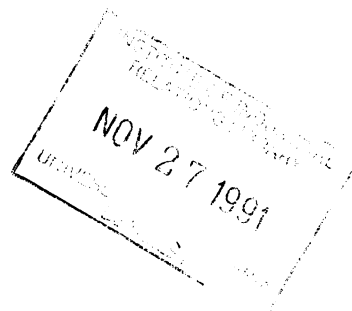


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GROUP NETWORKS
AND CENTRALITY

by

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GROUP NETWORKS AND CENTRALITY

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Abstract

The earlier methods of Stephenson-Zelen for calculating centrality based on information is adapted to group networks. Consider a network of points where every point belongs to one of k mutually exclusive groups. This is called a group network in contradistinction to a unit network which cannot be further decomposed. Two groups are said to communicate if members of the different groups have at last one communication episode. The strength of the communication between two groups is measured by the ratio of the number of communication episodes to the maximum number possible. This parameter is equivalent to the probability of communication between a pair of units, randomly chosen from each of two groups. An example of a group network from a high technology company is used to illustrate the calculations and to show the usefulness of group centrality.

1. Introduction and Statement of Problem

The increasing use of social networks has been principally devoted to describing and analyzing the inter-relationships between individuals. In an abstract formulation, individuals are depicted as points and when a pair of individuals communicates, a line joins them. The "weight" associated with the lines may correspond to the frequency of communication or some other measure of the importance of the communication between pairs of points.

Recently we have been interested in adapting the ideas associated with networks to organizations having well defined groups or departments. The major interest is concerned with communication between departments rather than communication between individuals. Referring to the abstract model of social networks comprised of points and lines, the model now is composed of sets (groups) where a set may be composed of one or more points. A pair of groups or sets is defined to communicate if a point from each set communicates.

It seems appropriate to distinguish between these two classes of networks. We shall use the term "unit network" to describe a network comprised of individuals. It shall be assumed that the unit cannot be further decomposed. In a unit network, the probability of a pair of units communicating with each other is either one or zero, that is, they either communicate or they don't communicate with each other. In contrast, the term "group network" refers to networks composed of groups where each group has at least one unit. The probability of two groups communicating with one

another is not restricted to be zero or one. This probability refers to a communication episode between two randomly drawn units from each group.

There are several marked differences between unit and group networks. Among these differences is that not every pair of points may communicate in a unit network whereas in a group network it is likely that all pairs of groups communicate. The larger the number of individual points in a group, the greater the opportunity for the group to communicate. In a group network setting, the number of individuals or units may be quite large. Hence random sampling may be used to collect data in order to estimate the probability that two groups communicate.

In applying these ideas to real situations, a unit and communication episode must be carefully defined. Ideally, the class of respondents should be restricted to the communication process to be studied, e.g. middle and/or upper management. A communication episode should be carefully defined in terms of frequency, importance, rate of communication, and other criteria. For example, one might wish to impose a threshold level such that a minimum number of contacts must occur before the communication is considered to be an episode.

This paper extends the ideas of Stephenson and Zelen (1989) for measuring centrality in group networks. Section 2 introduces the basic model; section 3 illustrates the calculations using data from a small high technology company; sections 4 and 5 discuss

practical issues and general conclusions. The appendix contains a detailed discussion of the model.

2. Notation and Formulation

Consider a group network composed of N units or individuals such that each individual is identified with a single group. Let there be k groups such that N_i = number of units in the i^{th} group with $i=1,2,\dots,k$. The number of pairs of units amongst groups i and j is $N_i N_j$. Clearly the larger the number of pairs the greater the opportunity to have communication between the two groups. The calculation of centrality will be carried out so that the centrality does not depend on the number of pairs. One way to normalize for the number of pairs is to define a measure of communication which is the number of episodes of communications per pair. If N_{ij} represents the number of pairs from groups i and j who communicate, then the number of communication episodes per pair is defined by

$$P_{ij} = \frac{(\text{no. of episodes between groups } i \text{ and } j)}{(\text{no. of pairs between groups } i \text{ and } j)} = N_{ij}/N_i N_j,$$

This also represents the probability that a pair of randomly chosen units from groups i and j communicate. If all units in a pair of groups communicate then $N_{ij} = N_i N_j$ and $P_{ij} = 1$.

