subjective Values of Expectancy

The subjective values of the five probability statements associated with JOB X in Experiment 1 are not in agreement with those obtained in previous research (Shanteau, 1974; Lichtenstein and Newman, 1967). In particular, the negative mean subjective value (-0.52) found to be associated with the probability statement "No chance" is inconsistent with both previous empirical results and theoretical assumptions. It has always been assumed (and confirmed) that the expectancy dimension is a unipolar scale which can take on values from 0.0 to +1.0. Within the framework of the multiplicative hypothesis of the expectancy model of motivation, this coding rule implies that when an outcome has a subjective probability of occurrence of zero as a result of engaging in some action (i.e. there is no chance that the outcome will occur), the outcome will have no influence whatsoever on the individual's motivation to perform the action, regardless of the valence of the outcome (force = (0.0 x V) = 0.0). The negative subjective value found to be associated with the statement "No chance" in Experiment 1 indicates that knowledge that an outcome has no chance of occurring as a result of engaging in some action does have an impact upon the individual's motivation to engage in the action. More specifically, it indicates that a subjective probability of zero of the occurrence of a negatively valent outcome as a result of engaging in an action results in increased motivational force to engage in the action (i.e. positive motivation resulting from subjective certainty of avoidance of a negative outcome). Conversely, a subjective probability of zero of the occurrence of a positively valent outcome as a result of engaging in an action results in decreased motivational force to engage in the action (i.e. negative motivation resulting from subjective certainty of avoidance of a positive outcome). These results then would indicate that, when attempting to predict the motivational force acting upon an individual to engage in an action, expectancy values should not be coded on a unipolar scale from 0.0 to +1.0, but rather should be allowed also to take on small negative values as the subjective probability of an outcome approaches zero.

The fact that the negative subjective value associated with the probability statement "No chance" is inconsistent with previous theory and empirical results suggests that care must be taken to explore possible alternative explanations for this result. It is possible that subjects in Experiment 1, rather than responding to the items exactly as instructed, were in fact making some implicit assumptions regarding the likelihood of the outcome associated with JOB X occurring in JOB Y, and vice versa for the outcome associated with JOB Y. If for example subjects were constantly assuming that the outcome associated with JOB X had some probability δ of occurring in JOB Y in all cases, and similarly that the outcome associated with JOB Y had some probability γ of occurring in JOB X, our model would become:

$$R_{xy} = ((E_x \times V_x) + \gamma V_y) - ((E_y \times V_y) + \delta V_x)$$

$$= (E_x - \delta)V_x - (E_y - \gamma)\ V_y$$

Such an assumption on the part of subjects in responding to the items would thus result in underestimates of the true subjective values of the probability statements associated with JOB X. Rather than valid estimates of subjective values, we would have estimates of subjective values minus $\delta((E_x - \delta)$ rather than $E_x)$. Thus, the "true" subjective probability value associated with the statement "No chance" could be 0.0, but the methodology employed yields the erroneous value of -0.52 which is really the true value minus the subject's implicit assumed probability value.

The date of Experiment 1 do not permit the drawing of any conclusions regarding the relative validity of this alternative interpretation of the subjective values obtained for the probability statements. It has not been conclusively shown that subjective values of probability statements should be allowed to take on negative values. At the same time the results do indicate the potential value of further careful reaserch designed to determine the relative validity of the competing explanations.

Validity of the Expectancy Model

Schmidt (1973) has pointed out that since the expectancy model hypothesizes a multiplicative relationship between expectancy and valence, valid predictions based upon the theory should require ratio scale values of expectancy and valence. It was thus hypothesized in Experiment 2 that subject's choices on the Job Choice Questionnaire between pairs of jobs would be better predicted from the ratio scale values of expectancy and valence derived in Experiment 1 than from any other arbitrary coding of expectancy and valence. The results did not support the hypothesis. In fact, predictions based upon the expectancy model employing individualized ratio scale values of expectancy and valence yielded the lowest mean correlation with observed responses to the Job Choice Questionnaire of the six alternative models tested. In order to begin to understand why these results were obtained, it is necessary to examine in greater detail the precise nature of the judgment tasks involved in the two experiments.

The model underlying subject's responses to the Job Preference Questionnaire items of Experiment 1 is as follows:

$$R_{xy} = F_x - F_y$$

$$= (E_x \times V_x) - (E_y \times V_y)$$

The levels of expectancy and valence associated with JOB X (the E_x and V_x) were different from the levels of expectancy and valence associated with JOB Y (the E_y and V_y). Thus, in responding to the items of the Job Preference Questionnaire in Experiment 1 it was never possible for the subject to simplify the cognitive processing task by extracting common elements.

In responding to each item the subject was forced to concentrate upon each alternative job separately and determine his preference for that job or the force acting upon him in the direction of that job. The results of Experiment 1 provide strong support for a model of this process which indicates that subjects perform this task by multiplying the value of E and V and take the difference of the two products to determine their response to an item.

The judgment task involved in responding to the items of the Job Choice Questionnaire in Experiment 2 differs in an important aspect. In Experiment 2 subjects were asked to rate their motivation or inclination to accept one of a pair of real full-time jobs. It is important to keep in mind that in computing predicted force scores, the <u>same</u> set of five outcomes (with associated valences) was employed in computing the motivational force associated with both jobs. Thus, our model of subjects' responses to the items of the Job Choice Questionnaire in Experiment 2 is as follows:

$$R_{jk} = F_j - F_k$$

$$= \sum_i E_{ij} V_{ij} - \sum_i E_{ik} V_{ik}$$

but

$$V_{ij} = V_{ik}$$

so

$$R_{jk} = \sum_i V_i (E_{ij} - E_{ik})$$

Thus, in responding to these items, it is possible for the subject to handle the judgment task by computing differences in levels of expectancy across outcomes, and using the valence of the outcome as a weighting factor. It is worthy of note that this type of choice process (and its associated model) is much more representative of actual choice processes in real world situations than the judgment task of Experiment 1. An outcome (or set of outcomes) is relevant to a choice situation, not to a particular alternative action within the feasible set for that choice situation. Thus, if an individual is faced with a situation in which he must make a choice between some set of alternative jobs which are available to him, there will be some common set of outcomes (e.g. salary, working conditions, status, vacations, fringe benefits, location, etc.) which that individual will employ in evaluating his motivation to accept each of the jobs. The set of outcomes employed will be specific to the situation of choosing among alternative jobs, but a specific and unique set of outcomes will not be employed for evaluating each individual job. Thus, the model proposed above for responses to the Job Choice Questionnaire of Experiment 2 constitutes a valid restatement of the expectancy model as applied to choices between alternative pairs of actions in real choice situations.

What are the implications of this reformulation of the expectancy model for prediction and theory in light of the results of Experiment 2?

