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WARREN L. SIMMONS

Bulletin Number 33

Industrial Relations Center
CALIFORNIA INSTITUTE OF TECHNOLOGY
Pasadena, California

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PREFACE

Operations Research has been available as a managerial technique since World War II, but it has not been adopted generally. Although its methods can assist every manager in every activity, this assistance is obtained only when the manager <u>uses</u> Operations Research. He cannot use it until he understands it.

The Industrial Relations Center of the California Institute of Technology is publishing this Bulletin to "remove the coefficient of mystification" from Operations Research. The Bulletin defines Operations Research, compares its approach to problems with that of the scientific method, and discusses accomplishments, limitations, and expectations in the field. With this Bulletin as an introduction, managers may be stimulated to further study and may proceed to use this technique for better management.

The Center is especially proud of this publication because its author, Warren L. Simmons, is a graduate student of the California Institute of Technology, receiving his Master of Science degree in Physics in June, 1961. His B.S. degree was obtained in 1959 at Syracuse University where he was elected to Phi Beta Kappa and graduated <u>summa cum laude</u>. The author received his commission in the United States Air Force and was sent for graduate work to Caltech where he was promoted from Second to First Lieutenant.

This paper also demonstrates how the California Institute of Technology gives training for business administration, not as a major but as part of the development of scientists and engineers. Lt. Simmons, who like all other candidates for the Masters degree was required to take a Humanities elective, selected Economics 106, Business Economics (Seminar).

As part of his work, he attended during the first term a special series of meetings on "Statistics for Management," conducted by Dr. Claude S. Brinegar, Manager, Economics Department, Union Oil Company of California. In this program, attended primarily by representatives of various business firms, Lt. Simmons prepared a special paper which was the first draft of this Bulletin. During the second term, in continuing his work, Lt. Simmons participated in another series of special meetings on "Report and Letter Writing," conducted by Frank E. Davie, Special Conference Leader in the Management Development Section. This helped in some of the revisions. Finally, during the third term, Lt. Simmons attended "Practicing Supervision," conducted by Thomas B. Riley, Manager, Technical Operations Department, Electro-Optical Systems, Inc. In this program, Lt. Simmons encountered many of the problems faced by first-line supervisors in industry.

The decision to publish this paper was based on the opinion that it would be of value to many present and future managers. We are pleased that it also illustrates the broad training given at Caltech. We hope that it will stimulate present and future students.

Robert D. Gray, Director
Industrial Relations Center

ACKNOWLEDGMENTS

The author acknowledges his indebtedness to the authors whose published material on the subject of Operations Research is listed in footnotes and References of this Bulletin.

Valuable encouragement and assistance was also received from Dr. Claude S. Brinegar, Manager, Economics Department, Union Oil Company of California, in both the initial draft of this material as a term paper for his class and in the final preparation of it for publication.

Helen Thompson, Assistant to the Director, Industrial Relations Center, assisted in the final editing and supervision of reproduction of this Bulletin.

Warren L. Simmons

June 9, 1961

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INTRODUCTION

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Operations research is, as its name suggests, research in regard to operations. Specifically, it is a method of research that is undertaken to improve a particular operation. In thinking of the word "research," one immediately envisions scientific research--test tubes, formulas scribbled on a blackboard, scientists in long white coats--and this is good, because one of the basic characteristics of operations research is that it makes full use of the scientific method. By scientific method I mean that process which has led to most of today's advances in science: observation, analysis, hypothesis, and experiment. That is, a phenomenon is observed; it is broken down into component parts; an attempt is made to explain what is happening (generally by reducing the phenomenon to mathematical formulas that relate the various factors involved); and experiments are conducted to check these explanations and to determine whether other phenomena implied by the present explanation actually do exist.

For instance, Newton was hit on the head by an apple--it hurt (observation). He thought for a moment--the apple started at rest, but somehow it started moving downward (analysis). Then he reasoned that things generally stay at rest unless a force tends to move them. Therefore, there must have been a force acting on the apple (hypothesis, an explanation of what happened). So Newton ran into his laboratory, started playing with falling objects (experiment), and discovered the universal law of gravitation.

Operations research progresses along similar lines. In fact, this scientific method of approach to the problems of management, production, marketing, the military, etc. is what makes an operations analyst (a person who works in this field) different from the usual management consultant, production or marketing analyst, or military planner.