In the real life setting, the values of $\{P_{ij}\}$ must be estimated from a sample. Incomplete observations may arise from a random sample of units in a group (information is not known for units not in the sample) or may be due to non-response. When the incomplete

information is due to non-response, it will be assumed that the reason for the non-response is independent of the number of communication episodes and of any other characteristics of the group. If this independence assumption is not true, then estimates based on the sample may be biased.

To account for sampling and/or non-responses define n_i as the number of respondents in a random sample from group i . Each sampled unit in group i is queried about communication episodes from the other groups. Referring to groups i and j , a sample of n_i units in group i actually represents a sample of $n_i N_j$ pairs of units from groups i and j . Similarly a sample of n_j units from group j represents a sample of $N_i n_j$ pairs from groups i and j . Since $n_i n_j$ pairs are counted twice, the total number of sampled pairs is

$$M_{ij} = N_i n_j + n_i N_j - n_i n_j.$$

Therefore if n_{ij} represents the number of distinct communication episodes in a sample from each group, the estimate of P_{ij} is

$$\hat{P}_{ij} = n_{ij}/M_{ij} \quad \text{for } i \neq j \quad (1)$$

A simple example of a group network will be discussed in order to illustrate the basic ideas. Figure 1 consists of a two group network composed of $N=18$ units. In this network, we can envision the points representing individuals in an organization and the lines representing their intergroup communications with one

another. Obviously, not all individuals communicate with one another. The two distinct sets or groups are designated Group 1 and Group 2 having $N_1 = 10$ and $N_2 = 8$ units in each group respectively. Between the two groups, there are $N_1 N_2 = 80$ pairs which are all the possible inter-group communication episodes. However, as depicted in the figure, there are actually eight communication episodes between groups 1 and 2. Thus, the theoretical probability of communication is:

$$P_{12} = N_{12}/N_1 N_2 = 8/80 = .10$$

In a real life setting, we do not know how many units are communicating with one another unless there is a full census. Excluding the full census situation, it is necessary to estimate P_{12} by collecting a random sample of units from each group. Another way of stating the problem is to estimate the probability of a unit from group 1 communicating with a unit from group 2. Figure 2 illustrates the situation arising from a random sample. It depicts a random sample of $n_1 = 5$ and $n_2 = 4$ from these two groups. These nine individuals are queried about their communication episodes with units from outside their group. The total number of pairs sampled is

$$M_{12} = N_1 n_2 + n_1 N_2 - n_1 n_2 = (10)(4) + (5)(8) - (4)(5) = 60$$

The random sample resulted in identifying 7 distinct episodes. (There are 4 and 5 episodes respectively from the two groups, however, two have been counted twice.) Hence the estimate of P_{12} is:

$$\hat{P}_{12} = 7/60 = 0.12$$

In adapting the Stephenson-Zelen method, the k groups constitute the network. The weights between groups i and j is \hat{P}_{ij} . Therefore using equations (13) and (14) in Stephenson and Zelen (1989), the calculation of centrality is obtained by defining the matrix B by

$$B = \begin{bmatrix} \hat{P}_{1.} & -\hat{P}_{12} & -\hat{P}_{13} & \dots & -\hat{P}_{1k} \\ -\hat{P}_{21} & \hat{P}_{2.} & -\hat{P}_{23} & \dots & -\hat{P}_{2k} \\ \vdots & & \ddots & & \\ -\hat{P}_{k1} & -\hat{P}_{k2} & -\hat{P}_{k3} & \dots & \hat{P}_{k.} \end{bmatrix} \quad (2)$$

where $\hat{P}_{i.} = \sum \hat{P}_{ij}$. The matrix B is singular and will have rank $(k-1)$. Following Stephenson and Zelen, form the non-singular matrix $(B+J)$ where J is a $k \times k$ matrix of unity elements. Since $(B+J)$ is non-singular, it will have an inverse. Define the matrix $C = (c_{ij}) = (B+J)^{-1}$. Then the Stephenson-Zelen measure of centrality for group i is given by

$$I_i = [c_{i1} + (T-2R)/k]^{-1}, \quad (i = 1, 2, \dots, k) \quad (3)$$

where $T = \sum c_{i1}$ and $R = \sum c_{ij}$ (Note that all row sums are equal for the C matrix). The groups in the group network can now be ordered

by $\{I_i\}$. Large values of I_i indicate greater centrality in the group network. The appendix outlines the derivation in some detail.