Given the results presented by Dawes and Corrigan (1974), it should not be surprising that the expectancy model employing ratio scaled expectancies and valences did not yield the highest correlations. Our model of subjects' responses in Experiment 2 involves a weighted sum of differences in expectancies associated with the two alternative jobs. The weights are the valences of the outcomes. In studying situations of this type, Dawes and Corrigan were able to demonstrate for five separate empirical studies that models employing randomly selected weights had, on the average, correlations with the criteria which were higher than those obtained from expert judges' models of the decision process. Models employing equal weighting performed even better than the random models. Other authors (Schmidt, 1971; Marks, 1966) cited by Dawes and Corrigan have shown that equal weighting may be superior to cross-validated optimal weighting schemes in situations in which the ratio of observations to predictors is less than approximately 20 to 1, due to the instability of beta coefficients.

Dawes and Corrigan pointed out that the equal weight linear model is able to perform extremely well in situations in which:

(a) the predictor variables have conditionally monotone relationships to criteria (or may easily be rescaled to have such a relationship), i.e. higher values on each predictor are associated with higher values on the criterion, regardless of the values of the remaining variables;

(b) there is error in the dependent variable;

(c) there is error in the independent variables;

(d) deviations from optimal weighting do not make much practical difference.

Such situations are pervasive. Linear models obviously do not work well in such situations by magic, but do so for the following reasons:

(a) linear functions are good approximations to conditionally monotone functions;

(b) the relative weights derived from a linear regression analysis are not affected by error in the criterion variable;

(c) conditionally monotone functions tend to become more linear in the presence of increasing error in the predictor variables;

(d) the problem of optimal weights is one that has a "flat maximum," i.e. linear models are robust over deviations from optimal weighting. Weights that are near to optimal lead to almost the same outputs as do optimal beta weights (p. 99, p. 103).

The judgment situation of Experiment 2 belongs to the general class of situations described by Dawes and Corrigan in which equal weighted linear models should be expected to perform well and the results of Experiment 2 are in accordance with the findings reported by Dawes and Corrigan:

(a) an equal weighted model (taking sign into account) can predict approximately as well as a model employing valences as weights, regardless of the coding of the predictor variables;

(b) an equal weighted model employing arbitrary equal interval coding of the predictor variables (expectancies) has a higher predictive validity than either an equal weighted model or a valence weighted model employing ratio scale values of expectancy derived for each individual subject. Thus, an equal weighted model with error in the predictors performs better than models employing more precise codings of the independent variables.

The validities of the equal weight and theoretical models cannot be meaningfully compared to the validities of the optimal linear model, since the latter were not cross validated. Similarly the absolute magnitude of the correlation coefficients cannot be meaningfully interpreted since the predictions were based on a set of five outcomes chosen from a predetermined list which may not have included the most important or salient outcomes for each subject.

The findings of Dawes and Corrigan and the results of Experiment 2 taken together lead to the conclusion that valid expectancy theory predictions of motivation to engage in an activity do not require ratio scales of expectancy and valence. In fact, they do not require scale values of valence at all. What is required is the following:

(1) a set of outcomes which are relevant and salient in the choice situation for the individual

(2) ordinal scale measures of the individual's expectancy of attaining each of the alternative actions

(3) assignment of +1, 0, or -1 to each of the expectancies according to the valence of each outcome for the individual (positive, indifferent, negative)

(4) computation of motivation or force scores for each alternative action by adding the appropriately signed expectancies.

It is critical that such predictions be made on a within subjects basis, and that the predictions be based upon a set of outcomes uniquely determined for each individual. The list of outcomes need not be particularly long. Meehl (1972, cited by Dawes and Corrigan, 1974, p. 105) has pointed out that "in most practical situations an unweighted sum of a small number of 'big' variables will, on the average, be preferable to regression equations". The key to the development of valid predictions of motivational force is captured by the conclusion reached by Dawes and Corrigan: "The whole trick is to decide what variables to look at and then to know how to add" (p. 105).

REFERENCES

Anderson, N.H. Application of an additive model to impression formation. Science, 1962, 138, 817-818. (a)

Anderson, N.H. On the quantification of Miller's conflict theory. Psychological Review, 1962, 69, 400-414. (b)

Anderson, N.H. A simple model for information integration. In R.P. Abelson, E. Aronson, W.J. McGuire, T.M. Newcomb, M.J. Rosenberg, and P.H. Tannenbaum (Eds), Theories of cognitive consistency: A sourcebook. Chicago: Rand McNally, 1968.

Anderson, N.H. Functional measurement and psychophysical judgement. Psychological Review, 1970, 77, 153-170.

Anderson, N.H. Integration theory and attitude change. Psychological Review, 1971, 78, 171-206.

Anderson, N.H. Algebraic models in perception. In E.C. Carterette and M.P. Friedman (Eds.), Handbook of perception. New York: Academic Press, 1973. (a)

Anderson, N.H. Cognitive algebra: Integration theory applied to social attribution. In L. Berkowitz (Ed.), Advances in experimental social psychology. Vol. 7, New York: Academic Press, 1973. (b)

Anderson, N.H. Information integration theory: a brief survey. In D.H. Krantz, R.C. Atkinson, R.D. Luce, and P. Suppes (eds.), Contemporary developments in mathematical psychology, Vol. 2. San Francisco: W.H. Freeman, 1974. (a)

Anderson, N.H. Algebraic models for information integration. Technical Report No. 45, Center for Human Information Processing, University of California, San Diego, 1974, (b)

Anderson, N.H. and Butzin, C.A. Performance = Motivation x Ability: An information - theoretical analysis. Journal of Personality and Social Psychology, 1974, 30, 598-604.

Anderson, N.H. and Shanteau, J.C. Information integration in risky decision making. Journal of Experimental Psychology, 1970, 84, 441-451.

Arvey, R.D. Task performance as a function of perceived effort-performance and performance-reward contingencies. Organizational Behavior and Human Performance, 1972, 8, 423-433.

Behling, O. and Starke, F.A. The postulates of expectancy theory. Academy of Management Journal, 1973, 16, 373-388. (a)

Behling, O. and Starke, F.A. Some limits on expectancy theories of work motivation. Proceedings, Midwest Meeting, American Institute for Decision Sciences, 1973. (b)

Bettman, J.R., Capon, N., and Lutz. R.J. Cognitive algebra in multi-attribute attitude models. Paper No. 10, Center for Marketing Studies, University of California, Los Angeles, 1974. (a)

Bettman, J.R. Capon, N., and Lutz, R.J. A multimethod approach to validating multi-attribute attitude models. Paper No. 16, Centre for Marketing Studies, University of California, Los Angeles, 1974. (b)

Bettman, J.R. Capon, N., and Lutz, R.J. Multi-attribute measurement models and multi-attribute attitude theory: a test of construct validity. Paper No. 22, Center for Marketing Studies, University of California, Los Angeles, 1974. (c)

Birnbaum, M.H. Reply to the devil's advocates: Don't confound model testing and measurement. Psychological Bulletin, 1974, 81, 854-859.

Campbell, J.P. Dunnette, M.D., Lawler, E.E., III, and Weick, K.E., Jr. Managerial Behaviour, performance, and effectiveness. New York: McGraw-Hill, 1970.