For example, suppose a soap company seeks to know whether its advertising budget for the coming year should be raised, lowered, or kept at its present

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level.[1] The company might call in a business consultant who, in this case, would be a specialist in marketing or advertising or both. He would have had considerable experience in the advertising and marketing of similar products, would have an abundance of related data available, and would surely know where to go to supplement this information, if necessary. He would examine his client's records, correlating sales with advertising expenses over the past years. He might consider data by states or marketing areas, or even broken down to the city level. He might compare his client's records with those of competitors. If the project was at all large, he would undoubtedly make a field survey, questioning a random sample of housewives about the type of soap they preferred and what advertisements they were aware of. With all this information assembled, he would determine the level of advertising expenditure appropriate to the forecast level of sales. Very likely the figure would turn out close to five per cent of the expected gross sales since our consultant would be well aware of the old maxim, "It takes five cents worth of advertising to sell a dollar's worth of soap," and since he would have little reason to deviate far from the standard norm. Besides, all the data he considered were in this range, except for a few marketing flops and, at the other extreme, advertising campaigns that were successful beyond all expectation or possible duplication. In short, he would arrive at a recommendation based solidly on, and varying little from, past experience.

On the other hand, the company might solicit the services of an operations analyst. Typically, this man or firm (much operations research is done by groups of people with varied backgrounds) would have no feeling for soap selling (the American Marketing Association reports that more than forty per cent of operations analysts are engineers by training, and another forty-five per cent are mathematicians, statisticians, or natural scientists). But to a good operations analyst, this lack of soap-selling experience would be no handicap. With his scientific

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[1] In preparing this expository article, I have drawn heavily on the published work of authors listed in my References. For this assistance I offer my thanks. See Reference 1.

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background he might formulate the problem this way:

"Let p_{xt} be the probability that the x^{th} household will buy my client's soap during the t^{th} week. Then p_{xt} is the product of two factors: p'_{xt}, the probability that they will buy any soap during the t^{th} week, and a_{xt}, the probability that if they buy any soap it will be my client's brand. Assume p'_{xt} cannot be influenced by any advertising of my client, but that a_{xt} can. In fact, suppose that the Weber-Fechner law applies[2] so that the rate of increase of a_{xt} with respect to advertising expenditure is inversely proportional to the level of advertising expenditure already attained, i.e.:

$$\frac{da_{xt}}{dE} = \frac{c}{E}$$

where E is the level of advertising expenditure and c is the constant of proportionality. Integrating this equation,

$$a_{xt} = c \log E + \log K = \log KE^c$$

where K is the integration constant (which can be determined from the data). Then $p_{xt} = p'_{xt} \log KE^c$, and the total expected sales in the t^{th} week (S_t) can be estimated by integrating this expression over the entire population (i.e., over all values of x from 0 to X, the number of households):

$$S_t = \log KE^c \int_0^X p'_{xt} dx$$

$$\text{or} \quad \frac{dS_t}{dE} = \frac{c}{E} \int_0^X p'_{xt} dx$$

Then if we find the value of E at which the profit per dollar's worth of sales (p),

[2] This supposition has to be investigated, but any model must start with some hypothesis.

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multiplied by this derivative, equals 1 (the cost of an additional dollar's worth of advertising), we will have the optimum level of advertising $\hat{E}$. That is, we must solve:

$$p \left. \frac{dS_t}{dE} \right|_{E=\hat{E}} = 1$$

$$p \frac{c}{\hat{E}} \int_0^X P'_{xt} dx = 1$$

$$\text{or} \quad \hat{E} = pc \int_0^X P'_{xt} dx \quad ."$$

In short, the operations analyst aims to arrive at a recommendation based on logical deduction from simple first assumptions which are a good approximation to the laws underlying the phenomenon in question.

The difference between these two approaches is clear, and it is also clear how the operations analyst utilized scientific methods, particularly analysis and hypothesis. Experiment follows when the analyst plugs in some numbers and determines whether the analytical model fits the actual data. For example, at this point the analyst must test the validity of assuming that the Weber-Fechner law applies. If it does, he extrapolates to the coming year's situation; if not, he starts over with a new theory and tries to make that fit the past and existing conditions. This is the scientific method, the approach of operations research.

BREAKDOWN: AN ANALYSIS OF THE METHOD OF OPERATIONS RESEARCH

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Thus the work of an operations analyst can be broken down in general into the following steps:

First he has to determine the objectives of a particular operation. In many cases this is not as easy as it may seem. In the case just given, the objective was simple: The company wanted to make money by selling soap. This means sell as much soap as possible while spending as little as possible to advertise. In another problem the objective might be to maximize the number of cars that go through a toll booth per unit of time while minimizing the number of people needed to run the booth. Or perhaps a manufacturer wishes to keep inventory at a minimum while still having enough on hand so he can supply dealers with little or no delay. In these cases the objective is easy to recognize and describe.