3. Example and Interpretation

The theory of the preceding sections will be illustrated by an example from an industrial setting. One of us (KS) gathered data in a high technology firm, which for reasons of confidentiality, will be called U.S.TEK. This company is composed of a number of independent subsidiaries. It is in one of these subsidiary companies (SOFTEK) where a survey on inter-departmental communications was carried out. A full discussion of this data can be found in Stephenson (1990). We shall illustrate the calculations by using a subset of the survey data.

At the time the survey was conducted, SOFTEK was comprised of 219 employees which were distributed among $k = 6$ departments. Table I summarizes the number of employees in each department (N_i) and the number of responses (n_i) in the survey by department. Table II is a summary of the \hat{P}_{ij} . The President of the company declined to respond. Nevertheless, we were able to estimate a sample of his communication episodes from the other respondents in the sample.

Using the values of Table II, we calculated the centrality (Equation 3). These are summarized in Table III. The department with the highest centrality score is the office of the President. The department with the lowest centrality score is Sales. One

would expect that the President, as leader of the corporation, would be the most central in terms of communications. Table III confirms this expectation.

In contrast to the singular office of the President, the Sales department consists of nearly half of the corporate population. The low centrality ranking of this department may indicate that Sales is not well integrated into the subsidiary as the other departments. The relative communication isolation of the Sales Department may be a consequence of the failure of electronic communications (fax, phone, or Email) to effectively close a geographical gap generated by field offices.

However, communication isolation is not necessarily predicated on geographical distance. Another example of communication isolation is found in the relative low centrality rankings of the Customer Service and Development departments. From personal interviews, KS observed that members of Customer Service generally communicated only with members of the Development group. On the other hand, Development typically avoided communication altogether. In one instance, K.S. noted that personnel in Sales were actively discouraged from speaking to their peers in Development. To be "informationally" isolated may handicap a group's collective effort to quickly and effectively communicate. This may compromise their performance. Consequently, the informational isolation of the Development and Customer Service departments led to a modus operandi of "creativity on demand" and resulted in depleted, exhausted and disillusioned groups. Tracy Kidder observed this

phenomena in the research group at Digital Equipment Corporation (1981).

The distribution of the centrality rankings shows a distinct clustering. It is of interest that a newly created department, Corporate Consulting Service (CCS) would rank relatively high in centrality along with Marketing and Administrative/Finance (Ad/F). The CCS department was responsible for developing new products targeted at new markets arising directly from customer consultation. Thus, members of CCS not only communicated with the external customer but also created internal communication ties which linked them to other departments within SOFTEK. By "networking", the members of this group ignored the boundaries of their own department and were able to constantly improvise and innovate by juggling information, ideas and ways of working. From direct observation, K.S. (1990b) noted that this department readily adjusted to a variety of situations beginning with its genesis as an ad hoc group to its final integration as an independent business unit.

4. Some Practical Issues

We have adapted the basic ideas from our earlier work (Stephenson and Zelen, 1989) to group networks. In most settings, the number of groups in such networks will be relatively small. As a result there should be no problem with numerical calculations. This is in contrast to unit networks which may consist of hundreds or even thousands of points. However, it is important to take note

of a number of practical problems in applying our theory to real life situations. These problems are particularly important in modern organizational environments, otherwise anomalies can arise.

(1) In modern organizational settings it is not unusual for an administrative assistant or secretary to be at the center of information trafficking. By default, this particular job requires frequent communications with other individuals and groups throughout the company. However this individual may not represent the class of communication episodes which is being studied. It is important to limit the surveys to classes of individuals having comparable status, such as support personnel, managers, technicians, salaried employees, etc.

(2) A convenient time frame should be chosen which represents a typical working environment. Avoid holidays or unusual periods of stress unless this is the time frame to be studied.

(3) It is necessary to distinguish between important and unimportant communications. The "importance" of communications should be identified by the respondents. Frequency of communication will also be a factor in defining importance; e.g. in the survey of U.S.TEK, a communication episode was counted if the respondents identified it as

"important" and if communication averaged at least one episode per week.