Dachler, H.P., and Mobley, W.H. Construct validation of an instrumentality - expectancy - task-goal model of work motivation: Some theoretical boundary conditions. Journal of Applied Psychology Monograph, 1973, 58, 397-418.

Dawes, R.M. and Corrigan, B. Linear models in decision making. Psychological Bulletin, 1974, 81, 95-106.

Galbraith, J., and Cummings, L.L. An empirical investigation of the motivational determinants of task performance: Interactive effects between instrumentality-valence and motivation-ability. Organizational Behavior and Human Performance, 1967, 2, 237-257.

Gavin, J.F. Ability, effort, and role perception as antecedents of Job performance. (Experimental Publications System Ms. No. 190A) Washington, D.C.: American Psychological Association, 1971.

Georgopoulos, B.S., Mahoney, G.M., and Jones, N.W. A path-goal approach to productivity. Journal of Applied Psychology, 1957, 41, 345-353.

Graen, G. Instrumentality theory of work motivation: Some experimental results and suggested modifications. Journal of Applied Psychology Monograph, 1969, 53, 1-25.

Hackman, J.R., and Porter, L.W. Expectancy theory predictions of work effectiveness. Organizational Behavior and Human Performance, 1968, 3, 417-426

Heneman, H.G., III and Schwab, D.P. An evaluation of research on expectancy theory predictions of employee performance. Psychological Bulletin, 1972, 78 1-9.

Holmstrom, V.L., and Beach, L.R. Subjective expected utility and career preferences. Organizational Behavior and Human Performance, 1973, 10, 201-207.

House, R.J. A path-goal theory of leader effectiveness. Administrative Sciences Quarterly, 1971, 16, 321-338.

Krantz, D.H., and Tversky, A. Conjoint-measurement analysis of composition rules in psychology. Psychological Review, 1971, 78, 151-169.

Lawler, E.E., III. A correlation-causal analysis of the relationship between expectancy attitudes and job performance. Journal of Applied Psychology, 1968, 52, 462-468

Lawler, E.E., III. Motivation in work organizations. Monterey, Calif.: Brooks/Cole, 1973.

Lawler, E.E., III, and Suttle, J.L. Expectancy theory and job behavior. Organizational Behavior and Human Performance, 1973, 9 482-503.

Lichtentein, S., and Newman, J.R. Empirical scaling of common verbal phrases associated with numerical probabilities. Psychonomic Science, 1967, 9, 563-564.

Mikes. P.S., and Hulin, C.L. Use of importance as a weighted component of job satisfaction. Journal of Applied Psychology, 1968, 52, 394-398.

Mitchell, T.R. Instrumentality theories: Conceptual and methodological problems. Technical Report 71-19, University of Washington, 1971.

Mitchell, T.R. Expectancy models of job satisfaction, occupational preference, and effort: A theoretical, methodological, and empirical appraisal. Psychological Bulletin, 1974, 81, 1053-1077.

Mitchell, T.R. and Albright, D.W. Expectancy theory predictions of the satisfaction, effort, performance and retention of naval aviation officers. Organizational Behavior and Human Performance, 1972, 8, 1-20.

Nebeker, D.M., and Mitchell, T.R. Leader behavior: An expectancy theory approach. Organizational Behavior and Human Performance, 1974, 11, 355-367.

Pritchard, R.D., and DeLeo, P.J. Experimental test of the valence-instrumentality relationship on job performance. Journal of Applied Psychology, 1973, 57, 264-270.

Pritchard, R.D., and Sanders, M.S. The influence of valence, instrumentality, and expectancy on effort and performance. Journal of Applied Psychology, 1973, 57, 55-60

Reinharth, L., and Wahba, M.A. Expectancy theory as a predictor of work motivation, effort expenditure and job performance. Proceedings of the Eastern Academy of Management, 1974.

Schmidt, F.L. Implications of a measurement problem for expectancy theory research. Organizational Behavior and Human Performance, 1973, 10, 243-251.

Schwab, D.P., and Dyer, L.D. The motivational impact of a compensation system on employee performance. Organizational Behavior and Human Performance, 1973, 9, 215-225

Shanteau, J.C. Component processess in risky decision making. Journal of Experimental Psychology, 1974, 103, 680-691.

Shanteau, J.C. and Anderson, N.H. Integration theory applied to Judgments of the value of information. Journal of Experimental Psychology, 1972, 92, 266-275.

Sheard, J.L. Intrasubject prediction of preferences for organizational types. Journal of Applied Psychology, 1970, 54, 248-252.

Sheridan, J.E. Slocum, J.W., Jr., and Min, B. Motivational determinants of job performance. Journal of Applied Psychology, 1975, 60, 119-121.

Sobel, R.S. Tests of preperformance and postperformance models of satisfaction with outcomes. Journal of Personality and Social Psychology, 1971, 19, 213-221.

Suttle, J.L. Antecedents to motivation: Some determinants of motivationally-relevent beliefs and values. Unpublished doctoral dissertation, Yale University, 1975.

Thurstone, L.L., and Jones, L.V. The rational origin for measuring subjective values. Journal of the American Statistical Association, 1957, 52, 458-471.

Vroom, V.H. Work and motivation. New York: John Wiley, 1964.

Wahba, M.A., and House, R.J. Expectancy theory in work and motivation: Some logical and methodological issues. Human Relations, 1974, 27, 121-147.

Winer, B.J. Statistical principles in experimental design (Second Edition). New York: McGraw-Hill, 1971.

FOOTNOTES

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TABLE 1

Probability Statements Employed with Associated Subjective Values from Shanteau (1974, pp. 681-686)

JOB X Probability Statement	JOB X Subjective Value	JOB Y Probability Statement	JOB Y Subjective Value
No chance	.00		
Somewhat unlikely	.27-.34	Unlikely	.18-.19
Not quite even chance	.45-.50	Better than even chance	.60-.66
Fairly likely	.69-.72		
Sure thing	1.00		

TABLE 2

Job Outcomes

1. Receiving a 50% salary increase during the first year.
2. Receiving frequent criticism from your boss in front of others.
3. Having frequent opportunities to work overtime for double pay.
4. Getting an important promotion during the first year.
5. Being transferred to a new location during the first year.
6. Going to sleep frequently on the job due to boredom.
7. Having an accident involving serious physical injury.
8. Having an attractive female supervisor.
9. Having frequent opportunities for foreign travel during the first year.
10. Being laid off during the first year.
11. Having coworkers who are difficult to get along with.
12. Receiving a $1000 bonus for outstanding performance during the first year.
13. Becoming totally absorbed in your job 24 hours a day.
14. Receiving frequent constructive feedback from your supervisor.