However, in a problem that arose during World War II in the British Artillery, it was not that simple. The British were trying to determine the most effective type of shell to fire at enemy troops in the field. But when they started to make an evaluation of "effectiveness," they didn't know what was or wasn't "effective." Was it the total number of enemy troops killed by an exploding shell? Or, perhaps a shell killing fewer but wounding more? Then too, the sound of exploding shells has a great demoralizing effect on enemy troops--hence a loud, practically harmless shell might be very effective. After many long, top-level discussions they decided a compromise between lethal and loud shells was best. They tested various types of shells and fuses by "attacking" wooden dummies that resembled men. A hole completely through a dummy "killed" it, while "wounds" were inflicted by any piece of shell striking the dummy. It was found that shells which burst upon hitting the ground would generally kill all the people in the immediate area, but rough terrain would protect others nearby. A shell bursting a number of feet up in the air, although it didn't kill as many as a direct hit, was louder and far more effective in wounding large numbers. Thus the operations

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research suggestion--to reset fuses so that shells exploded before hitting the ground--increased the "effectiveness" of British artillery by factors of two and three (those are British figures).

Once he has in mind what an operation is trying to perform (scientific method--"observation"), the operations analyst tries to formulate a model (scientific method--"analysis"). In many cases the model consists of mathematical symbols which designate the various factors or parameters with which the problem deals. "Let p_{xt} equal the probability . . . " Or perhaps N is the number of cars that pass a given toll booth in the time t. The parameters must lend themselves to quantitative evaluation, or in general they will be useless. The degree of accuracy to which a parameter can be evaluated will be reflected in the accuracy of the final recommendation. It is often desirable to keep the number of parameters at a minimum, approximating slowly varying factors by constants, so that the final equations are less formidable in solution.

In other cases the model to be set up is physical, like the wooden dummies the British shot up in their experiments. Here again the model is simple and can be evaluated quantitatively (a hole completely through is a "kill").

Now with the various parameters clearly in mind, the operations analyst sets up the relationships that exist between them (scientific method--"synthesis"). Here is the crux of the whole process, for here is where the assumptions are made. Is there actually an inverse relation between the level of advertising expenditure and the rate of change of probability that a household will buy brand "X" soap? Experiments may help one to find out and are used extensively in most operations research projects.

For instance, during World War II the United States Air Force made extensive studies on bombing techniques and efficiencies, especially in the Eighth Bomber Wing, which engaged in the bombing of Germany. The operation objective was simple: to determine what types of bombs and means of delivery would produce the greatest destructive effect on all

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types of German industrial and military strength. The parameters were quickly established: the number N_a, $N_b \dots$ of planes of type a, b ..., the number n_1, $n_2 \dots$ of type 1, 2, 3... bombs they carried, speeds of planes V_a, $V_b \dots$, altitude of attack h, bomb weights W_1, $W_2 \dots$, plane carrying capacities W_a, $W_b \dots$, expected fractional loss of type a, b ... bombers to enemy fighters, L_a, $L_b \dots$, to enemy antiaircraft guns L'_a, L'_b ..., and so forth.

Some relations were immediately clear. For instance: $W_a = n_1 W_1 + n_2 W_2 + \dots$, or the number of planes reaching the target $N = (1-L_a-L'_a)N_a + (1-L_b-L'_b) N_b + \dots$, etc. L_a is clearly some function of bomber type a, of speed V_a and of the number of planes in the formation $N_a + N_b + \dots$. L'_a is dependent upon these same factors as well as on h. The type of formation flown (position of planes in relation to one another) affects both L_a and L'_a as well as the accuracy that can be accomplished in the bombing.

A scale for evaluating the bombing accuracy was not so easy to obtain. After examination of extensive numbers of combat photographs taken for the purpose of the study, the operations analysts decided to measure the size of the bomb pattern for each formation, convert it to an equivalent rectangle, and measure the distance from the center of the rectangle to the center of the target. Many men risked their lives flying their planes in straight lines after dropping their bomb loads waiting for the cameras to grind out a record of the bombs falling and exploding, when they might have preferred to peel off and head for home. Extensive research had to be done to evaluate the effects of various type bombs on various type targets. With this tremendous volume of information and empirical relationships, it was a straightforward, though tedious, job to determine the best way to get the best results. Out of this work evolved the proximity fuse, new training manuals, better bomb-run tactics, and salvo bombing (letting the whole load go at once to reduce the length of the bomb pattern).

It suffices to say that the results justified the operations research. Actual improvement was from an estimated fifteen per cent of all bombs dropped falling within 1,000 feet of the target at the beginning

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of the project to better than sixty per cent toward the end of the war. To this can be added the immeasurable increase in destruction that resulted from the development of new types of bombs and proper fusing.

With the whole problem now in front of him in mathematical or equivalent form (equations, inequalities, block diagrams, logical sequences), the analyst attempts to reach some optimum solution in terms of the previously established objectives. I say "attempts," because many times the problem must be simplified in numerous ways before a solution can be obtained. The analyst must draw upon many facets of his scientific background--the methods of algebra, calculus, differential equations, probability theory, statistics, numerical analysis--as well as upon his imagination and judgment. Many problems can be solved only on a gigantic computing machine, and there are other times when a whole new method of solution must be invented.