(4) In large organizational settings it may be necessary to collect data from a random sample of units within a department. In other situations a complete sample may be feasible. In either case it is important to strive for a 100% response rate from all individuals designated to submit data. A high proportion of non-responses could possibly bias the results. The methods discussed here are only appropriate when non-response is independent of any network characteristics. However as a practical matter, if the non-response is low, it is unlikely to effect the network results, even if the reasons for non-response are network related.

(5) It is important to note that when querying respondents, one is not measuring actual communication episodes. Rather, one is asking respondents to report their perception of communication to others. Consequently, only one of the two participants in a communication episode may report that communication has occurred. In this way, the data reflects individual perceptions of communications. Our method of sampling requires that only one of the two participants in an episode be in the sample. If two units from different groups are in the sample, it is possible that only one will report a communication episode.

By allowing respondents the opportunity to choose categories such as frequency or importance, one may study various aspects of the communication process. For instance, in some cases frequent but unimportant communications may consist of routine office "small talk". There may be instances in which a respondent's frequent but unimportant communications may be symptomatic of "veiled" important communications. An example of a "veiled" important communication would be when members in one department characterize inter-departmental communications with other departments as unimportant because of inter-departmental conflict. In some instances there may be some merit in investigating networks of frequent but unimportant communication episodes.

5. Discussion and Conclusions

Measures of centrality are "descriptive" measures of unit and group networks. The useful application of these measures depends on how centrality may be used to characterize communication networks in organizational settings. For example, one can study networks before and after a major intervention such as the implementation of a new policy, a department or company reorganization or a period of stress. Another example is to periodically survey an organization to determine trends in inter-group communications.

The motivation behind this extension is the study of existing groups or departments in organizational settings. One marked characteristic of studying modern organizational environments is

that the existence of all individuals are known. Hence it is possible to apply random sampling techniques to gather data as the sampling frame is known.

The primary aim in this paper is to define and estimate a centrality measure associated with groups. Our approach is different from calculating graph centralities (Freeman, 1979). Freeman defines graph centralities as the average of individual unit centralities for a specific graph where a graph may represent a group. Our method is conceptually different as we only count intergroup communication episodes.

In formulating the group problem we define the probability of units in different groups communicating with each other and show how this parameter can be estimated. This parameter, by itself, may be worth studying. For example, the larger the group, the more likely communication will be intra-group and the less need to communicate outside the group. This empirical generality was noted at the turn of the century by Werner Sombart - now recognized as Sombart's Law (Kuznets (1967), (Leamer and Stern, 1970)). It states that as groups grow in size they will differentiate from within and increasingly rely on some form of intra-group communication rather than inter-group communication. This phenomenon can be studied using the $\{P_{ij}\}$ parameters.

In our development of inter-group communication, it is possible to define the probability of communication within each group, i.e., this probability refers to the probability of communication when choosing two units at random within the same

group. In some instances, the intra-group probabilities may be of little value as they may all be unity. This is the case if the class of respondents refers to individuals of similar status who are in regular contact with each other by virtue of their responsibilities; e.g. middle managers, faculty members in a department, etc.

In conclusion, we have extended earlier ideas of calculating information centrality among individuals (Stephenson and Zelen, 1989) to group networks. Group networks may be composed of relatively few groups but may consist of hundreds of individuals. As a result, the monitoring of communications may have to be carried out by random sampling of units within groups. We have incorporated the consequences of random sampling in our model.

Fundamental to our model is the probability that two units randomly chosen from two different groups will communicate. In a unit network, the probability that any pair of individuals will communicate will either be zero or one. However for group networks, this probability may be other than zero or one. Another aspect of group networks is that the individuals comprising a group is known in advance.

One open question is the uncertainty of the estimates of centralities in group networks. It is a straightforward but complicated algebraic problem to calculate the standard deviation of the estimates associated with random sampling. However, a more complex model is required to account for the variation in communication patterns over time, i.e., no organization is stable

with respect to personnel and responsibilities over time. What are suitable models characterizing these instabilities?