Table 3

Arbitrary Codings of Expectancy and Valence

Expectancy		Valence	
Probability statement	Coding	Outcome	Coding
No chance	.00	Most positive	+2
Somewhat unlikely	.25	Moderately positive	+1
Not quite even chance	.50	Indifferent	0
Fairly likely	.75	Moderately negative	-1
Sure thing	1.00	Most negative	-2

Table 4

Mean variance explained

Source	ω^2
Main effects	.366
E_x	.024
V_x	.143
E_y	.026
V_y	.173
2-way interactions	.476
$E_x \times V_x$	.245
$E_x \times E_y$	.001
$E_x \times V_y$	.004
$V_x \times E_y$	.003
$V_x \times V_y$	.011
$E_y \times V_y$	.212
3-way interactions	.026
$E_x \times V_x \times E_y$	.002
$E_x \times V_x \times V_y$	.012
$E_x \times E_y \times V_y$	.003
$V_x \times E_y \times V_y$	.009
4-way interaction	.004

Table 5

Percentage of predicted 2-way interactions concentrated in the bilinear component

	Percent Bilinear			Percent Bilinear	
Subject	$E_x \times V_x$	$E_y \times V_y$	Subject	$E_x \times V_x$	$E_y \times V_y$
01	98.1	99.9	16	89.1	15.7
02	71.1	95.7	17	93.5	95.7
03	85.7	96.4	18	99.4	98.1
04	91.6	95.9	19	89.5	97.1
05	93.6	99.3	20	94.8	93.7
06	93.9	80.3	21	88.8	99.6
07	96.7	94.9	22	90.0	97.6
08	94.4	99.6	23	98.7	85.8
09	63.4	45.0	24	85.5	84.6
10	90.8	99.1	25	87.7	85.8
11	72.0	00.1	26	78.4	93.5
12	98.0	99.8	27	87.2	55.6
13	98.2	71.1	28	72.4	97.5
14	97.9	85.8	29	96.5	98.4
15	86.7	99.8	30	68.1	69.9
			31	93.4	99.3

Table 6

Subjective values of expectancy of the five probability statements associated with JOB X

Subject	Subjective value (Expectancy) (1) No chance	(2) Somewhat unlikely	(3) Better than even chance	(4) Fairly likely	(5) Sure thing
01*	0.0	0.20	0.82	0.94	1.00
02	-0.43	-0.27	0.63	0.93	1.00
03	-0.60	0.04	0.31	0.66	1.00
04	-0.27	0.33	0.41	0.66	1.00
05	-0.59	0.24	0.58	0.66	1.00
06	-0.42	0.18	0.26	0.68	1.00
07*	0.0	0.56	0.84	0.83	1.00
08*	0.0	0.70	0.85	0.88	1.00
09	-1.12	-0.67	0.48	1.00	0.93
10	-0.50	-0.13	0.29	0.79	1.00
11	-1.05	0.08	0.28	0.60	1.00
12	-0.46	0.03	0.46	0.51	1.00
13	-0.72	-0.03	0.47	0.72	1.00
14	-0.81	0.07	0.42	0.66	1.00
15*	0.0	0.31	0.61	0.84	1.00
16*	0.0	0.24	0.52	0.80	1.00
17	-0.65	-0.13	0.40	0.72	1.00
18*	0.0	0.36	0.61	0.62	1.00
19	-0.78	-0.34	0.04	0.66	1.00
20*	0.0	0.51	0.66	0.68	1.00
21	-0.63	0.13	0.39	0.79	1.00
22*	0.0	0.40	0.46	0.69	1.00
23	-0.70	-0.23	0.01	0.52	1.00
24	-0.10	0.27	0.37	0.91	1.00
25	-0.93	-0.36	0.22	0.75	1.00
26	-1.10	-0.28	0.35	0.61	1.00
27	-1.66	0.79	0.80	0.89	1.00
28	-0.48	-0.03	0.36	0.92	1.00
29	-0.83	-0.50	0.59	0.77	1.00
30	-0.78	-0.66	0.38	0.97	1.00
31	-0.38	0.26	0.45	0.88	1.00

* No significant crossover. Zero point taken as mean response for Expectancy = "No chance"

Table 7

Average subjective values for the five levels of Expectancy X

Probability Statement	Subjective Value
(1) No chance	-.52
(2) Somewhat unlikely	.06
(3) Not quite even chance	.46
(4) Fairly likely	.76
(5) Sure thing	1.00

Table 8

Subjective values of valence of the five outcomes associated with JOB X for each subject

Subject	Subjective value (Valence) (1) Most positive	(2) Moderately Positive	(3) Indifferent	(4) Moderately Negative	(5) Most Negative
01	+10.00	+ 9.07	-21.86	-119.38	-187.47
02	+ 2.49	+10.00	- 1.33	- 18.05	- 11.17
03	+10.00	+ 5.68	- 5.01	- 2.24	- 11.49
04	+ 9.78	+10.00	- 0.86	- 2.48	- 11.86
05	+10.00	+ 6.96	+ 4.42	- 10.58	- 11.21
06	+10.00	+ 9.08	+ 1.59	- 12.29	- 16.15
07	+ 9.83	+10.00	+ 0.66	- 6.15	- 1.46
08	+ 8.91	+ 7.91	+10.00	- 18.32	-277.27
09	+ 5.11	+10.00	- 0.85	- 20.87	- 22.16
10	+10.00	+ 6.04	-10.42	- 12.75	- 13.36
11	+ 7.46	+10.00	+ 4.21	- 10.07	- 13.12
12	+10.00	+ 8.09	+ 0.77	- 29.10	- 33.02
13	+ 9.98	+10.00	- 4.60	- 7.38	- 9.58
14	+10.00	+ 9.75	- 8.05	- 19.25	- 20.81
15	+10.00	+ 4.04	-11.30	- 30.43	- 85.48
16	+ 5.64	+10.00	-50.94	- 89.20	-198.73
17	+ 8.23	+10.00	+ 0.10	- 8.68	- 13.13
18	+10.00	+ 5.03	- 1.15	+ 4.40	- 77.78
19	+10.00	+ 6.18	- 3.06	- 7.28	- 15.15
20	+ 8.08	+10.00	+ 1.34	- 0.66	- 1.55
21	+ 9.22	+10.00	- 1.00	- 8.54	15.90
22	+ 3.42	+10.00	-19.26	- 13.05	- 17.28
23	+10.00	+ 4.79	-11.40	- 12.10	- 11.92
24	+10.00	+ 6.19	+ 3.25	- 9.62	- 8.04
25	+ 9.50	+10.00	- 2.04	- 12.25	- 11.38
26	+ 7.67	+10.00	- 5.27	- 10.67	- 30.67
27	+10.00	+ 8.95	+ 1.05	- 11.41	- 11.77
28	+ 9.86	+ 9.47	+10.00	- 6.63	- 8.40
29	+10.00	+ 6.09	- 4.87	- 6.90	- 7.08
30	+10.00	+ 8.77	- 4.82	- 9.19	- 11.55
31	+10.00	+ 9.51	- 9.52	- 16.28	- 19.32