With a solution in hand that is consistent with existing data and past experience, the analyst reconverts his symbols to words and makes a recommendation to the organization that hired him.

ACCOMPLISHMENTS: BRIEF ACCOUNTS OF SOME IMPORTANT RESULTS IN OPERATIONS RESEARCH

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Operations research was being performed long before anyone decided to call it by that name. Ever since the third century, B.C., when Hieron, King of Syracuse, asked Archimedes to devise means to break the Roman naval siege of his city, political and military leaders have consulted scientists for solutions to the problems of war. During World War I, Thomas Edison worked with the Naval Consulting Board, studying antisubmarine warfare. It was not until World War II, however, that the military fully realized the usefulness of operational research. First in Great Britain, then in America, operations research teams began to appear in the military hierarchy, at all levels of command. Some typical results of their work might help to clarify the approach of operational research:

During the planning of the first low-level bombing attack against the Ploesti oil fields in Rumania, there was considerable concern that bombers might fly into the cables of barrage balloons that were known to be in use over the target. An analyst at Fifteenth Air Force Headquarters made some calculations and predicted that if this happened, the balloon cable would break before the aircraft wing would shear off. The accuracy of his prediction was dramatically demonstrated when aircraft returned from the mission with the imprints of balloon cables on the leading edges of their wings.

At another time the Fifteenth Air Force was considering an attack on a target of underground oil storage tanks. When intelligence reported that the underground tanks were covered by heavily reinforced concrete slabs plus ten to fifteen feet of earth, there was great doubt that bombs could penetrate the barrier and reach the target; consequently, consideration was given to abandoning the attack. Again, after making a few calculations, an analyst pointed out that the cost of such a protective structure would be much more than the value of the tanks and oil combined. Therefore, it was reasoned that no such protection existed and that the intelligence had been deliberately faked in

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hopes of averting an attack. On the basis of this study the attack was made and was quite successful.

The Navy made extensive use of its operations research teams, who came up with plans for the most efficient use of radar and sonar, diagrams for least vulnerable fleet formations, and plans for the size of merchant convoys that would minimize loss to submarine action while minimizing escort protection. They made a study on "killing" submarines with depth charges dropped from dive bombers. Prior to the study, the charges were set to explode at depths of at least 100 feet, where they would be well "tamped" by the water. Upon collecting data, however, the analysts found that most attacks were made against submarines at or near the surface; planes sighting a submarine tended to move in for the kill immediately, since a submarine on the surface or barely submerged was a more favorable target than one at a greater depth. Since the lethal range of the charge was approximately 20 feet, it was only on rare occasions that a U-boat would reach the danger depth in time for the detonation; the enemy subs were obviously not receiving the full punch of the depth bomb explosions. The analysts recommended resetting fuses so that the charges went off 20 to 25 feet below the surface. The German crews soon were reporting that "new and more powerful" bombs were being used against them; the increase in destruction to submarines was estimated at over 500 per cent.

While it is true that operations research received its name and initial growth during World War II, it really "blossomed" in the peace that followed. One aspect of the field has come to be known as queuing theory--the method of tackling problems having to do with a queue or waiting line. The problems applicable to such an approach are numerous: airplanes waiting to land or be serviced at an airport, tankers or cargo ships waiting to be unloaded at a seaport, autos waiting to pass through a toll gate, housewives waiting to check out of a supermarket, a telephone subscriber waiting to place a call, locomotives waiting to be serviced at the depot; all of these and many more. The problem in each case is basically the same: how can the most people

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be serviced satisfactorily by the fewest number of servicing facilities, keeping queues reasonably short? Without going into detail, this is a problem for the computers. One solution is called the Monte Carlo method, where the results of twenty years of random arrivals at an airport, dock, or toll booth can be ground out in minutes by plugging random numbers into a computer. The average length of the queue and the total number of idle servicing-facility man-hours falls out immediately, and from these data it is easy to determine whether more or fewer facilities are needed. The effects of scheduled arrivals can be incorporated, rush-hour loads added, strikes by dock workers included--and in the end the most efficient use of present or planned facilities can be evaluated.

Industry has received tremendous benefits from what is known as combinatorial or sequencing theory, where the operations research recommendation concerns the order in which operations in some complicated process are to be performed. There are undoubtedly numerous ways to put a wrist watch together, but there is only one way (or a few equivalent ways) to assemble it most efficiently (in terms of speed and ease). It is clear that the best way is <u>not</u> to put the crystal and the case together, then, with tiny tweezers, fish the workings in through the hole where the stem eventually protrudes. It is obvious that you should put your socks on before your shoes, but not so obvious whether the socks should precede the trousers. In the shop, if the same machine is to stir black paint and white paint, it is better to stir the white first. In a moving production line, not only the order of operations is important but also that each operation takes approximately the same amount of time. Thus the operations analyst reduces a complex procedure into component operations, assigns values to sequences of operations in relation to other sequences, and optimizes a series of sequences into the best over-all procedure.