Finally we wish to state that our model for group networks only relates to units which belong to a single group. A more general formulation of this problem is to consider group networks in which an individual not only belongs to a group which is organizationally defined, but has personal characteristics which are attributes of the individual (gender, age, race, etc.). It would be useful to incorporate these characteristics in models of inter-group communications.

TWO GROUP NETWORK

Group 1 has 10 units; Group 2 has 8 units.

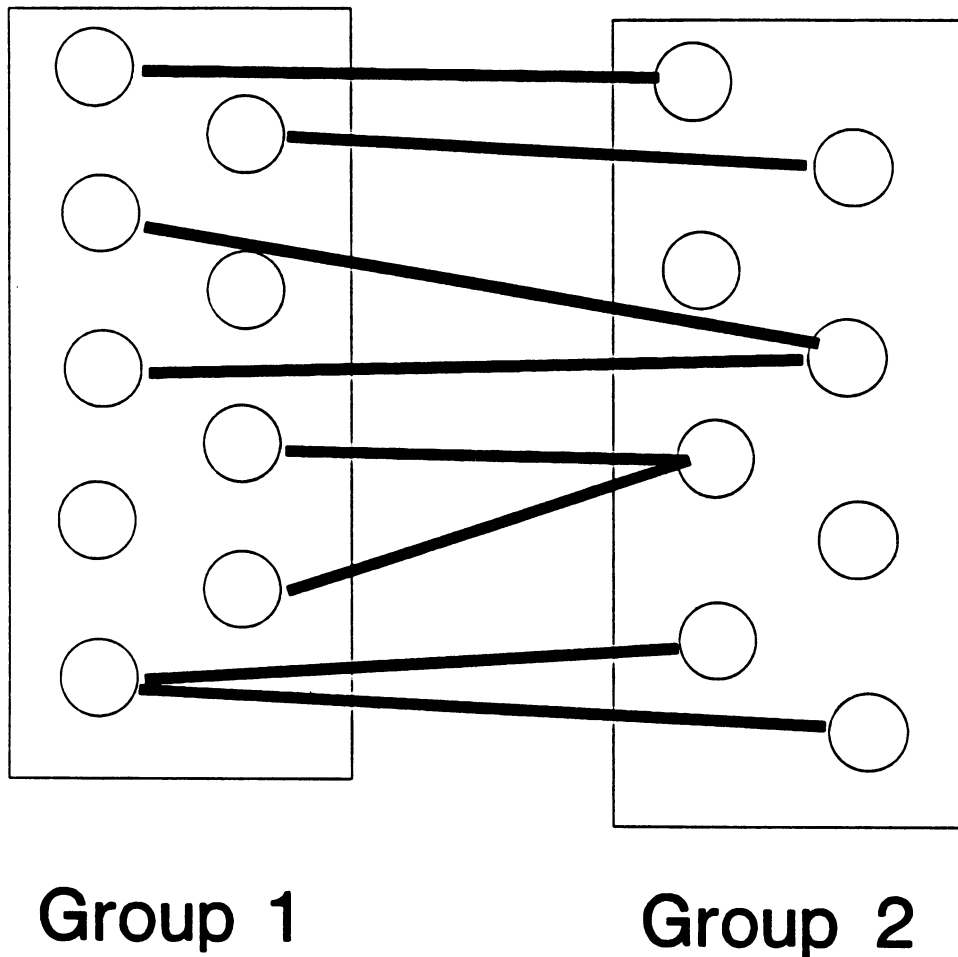


Figure 1. Two group network

(Value of $P_{12} = 8/80 = .10$)