Table 9

Correlations of Predicted and Observed Responses to the Job Choice Questionnaire

Subject	Individualized Ratio Scale Coding: Theoretical Model (ΣExV)	Individualized Ratio Scale Coding: Equal (unit) Weight Model (ΣE)	Individualized Ratio Scale Coding: Optimal Linear Model (ΣbE)	Arbitrary Coding: Theoretical Model (ΣExV)	Arbitrary Coding: Equal (unit) Weight Model (ΣE)	Arbitrary Coding: Optimal Linear Model (ΣbE)
01	.43	.32	.96	-.01	.03	.96
02	-.34	-.15	.71	-.02	-.26	.71
03	.82	.85	.87	.84	.85	.87
04	-.25	-.19	.91	-.39	-.34	.91
05	.58	.58	.89	.66	.72	.89
06	.77	.75	.90	.81	.79	.90
07	-.36	-.30	.92	.34	.11	.92
08	.43	.68	.94	.88	.86	.94
09	.43	.72	.98	.66	.60	.98
10	.81	.74	.96	.73	.73	.96
11	.85	.85	.96	.92	.92	.96
12	.79	.82	.98	.91	.89	.98
13	.59	.56	.94	.60	.58	.94
14	.36	.14	*	.14	-.18	*
15	.65	.85	.97	.72	.86	.91
16	.25	.20	.91	.31	.22	.91
17	.73	.68	.97	.75	.70	.97
18	.32	.44	*	.47	.62	*
19	.70	.79	.97	.67	.79	.97
20	.44	.42	.67	.36	.33	.60
21	.88	.88	.95	.84	.86	.95
22	.84	.84	*	.90	.90	*
23	.83	.56	.99	.73	.63	.99
24	-.07	-.19	.96	.30	.18	.96
25	.79	.76	.96	.78	.76	.96
26	.51	.57	.99	.50	.64	.99
27	-.08	-.07	.67	.04	-.01	.67
28	.55	.38	.88	.55	.36	.88
29	.24	.72	.91	.74	.71	.91
30	.65	.63	.99	.66	.62	.99
31	.45	.20	.90	.30	.11	.90
Mean	.47	.48	.91	.54	.50	.91

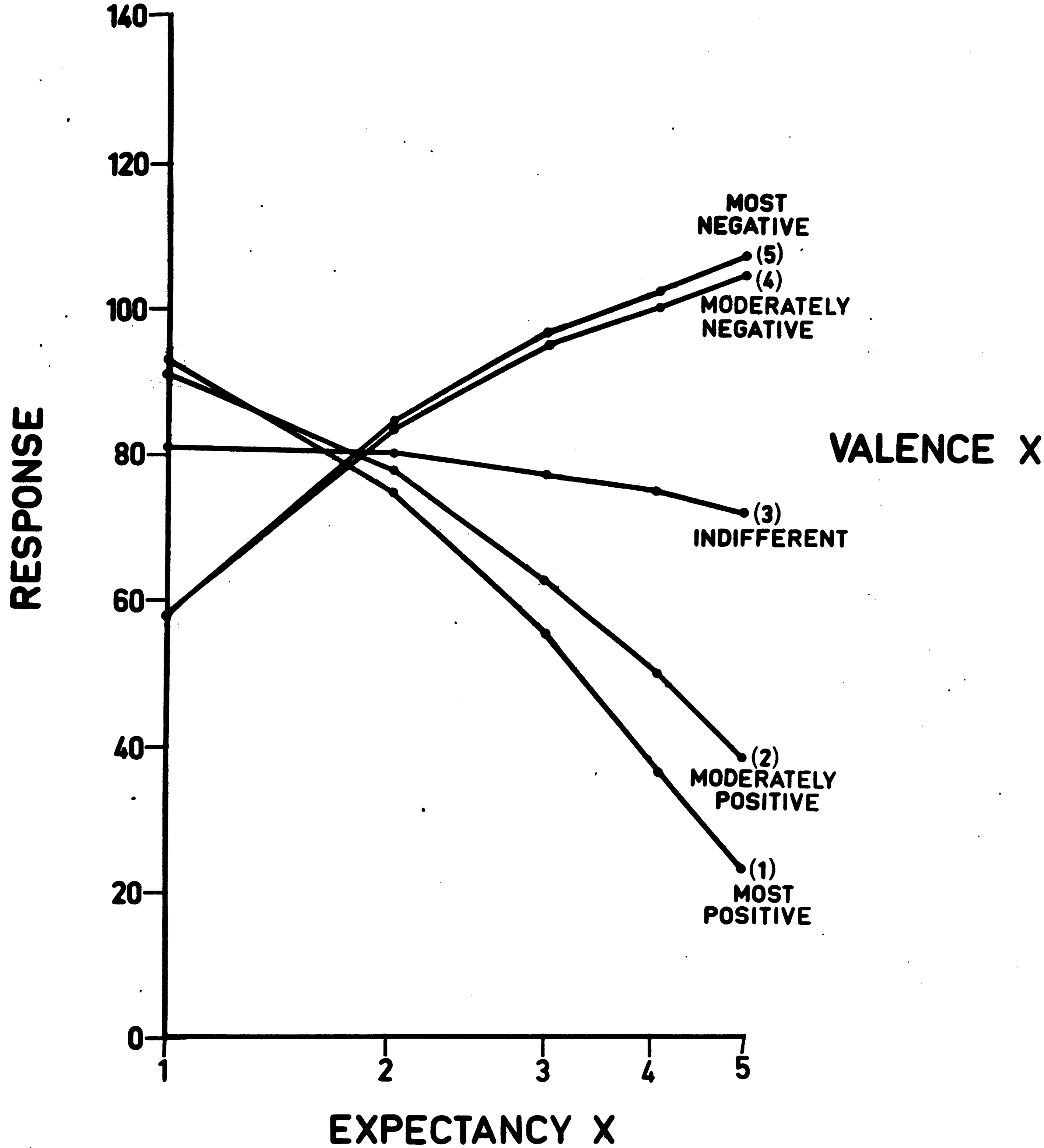
* Optimal weights could not be obtained due to interdependence of independent variables