Also in industry, research into production methods has proved very useful. Consider the case of the firm that produced steel bars of standard lengths with small allowable tolerances. Each bar was rolled from a billet whose weight was brought as close as possible to a fixed standard; but because of

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fluctuations in the size of the billets, the actual as-rolled lengths of the final bars varied over a range of several feet. A bar exceeding the required length had to be trimmed to size and the pieces cut off were usable only as scrap; a short bar had to be scrapped altogether. The foreman of the bar mill made such a fuss when a bar came out short, that the billet cutter made sure his billets were large enough so a short bar was a very rare occurrence.

An operations analyst investigating the operation measured the as-rolled lengths of a large number of bars and found there was a normal distribution of lengths as in Figure 1:

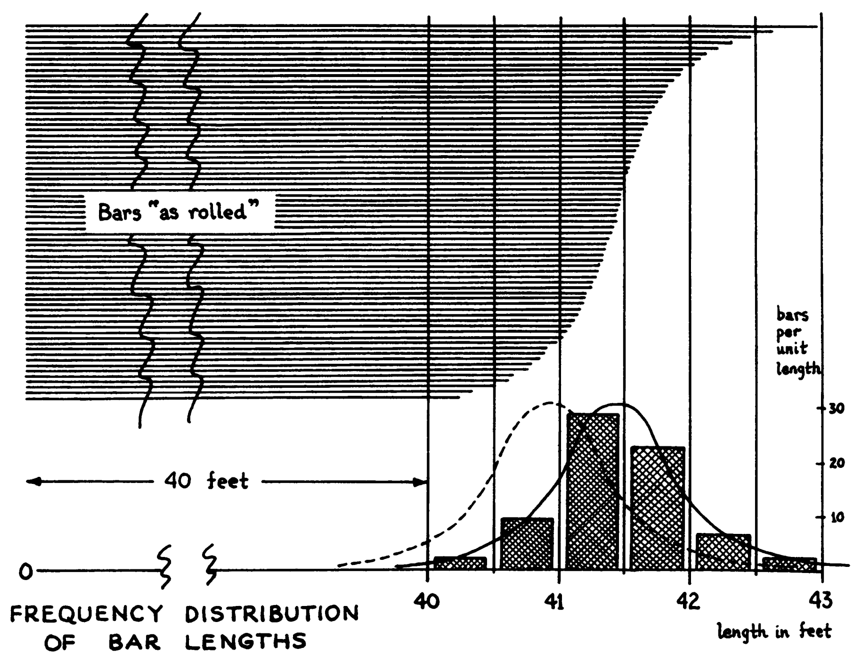

Figure 1.

The histogram shows the number of bars in each 6-inch length interval. In this example there is an average waste of 1 foot 6 inches arising from the trimming of over-length bars, in other words a 3.6 per cent loss. The operations analyst showed the loss could be reduced if the average billet size were reduced so that the distribution curve moved to the left--to the position illustrated by the dotted curve. The minimum loss in this case corresponds to a position of the curve where 1 per cent of the bars is scrapped for being too short, but simultaneously only about 1

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foot per bar, or 2.4 per cent, is lost in trimming. The total loss then would be 3.4 per cent as opposed to 3.6 per cent. By decreasing the billet cutter's settings to give this new average bar length, a valuable increase in yield (.2 per cent) was effected without any other changes in the process.

Merchandising as well as industry has found operations research valuable in regulating inventory policies. The techniques can be divided into three classes in order of increasing sophistication: static analyses, stationary state analyses, and dynamic analyses. Let me cite an example from the first, referring readers interested in the latter two techniques to References 1 and 3. Static analyses deal with isolated periods of time and average behavior during these periods. A typical result known as the "square root law", at best a crude approximation to actual practice, can be derived as follows:

Suppose that the average weekly requirement for a particular inventoried item is n units, that it costs \$c per unit per week to store such an item, and that the cost of placing an order over and above the cost of the goods ordered is \$k. Suppose further that reorders are placed in time so that the new stock arrives just as the old stock is exhausted (note the inapplicability of this approach to states where there is delay between ordering stock and receiving delivery of it). Then the only decision that has to be made is how many items to order each time an order is placed. Call this number x. On these assumptions, $\frac{n}{x}$ is the average number of orders made per week, at a cost of \$ $\frac{kn}{x}$ per week. Inventory runs from x units immediately after replenishment to zero just before, making the average inventory $\frac{x}{2}$ units costing \$ $\frac{cx}{2}$ in carrying charges per week. Then the total cost of maintaining an inventory under these circumstances is \$ $(\frac{kn}{x} + \frac{cx}{2})$ per week. To find the optimum value of x, we differentiate this cost with respect to x and set the result equal to zero:

$$\frac{d}{dx}\left(\frac{kn}{x} + \frac{cx}{2}\right) = -\frac{kn}{x^2} + \frac{c}{2} = 0$$

$x = \sqrt{\frac{2kn}{c}}$. Thus the optimum size of

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order (and of average inventory) varies in proportion to the square root of 1) sales or consumption n, and 2) the ratio of ordering cost to carrying cost, $\frac{k}{c}$. The analytical approach of operations research is clear.