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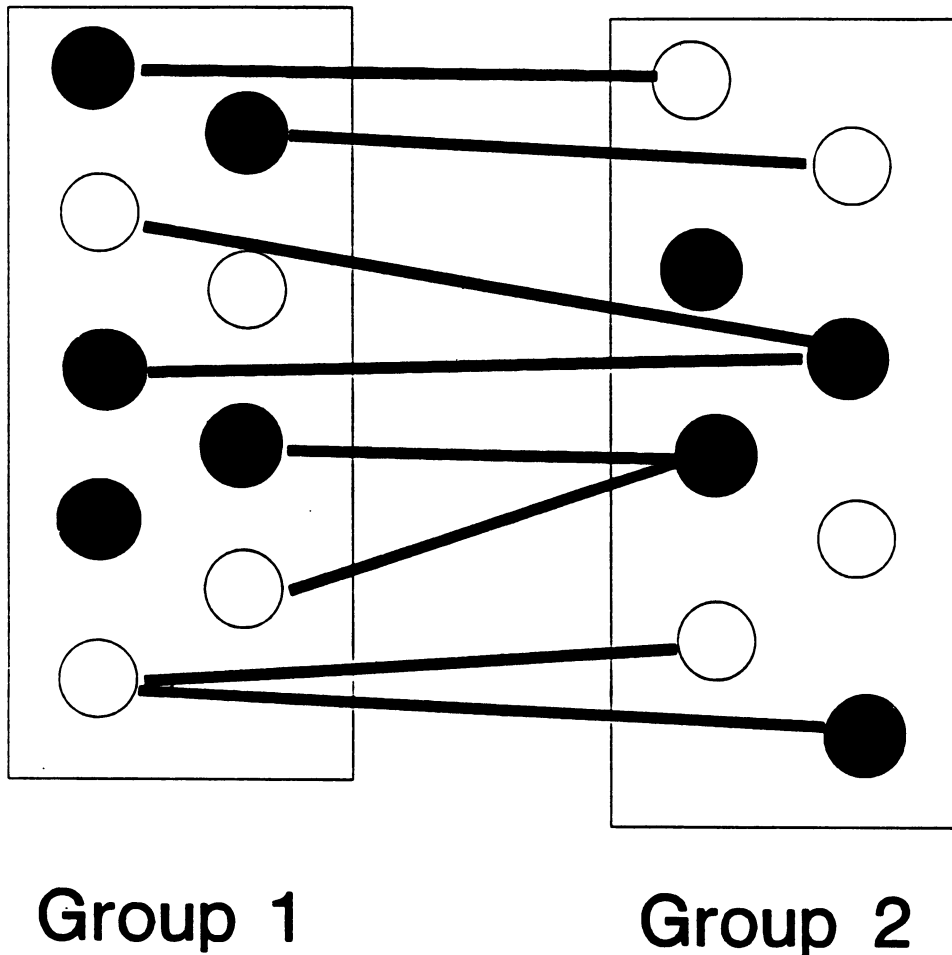


Figure 2. Two group network

(Solid circles represent units in random sample)

Table I. Summary Statistics of October 1988 Survey.

Department	Number in Department (N_i)	Number Responding (n_i)
Sales	98	57
Marketing	19	14
Admin/Finance	23	16
Development	33	22
Corporate Consulting Service (CCS)	6	6
Customer Service	40	28
President	1	0
TOTAL	220	143

Table II. Summary of Number of Communication Episodes per Pair by Department*

		Group*						
		1	2	3	4	5	6	7
SALES	1	-	-	-	-	-	-	-
MKT	2	.021	-	-	-	-	-	-
ADM/FIN	3	.032	.067	-	-	-	-	-
DEVELOP	4	.006	.073	.018	-	-	-	-
CCS	5	.032	.079	.043	.061	-	-	-
CUST SERVICE	6	.013	.044	.044	.045	.008	-	-
PRESIDENT	7	.036	.214	.250	.045	.167	.035	-

* Entries are $\hat{P}_{ij} = \hat{P}_{ji} = n_{ij}/M_{ij}$.

Table III. Centrality Estimates.

Rank	Department	Centrality	Normalized* Centrality
1	President	.253	.19
2	Marketing	.228	.17
3	Adm/Fin	.217	.16
4	CCS	.208	.15
5	Development	.170	.13
6	Cust. Serv.	.149	.11
7	Sales	.122	.09

* Normalized centralities are the actual centralities divided by the sum of all the centralities.

Appendix

This appendix outlines the modeling considerations which lead to the calculation of the centralities discussed in the main body of the paper. The theory developed in Stephenson and Zelen for unit networks can be adapted to group networks. The essential theory is contained in section A.5 (weighted centrality) of our paper.

The network is envisioned as containing k points (corresponding to the groups). If there is communication between a pair of groups a line connects them. The weight associated with a line connecting two points is the probability of communication between the two points. If the probability associated with a line is zero, then there is no communication between the pair of groups connected by the line. Hence, one can consider a network of k points and $k(k-1)/2$ lines connecting each pair with a probability associated with each line. The degenerate case is when the probability is null associated with a line connecting a pair of points.