FIGURE CAPTIONS

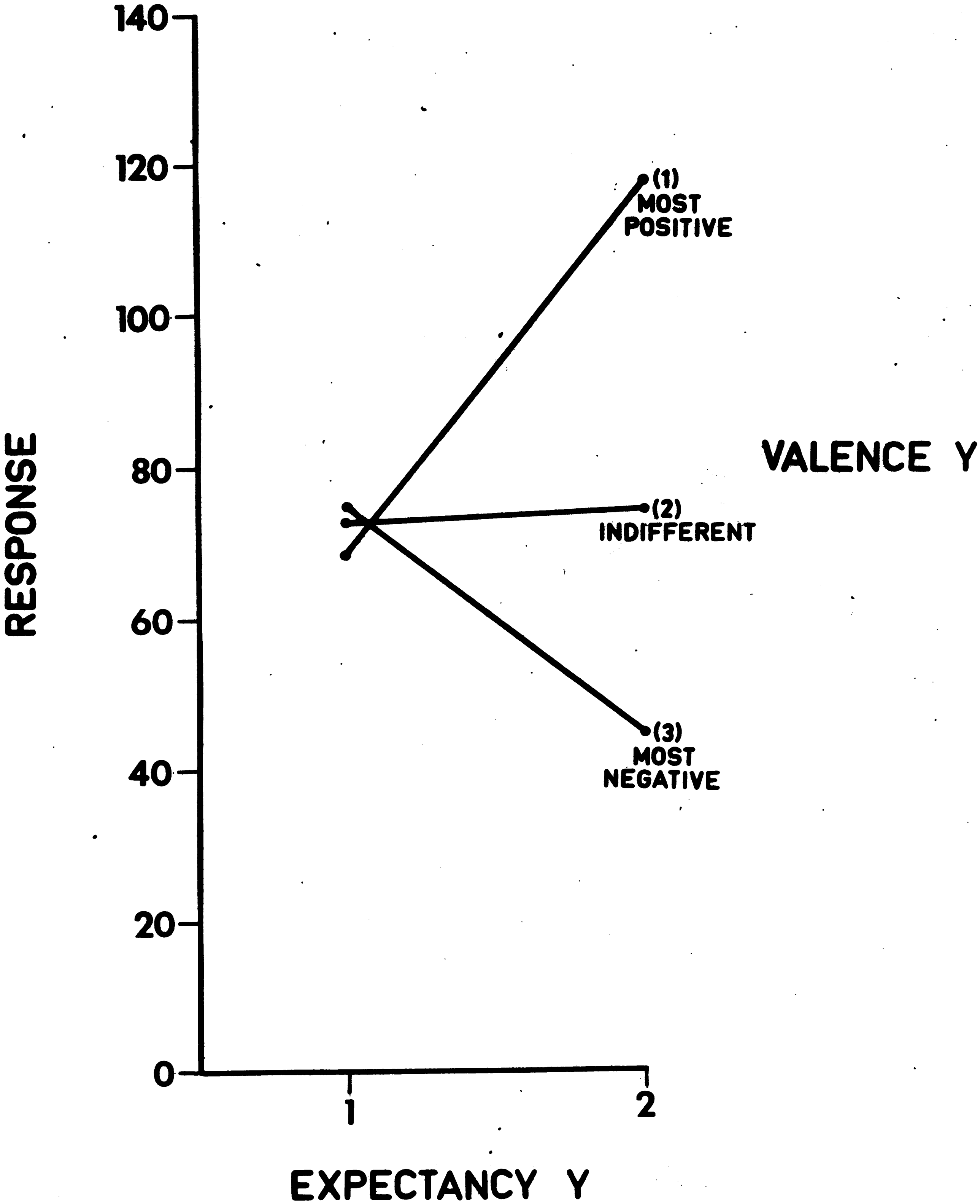
FOR EACH ITEM PLACE A MARK ON THE LINE BETWEEN JOB X AND JOB Y AT THE POINT WHICH BEST INDICATES YOUR INCLINATION TOWARD TAKING ONE OR THE OTHER JOB.

JOB X IS <u>A SURE THING</u> TO LEAD TO <u>RECEIVING A 50% SALARY INCREASE DURING THE FIRST YEAR</u>.	JOB Y HAS A <u>BETTER THAN EVEN CHANCE</u> OF LEADING TO <u>HAVING COWORKERS WHO ARE DIFFICULT TO GET ALONG WITH</u>.

JOB X ———————————— JOB Y



RESPONSE
140
120
100
80
60
40
20
0
1
2
3
4
5
EXPECTANCY X
VALENCE X
MOST NEGATIVE
(5)
(4)
MODERATELY NEGATIVE
(3)
INDIFFERENT
(2)
MODERATELY POSITIVE
(1)
MOST POSITIVE


140
120
100
80
60
40
20
0
RESPONSE
(1)
MOST
POSITIVE
VALENCE Y
(2)
INDIFFERENT
(3)
MOST
NEGATIVE
1
2
EXPECTANCY Y

Appendix 1

Cognitive Integration Theory and Functional Measurement

The theory of information integration developed by Anderson (1962a, b, 1968, 1970, 1971, 1973a,b, 1974a) employs the basic conception of the organism as an integrator of stimulus information. Two central aspects of the information integration approach deserve close attention:

(1) Algebraic models. The approach employs simple algebraic models for the description of cognitive information integration processes. Empirical applications of the techniques have shown that algebraically simple adding, subtracting, averaging, and multiplying models can successfully yield detailed quantitative accounts of various complex cogntive processes (Anderson, 1974a, p. 237).

(2) Functional measurement. The algebraic models employed are expressed in subjective metrics or psychological values of the response and of the stimuli. Confounding is introduced when the need for subjective metrics is ignored and only arbitrary scales are employed. An adequate test of the validity of a model of cognitive processing is impossible without the psychological values (Anderson, 1974a, p. 238).

Obtaining the psychological values obviously requires a theory of measurement. The guiding idea of the functional measurement approach is that measurement and model testing go hand in hand, that measurement scales are derivative from substantive theory. This position obviously differs from more traditional approaches to scaling which view the establishment of scales of measurement as logically prior to model testing. The functional measurement approach views the development of the theory and of the scale as integrally and intimately related. The final validation of the scale is seen as dependent upon the empirical validity of the theory. The first step in this joint validation process then becomes the development of an experimental base of support for a quantitative law of behavior (Anderson, 1970, pp. 153-154).

With regard to the expectancy model of motivation, it might be argued that the empirical results to date do not constitute a sufficiently strong base of support for the hypothesized quantitative law to permit a useful application of functional measurement for scale development and model testing. Although it must be admitted that results reported in the organizational literature do not constitute a strong basis of support, there does exist experimental support for multiplicative models of cognitive processing in the literature of utility theory and decision theory. Recently there have also appeared successful applications of the functional measurement methodology to the study of cognitive multiplying processes.

Anderson and Shanteau (1970) and Shanteau (1974) have studied the information integration processes involved in risky decision making. They varied the probability of winning and losing various amounts of money in single and duplex bets and asked subjects to estimate the "subjective worth" of each bet. Their results strongly support the hypothesized multiplying relationship. Anderson and Butzin (1974) examined the validity of the Performance = Motivation x Ability hypothesis. Motivation and ability of stimulus persons were varied in a two-way factorial design and subjects were asked to estimate the performance of the stimulus person. Support for the multiplying process was obtained, although the results of this study were not unequivocal. Bettman, Capon, and Lutz (1974a,b,c) have also applied the methodology to the study of the validity of multi-attribute attitude models with some success.

These results provide a base of support for the validity of multiplicative cognitive models of information integration and suggest that the techniques might be fruitfully applied to the study of the information integration process hypothesized by the expectancy model of motivation.

Application of the Methodology to the Expectancy Model

In addition to the general approach of joint validation of theory and scale, and the existence of some degree of theoretical success in accounting for actual data, one obviously also requires a specific set of techniques for the testing of models and the derivation of scales. A detailed discussion of the nature of these techniques is presented by Anderson (1970, 1974a,b). Immediately following is a discussion of the specific techniques developed by Anderson for model testing and scale development as applied to the hypothesized multiplicative process of the expectancy model of motivation.

The algebraic form of the model to be tested is stated as follows:

$$F_i = F_i \left[\sum_{j=1}^{n} (E_{ij} V_j) \right] \qquad (i=n+1, \ldots, m)$$

F_i = force to perform act i

F_{ij} = strength of expectancy ($0 \leq E_{ij} \leq 1$) that act i will be followed by outcome j

V_j = valence of outcome j.