Operations research is the basic attack of the much publicized RAND Corporation in their requirements studies, investigations into the future of modern warfare, and other interesting subjects. As a result of some of their work, the theory of games--or game theory--was added to the tools of the operations analyst, and it is a fascinating technique. A short and simple (as game theory goes) example might be the following:[3] Suppose you are resting comfortably on the deck of a Las Vegas pool, and you are approached by a stranger who suggests matching coins. You, however, feel it is too hot for such exercise and say so. The stranger then says, "Well, let's just lie here and simultaneously speak the words 'heads' or 'tails'--and to make it interesting, I'll give you $30 when you call 'heads' and I call 'tails,' and $10 when it's the other way round. And, just to make it fair, you give me $20 when we match."

Alerted by the environment (Las Vegas), you suspect you should have the man arrested rather than play his silly game. But you are a gambling man, and, if the odds are right, you might make some money. So you make the following game theory calculation: In four boxes appropriately labeled you place the value of the four possible "tosses."

		stranger calls	
		heads	tails
you call	heads	−20	30
	tails	10	−20

(values in dollars)

[3] For more complete explanation, see Williams, J. D., *The Compleat Strategyst* (New York: McGraw-Hill Book Company, Inc., 1954), Chapters 1 and 2, especially pages 50-51, which include the example.

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You look for what is known as a saddle point--if it turns out that the best of the worst you can do is the same as the worst of the best he can do, the game is solved--you would both have a definite way to play the game.

		stranger calls: heads	stranger calls: tails	your worst
you call	heads	-20	30	-20
	tails	10	-20	-20
	his best	10	30	

either is best worst

worst best

It is clear that his worst best ($10) is not equal to your best worst (-$20), so the solution is not simple. You continue by determining the best way that each of you can play the game; that is, by finding what ratio of the time you and he should call "heads" or "tails" so as to make the most (or lose the least) amount of money. This is done by taking the differences of the values in the rows and columns in the following way:

		stranger calls: heads	stranger calls: tails	
you call	heads	-20	30	10–(–20)=30
	tails	10	-20	30-(-20)=50
		30-(-20) = 50	10-(-20) = 30	

...and by calling "heads" or "tails" in that proportion.

So the best way you can play the game is by calling 30 "heads" for every 50 "tails" (the stranger, it is seen, should call 50 "heads" for each 30 "tails"). At first sight it would seem that you should call more "heads" than "tails" since you stand to win $30 or lose $20 rather than win only $10 or lose $20. But it turns out that the stranger knows this and will play accordingly, calling "heads" (costing you $20 if you also call "heads") more often than he calls "tails." Hence the

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solution is correct (it can be shown mathematically), but you still don't know who is going to win money in the long run. So you compute the "value" of the game:

If you played "heads" all the time, you would win (assuming the stranger plays correctly):

$$\frac{50 \times (-\$20) + 30 \times \$30}{50+30} = \frac{-\$1000 + \$900}{80} = -\$1.25$$

Note that this is the same as if you had played "tails" all the time:

$$\frac{50 \times \$10 + 30 \times (-\$20)}{50 + 30} = -\$1.25$$

It is also what it would cost you on the average if you played any combination of heads or tails; that is, it is the "value" of the game. The best you can do is lose \$1.25 per toss. You now know the stranger is a shyster and hand him over to the nearest policeman.

This is only the very simplest type of game. Problems where the box diagrams resemble checkerboards have been devised and solved, but at this level it becomes necessary to use computers again. The problems in game theory that were first solved dealt mainly with the strategy of warfare, but the theory has since found many useful applications in business and government.

I have made numerous references to high-speed computing machines and perhaps should say a further word about one of their most important roles--in the field of linear and non-linear programming. The problem is to discover the best way to allocate a limited number of resources so as to obtain maximum possible attainment of objectives. As an example of linear programming, let us consider a highly simplified version of the scheduling of oil refinery operations.