If we consider the special case in which each act has only a single outcome, and set aside for the moment the precise form of the function f_i (but maintain the constraint that it be monotonically increasing), the model becomes:

$$F_{ij} = E_{ij} V_j$$

F_{ij} = force to perform act i for outcome j

E_{ij} = strength of expectancy ($0 \leq E_{ij} \leq 1$) that act i will be followed by outcome j

V_j = valence of outcome j.

Taking into account the fact that expectancy is hypothesized to be unipolar, varying from zero to one, and that valence is hypothesized to be bipolar, varying from negative to positive values, the form of the relationship among force, expectancy, and valence should be as depicted in Figure A (an asymmetric multiplying model).

Insert Figure A about here

It is possible to study the validity of this model by treating expectancy and valence as factors in a two-way factorial design, employing force as the dependent variable, and applying the analysis of variance to the results. As many levels as desired can be employed on each factor. Scaling of the levels is not an issue at this stage, since the validity of the ANOVA method is independent of the scaling of the independent variables. For each subject, measures would be obtained of "force" in each cell of the design. At least two observations per cell would be required for each subject in order to obtain a within-replicates error term to be used in testing the significance of the effects of the factors.

Once the data from such a design are in hand, both a graphical and an exact test of goodness of fit of the model can be carried out. If we assume that levels of expectancy correspond to the rows in our factorial design, and levels of valence to the columns, then $F_{i.}$ denotes the row mean for expectancy level i, $F_{.j}$ denotes the column mean for valence level j, and $F_{..}$ denotes the grand mean.

For the graphical test of fit, each level of expectancy E_{ij} is assigned the provisional empirical value $F_{i.}$ (the rationale for this provisional estimate is presented below). These estimates can then be employed as abscissa values, and a curve plotted for each level of valence based on the data from each column of the design (as in Figure A). These curves should form a diverging fan of straight lines with common intersection, except for sampling errors. Any systematic discrepancy of this set of curves from the general pattern of Figure A would constitute evidence counter to the multiplicative model (Anderson, 1970, p. 157).

In addition to this graphical test, an exact test of goodness of fit is available from the analysis of variance. Since the curves in Figure A are nonparallel, the row x column interaction is theoretically nonzero. If the hypothesized multiplying model is valid however, all of this interaction should be concentrated in a single degree of freedom corresponding to the bilinear (linear x linear) component (the two-way ExV interaction effect in the analysis of variance contains all combinations of E and V, e.g., ExV^2, E^2xV^2, E^4xV^3, etc.; we are interested only in the linear x linear component corresponding to E^1xV^1). In order to obtain this bilinear component we require certain comparison coefficients c_{ij} (c_{ij} = $c_i c_j$). The c_i are the coefficients of trend comparisons corresponding to the main effect for rows, and the c_j similarly for columns. If it can be assumed that the levels of the factors are equally spaced along their underlying continua, these coefficients are defined by the appropriate coefficients of orthogonal polynomials corresponding to the number of levels of each factor (Winer, 1971, p. 389). Such an assumption of equal spacing is not acceptable however, since we have no way of knowing that the subjective values of expectancy and valence operationalized in our design are in fact equally spaced. Hence we must obtain estimates of the c_i and C_c which will be proportional to the orthogonal polynomial coefficients but which will take into account the proper scaling of the subjective values of expectancy and valence. Such estimates are obtained as follows:

$$c_i = (F_{i.} - F_{..})$$

$$c_j = (F_{.j} - F_{..})$$

$$c_{ij} = c_i c_j = (F_{i.} - F_{..})(F_{.j} - F_{..}).$$

Here c_i is just the deviation of the mean for Row i from the grand mean. Except for sampling error of these estimates, the c_i are proportional to the linear orthogonal polynomial coefficients for the row effect, relative to the correct underlying scale of subjective values. The same is true of the c_j for the column dimension. The c_{ij} thus provide an estimate of the linear x linear component of the interaction. The sum of squares for this bilinear component is then:

$$SS_{LxL} = \frac{(\Sigma c_{ij} T_{ij})^2}{n \Sigma c_{ij}^{\ 2}}$$

where T_{ij} is the total of the n scores in Row i, Column j, and the sums are over all cells in the design. The critical term is then the residual interaction obtained by subtracting this bilinear sum of squares from the total interaction. This residual is zero in principle and hence should be nonsignificant in practice (Anderson, 1970, pp. 157-158).

One alternation to this computational procedure is required when both positively and negatively valent outcomes are employed in the design. In this case, the combination of positively and negatively sloped curves (as in Figure A) tend to cancel one another out and yield under-estimates of the true bilinear component. This can be clearly seen with regard to Figure A. Since both positive and negative valences were employed in constructing this graph, the mean for every level of expectancy is equal to the grand mean (zero in this case). Thus, the c_i computed from these data would all be zero and the bilinear component would be zero, even though the data are based upon a pure multiplying model. When both positively and negatively valent outcomes are employed in the design, and a priori grounds thus exist for expecting certain of the curves to be negatively sloped, it is still possible to obtain an accurate estimate of the bilinear component by complementing each column of the design expected to have a negative slope (i.e., each column corresponding to a negatively valent outcome) about some convenient arbitrary value (e.g., the upper end-point of the response scale). The result of this complementation is a positively sloped curve for each level of valence (each column of the design). This new matrix is then employed in determining the c_i values. The c_j values are computed from the original matrix of observations and the remainder of the analysis is upon the original data. Thus, in determining the bilinear component from a design employing both positively and negatively valent outcomes, we would employ:

$$c_i' = (F_{i.}' - F_{..}')$$

$$c_j = (F_{.j} - F_{..})$$

$$c_{ij}' = c_i' c_j = (F_{i.}' - F_{..}') (F_{.j} - F_{..})$$

$$SS_{LxL} = \frac{(\Sigma c_{ij}' T_{ij})^2}{n \Sigma c_{ij}'^{\,2}}$$

where $F_{.j}$ and $F_{..}$ are obtained from the original matrix of observations and $F'_{i.}$ and $F'_{..}$ are obtained from the appropriately complemented matrix. This procedure will then yield a valid estimate of the bilinear component. It is of course critical that this complementation procedure be carried out purely on a priori theoretical grounds. Complementation following inspection of the data is not appropriate.

This test of the bilinear component of the two-way interaction is the critical test of the validity of a multiplicative model of cognitive processing. It should also be kept in mind that a pattern of results generated by a pure multiplicative model (such as those in Figure A) will also exhibit main effects in the analysis of variance. In our example design employing expectancy and valence as factors in a two-way factorial design, main effects for valence will always be predicted by the multiplicative model. In addition, to the extent that levels of valence are employed which are not perfectly symmetric about the true zero point of valence, main effects for expectancy will also be predicted. These main effects do not constitute evidence for a combined adding/multiplying model of cognitive processing. The main effects for valence are an artifact of the combination in the expectancy model of a unipolar scale for expectancy and a bipolar scale for valence, and will always be predicted by the model. Any observed main effects for expectancy are an artifact of the lack of symmetry of levels of valence about their true zero point. This interpretation of the main effects only makes sense of course when the analysis yields a significant two-way interaction concentrated in the bilinear component. When such a bilinear interaction is obtained, a simple multiplying model expressed in the appropriate subjective metric can always fit the data as well as a combined adding/multiplying model, and hence provides the most parsimonious theoretical interpretation of the underlying cognitive process.

If and only if the results of the analysis of variance conform to the pattern predicted by the model (significant two-way interaction concentrated in the bilinear component, significant main effect for valence, main effect for expectancy), then the provisional estimates of scale values employed in the graphical test of fit take on psychological meaning as estimates of the subjective values of the stimuli. In carrying out our graphical test of fit, the marginal means of the matrix of observed responses were employed as estimates of subjective values of the stimuli. The psychological meaning of these marginal means can be seen from the following discussion.

Averaging over columns to obtain Row means yields:

$$F_{i.} = k+E_{i.}V_{.}$$

Similarly, averaging over rows to obtain Column means yields:

$$F_{.j} = k+E_{.j}V_{j}$$

The dot subscript denotes an average over the corresponding index.

Our first equation says that the theoretical Row mean is a linear function of the subjective values of expectancy, $E_{i.}$. It then follows that the observed Row means provide estimates of the subjective values of expectancy on an interval scale. In a similar manner, the second equation shows that the Column means provide an interval scale of the subjective values of valence (Anderson, 1974b, pp. 22-23).[4]

Deriving ratio scales for expectancy and valence requires an estimate of the constant k. For a multiplying model, such an estimate is feasible since the observed response F_{ij} must be zero when expectancy (E_{ij}) and/or valence (V_j) is zero. Thus, if we take care to include stimulus levels of expectancy and valence in our design which we would expect to take on subjective values of zero, then the observed point of intersection of the curves provides an estimate of the functional zero point of the scales, and permits the transformation from interval to ratio scales.

It must be stressed again that the estimates of subjective values of expectancy and valence derived by the above procedures only take on psychological meaning (and hence can only be considered valid estimates of subjective values of the stimuli) when the model has already passed the exact test of fit in the analysis of variance. If the model appears to be seriously in error, then the marginal means cannot be meaningfully interpreted.

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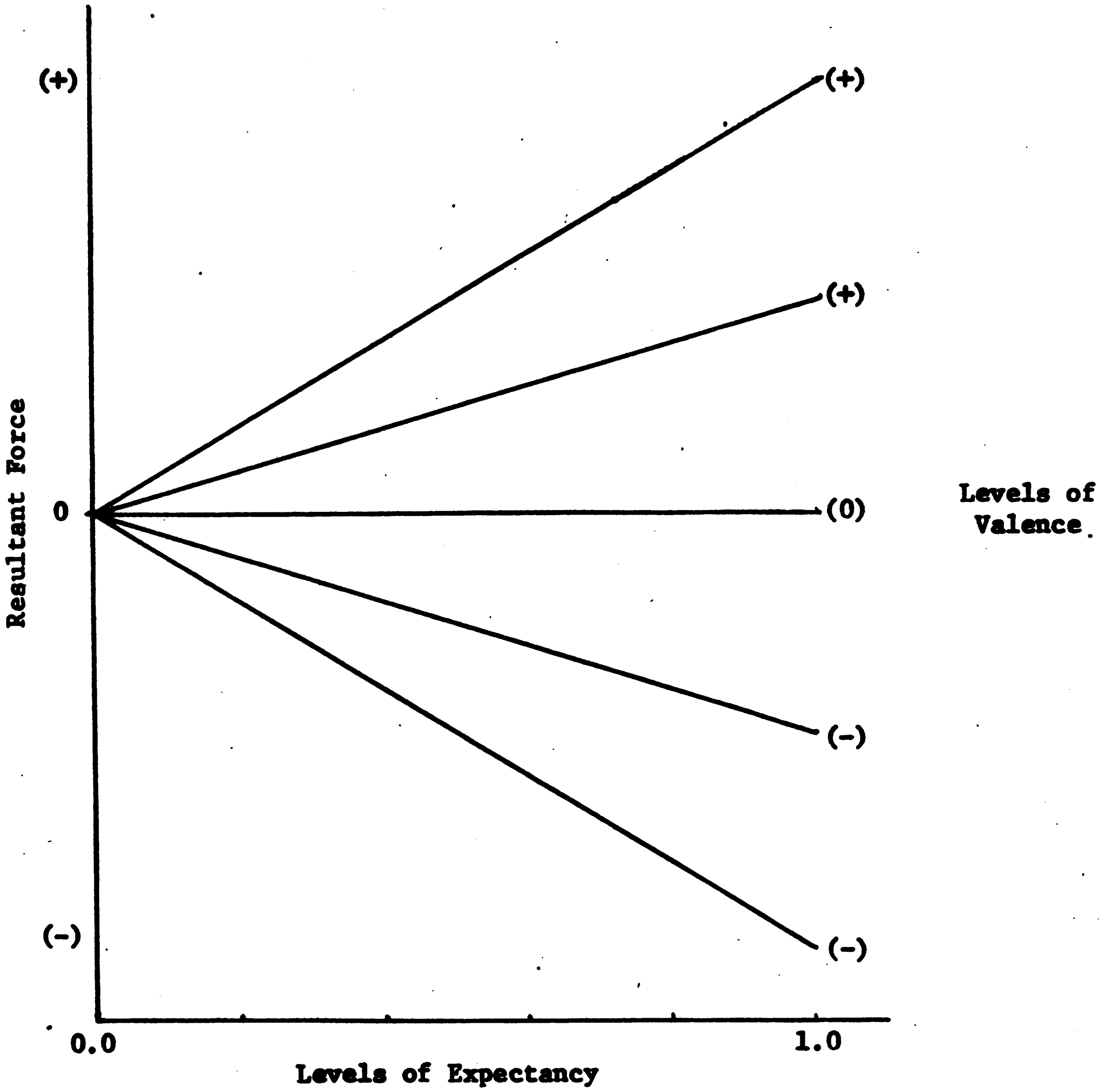
[4]It should be pointed out that if both positively and negatively valent outcomes are employed in our design, a technical problem will arise in obtaining estimates of the subjective values of expectancy from the Row means $F_{i.}$ which is identical to the problem we faced in obtaining estimates of the c_i for the computation of the bilinear component of the two-way interaction. The solution is identical and involves complementation of those columns predicted on a priori grounds to exhibit negatively sloped curves. The complemented matrix employed in obtaining estimates of the c_i is also employed to obtain estimates of the subjective values of expectancy $F_{i.}$ (cf. Anderson, 1974b, p. 24).

Figure Caption

Figure A. Asymmetric multiplying model predicted by expectancy theory



Resultant Force
(+)
0
(-)
(+)
(+)
(0)
(-)
(-)
Levels of Valence
0.0
1.0
Levels of Expectancy