Suppose a refinery has N units of blending capacity which it uses to make regular and premium gasoline (to be denoted by subscripts 3 and 4 respectively) from two grades of blending stocks (indicated

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by subscripts 1 and 2). We introduce four variables x_{13}, x_{14}, x_{23}, x_{24}, where x_{ij} represents the number of barrels of i^{th} blending stock devoted to production of gasoline j. The total input of blending stock 1 is $x_{13} + x_{14}$ and of 2 is $x_{23} + x_{24}$. If the stocks cost $\$C_1$ and $\$C_2$ per barrel respectively, the total cost of fuel inputs is $\$\left[C_1\ (x_{13} + x_{14}) + C_2\ (x_{23} + x_{24})\right]$.

The total output of regular gasoline is $x_{13} + x_{23}$, that of premium is $x_{14} + x_{24}$. If these are sold for $\$S_3$ and $\$S_4$ respectively, the value of the total output is $\$\left[S_3\ (x_{13} + x_{23}) + S_4\ (x_{14} + x_{24})\right]$. Subtracting we get the profit to be

$$\$\left[(S_3 - C_1)\ x_{13} + (S_3 - C_2)x_{23} + (S_4 - C_1)x_{14} + (S_4 - C_2)\ x_{24}\right]$$

The objective is to make this number as large as possible.

Now suppose that the only quality specification to be considered is octane rating. The octane rating of any blend is the weighted average of the ratings of its components. Thus suppose the octane rating of blend 1 is 0_1 and of 2 is 0_2. Then the octane rating of the regular gasoline is:

$$\frac{0_1 x_{13} + 0_2 x_{23}}{x_{13} + x_{23}}$$

Similarly for premium the rating is:

$$\frac{0_1 x_{14} + 0_2 x_{24}}{x_{14} + x_{24}}$$

If regular gas must have an octane rating equal or greater than some standard 0_3 and premium a rating equal or greater than 0_4, then it follows that:

$$\frac{0_1 x_{13} + 0_2 x_{23}}{x_{13} + x_{23}} \geqslant 0_3 \qquad \text{and} \qquad \frac{0_1 x_{14} + 0_2 x_{24}}{x_{14} + x_{24}} \geqslant 0_4$$

Finally, let us say it requires n_3 units of blending capacity to produce a barrel of regular fuel, and each barrel of premium requires n_4 units. Then since there are N units available, we have a capacity constraint: $n_3\ (x_{13} + x_{23}) + n_4\ (x_{14} + x_{24}) \leqslant N$.

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Our problem becomes one of maximizing

$$(S_3 - C_1)x_{13} + (S_3 - C_2)x_{23} + (S_4 - C_1)x_{14} + (S_4 - C_2)x_{24}$$

subject to the restrictions

$$\frac{0_1 x_{13} + 0_2 x_{23}}{x_{13} + x_{23}} \geqslant 0_3 \; ; \qquad \frac{0_1 x_{14} + 0_2 x_{24}}{x_{14} + x_{24}} \geqslant 0_4$$

and $n_3(x_{13} + x_{23}) + n_4(x_{14} + x_{24}) \leqslant N$. The rest is arithmetical calculation, since 0_1, 0_2, 0_3, 0_4, n_3, n_4, C_1, C_2, S_3, S_4 and N are all known.

In actual practice things are not quite this simple. Instead of two input stocks, there may be hundreds. Octane rating is not the only quality characteristic. Prices vary so that the problem must be continually reworked with new numbers. Blending capacity breaks down or various stocks become unavailable. However, the basic principle that all relations are linear is not violated, and with a large enough computer, almost any situation can be evaluated.

An even more common application of linear programming than the refinery blending problem is the so-called "transportation problem," which arises whenever a commodity can be shipped from a number of sources to a variety of destinations. The problem becomes involved when, aside from just the distances separating source and market, there are differences in production cost at various supply plants, limitations on the capacity of shipping routes, limitations in warehouse space, and other similar conditions. But all relationships are expressible as sums and differences (linear equations), and the computers can handle the most complicated situations.

Another simple linear programming problem appeared in the "games" section of a recent Scientific American magazine[4]. Consider the following situation. Two freight cars A and B are sitting at positions 1 and 2 in Figure 2:

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[4] *Scientific American,* Vol. 203, No. 4, October, 1960, p. 174.

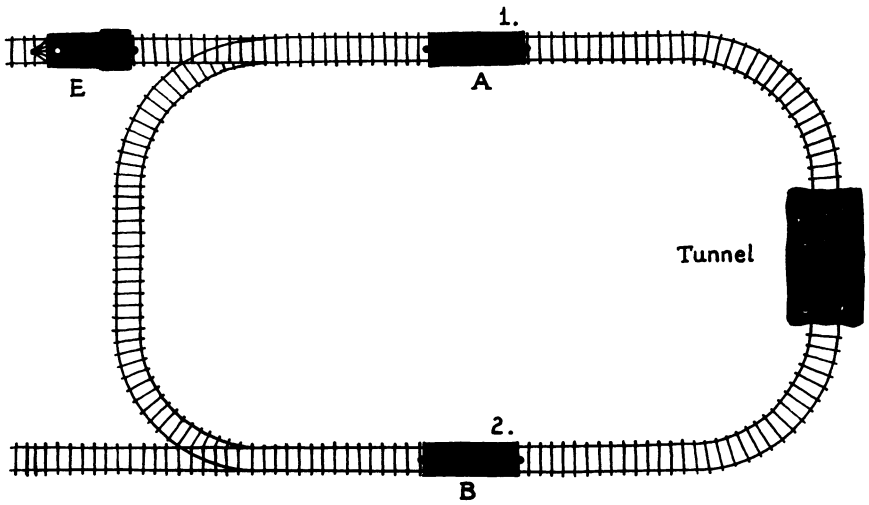

Figure 2.

At the left is a switching engine E, at the right a tunnel. However, the tunnel is small and while the engine passes through it freely, neither of the freight cars can. The problem is for the switching engine to interchange the positions of cars A and B and return to its present location in the fewest possible number of operations, with an "operation" defined as any action which results in the engine coming to a stop. The switches work automatically, but any reversal of direction by the engine is necessarily an operation, and it also must come to a stop to couple or uncouple either car or both. Like a computer, to solve this problem one must try all, or at least a large number of, solutions to determine the minimum number of operations necessary. To work this problem, you might let the engine and cars be a dime, nickel, and penny on a diagram similar to that above. The answer is 16.

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LIMITATIONS: RESTRICTIONS ON THE SCOPE OF OPERATIONS RESEARCH

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Having taken a look at some of the major areas where operations research has made its mark, it is time for us to talk about limitations. Perhaps the best approach is to look back at the generalized steps that go into the process of operations research. If the first step is to be fulfilled, operations research can be effective only in cases where the objectives of a particular operation are, or can be, defined. Looking further, we realize that techniques in operations research are limited to cases where the operation is describable by a reasonably manageable model. It must be expressible in symbols or physical objects which can be numerically evaluated. Operations research fails when major factors in the process are absolute uncertainties; they must at least be expressible in terms of a probability that one thing or another will occur. In evaluating the parameters défined by a problem, data must be available, or it must be obtainable from specially conducted surveys or testing. Operations research is limited in that it does not take account of unexpected turns of events. As long as the analyst is able to conceive of every possible mishap that could occur, his results will be sound. But there is nothing in the process of operations research that can account for the totally unforeseen development. Finally, for the costly operations research process to be warranted, the current operating practice must exhibit considerable room for improvement.

Many times it is not clear what operations research will produce for a solution--perhaps it will find that the process is working near peak effectiveness and that the analysis project was a waste of time and money. However, in just the opposite sense, one of the very useful by-products of operations research is the benefit an operation will receive merely by being questioned: the management may learn a great deal by answering the analyst's questions, supplying data, and checking over the model. It is possible that, even if the analysis comes up short of suggestions, the self-analysis, like psychoanalysis, will be helpful.

EXPECTATIONS: A DISCUSSION OF THE FUTURE OF OPERATIONS RESEARCH

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What is to come in the future?

Advances will undoubtedly be made by applying known methods for solving existing problems to other and different kinds of problems. That is, what works in one case may well work in other similar, or even entirely different, cases. But this type of approach might better be called operations "engineering", in analogy to the case of a pure science (like physics) versus the application of that science (engineering). Advances in operations research, then, must be more fundamental, not merely the application of an old technique to a new problem.

It is certain that problems for operations research will become more complex in the future. They will probably widen in scope, and be tackled at higher organizational levels. Thus, it seems, they will be harder to solve, requiring bigger research teams, larger expenditures of funds, and more complex technologies. The smaller kinds of problems will probably be left to the engineers.

Major areas of interest will certainly include defense and industry. The RAND Corporation will continue its war games, and corporations like General Motors, Chrysler, General Electric, etc., will continue to investigate production and marketing. There is a definite need for operations research in the government. Some work has been done in the postal department, but more is needed. Our country's survival may depend on a fresh approach to State Department problems in national and international affairs. In the Department of Agriculture, in fact all over the world, studies of food supplies and farming methods are badly needed. It is strange that agriculture has been so little studied by operations research, in spite of the fact that so much information is available (from the United Nations, for instance) to synthesize into problem solutions. In charity work, the administrative cost of collecting from small donors sometimes is a very high proportion of the actual donations; and there often appears to be little correlation between

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community needs, services available in the community from other sources (i. e., local or state government), and the allocation of collected funds. Operations research in this field would be very worthwhile.

I could mention medicine, law enforcement, education, and almost every other phase of modern life -- but I think the point is clear. Operations research has grown rapidly from its infant state during World War II, until today it is a recognized profession with a tremendous future in a wide variety of fields.

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