Using the notation of Stephenson and Zelen, define

$$n_{i\alpha} = \begin{cases} 1 & \text{if the } \alpha^{\text{th}} \text{ line intersects point (group) } i \\ 0 & \text{otherwise} \end{cases}$$

$$f_{\alpha} = \text{weight (probability) associated with line } \alpha$$

where $i = 1, 2, \dots, k$ and $\alpha = 1, 2, \dots, k(k-1)/2$. Then the equations given in (14) of Stephenson and Zelen can be written

$$r_i^* = \sum_{\alpha} n_{i\alpha} f_{\alpha} = \text{sum of weights of lines intersecting group } i = \sum_{j=1}^k P_{ij} = P_i.$$

$$\lambda_{ij}^* = \sum_{\alpha} n_{i\alpha} n_{j\alpha} f_{\alpha} = \text{weight associated with lines connecting groups } i \text{ and } j = P_{ij}.$$

Note that $r_i^* = \lambda_{ii}^*$. Then the matrix $B = (b_{ij})$ of Stephenson and Zelen is

$$b_{ii} = r_i^* = P_i.$$

$$b_{ij} = -\lambda_{ij}^* = -P_{ij}, \quad (i \neq j).$$

Therefore, the matrix $C = (c_{ij})$ (cf. equation 9 of Stephenson and Zelen) is $C = (B + J)^{-1}$ where J is a $k \times k$ matrix of unity elements. Hence, the centrality (Stephenson and Zelen, equation 12) is

$$I_i = [c_{ii} + (T - 2R)/k]^{-1}$$

$$\text{where } T = \sum_{j=1}^k c_{jj} \text{ and } R = \sum_{j=1}^k c_{ij}.$$

It remains to find P_{ij} . It will be assumed that a communication episode is well defined (e.g., importance, frequency, time span, etc.). The value of P_{ij} may be obtained by a complete enumeration of the patterns of communication between all groups or it may be estimated by a random sampling of units.

Define N_i as the number of units in the i^{th} group ($i = 1, 2, \dots, k$) and define N_{ij} as the number of communication episodes between units in groups i and j . The probability that a randomly chosen pair communicate is defined by $P_{ij} = N_{ij}/N_i N_j$ where the pair has one member each from groups i and j .

If there is a complete enumeration of all communication episodes, the quantity P_{ij} is completely defined. Alternatively, a complete enumeration may not be feasible and P_{ij} will have to be estimated by drawing a random sample of units from each group. Each unit in the random sample is monitored or queried about all inter-group communication episodes in which that individual is a direct participant. It is important to note that this method of sampling requires that only one of the two participants in an episode be in the sample.

The sampling plan consists of drawing a random sample of n_i units from group i . Since there are N_i units in group i , the probability of drawing any unit in the sample is $p_i = n_i/N_i$. The sampling scheme enumerates $M_{ij} = n_i N_j + N_i n_j - n_i n_j = N_i N_j [p_i + p_j - p_i p_j]$ pairs of units between any two groups (say groups i and j).

Let n_{ij} be the number of distinct communication episodes between groups i and j in the sample. Then the estimate of P_{ij} is $\hat{P}_{ij} = n_{ij}/M_{ij}$. We shall show that it is an unbiased estimate. For this purpose, define

$$\begin{aligned} \delta(\alpha|i) &= \begin{cases} 1 & \text{if } \alpha^{th} \text{ unit in } i^{th} \text{ group is in sample} \\ 0 & \text{otherwise} \end{cases} \\ A(\alpha, \beta|i, j) &= \begin{cases} 1 & \text{if } \alpha^{th} \text{ and } \beta^{th} \text{ units in groups } i \text{ and } j \text{ communicate} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

where $i \neq j = 1, 2, \dots, k$; $\alpha = 1, 2, \dots, N_i$ and $\beta = 1, 2, \dots, N_j$. Then the relations amongst these quantities are:

$$\begin{aligned} n_i &= \sum_{\alpha} \delta(\alpha|i), \quad N_{ij} = \sum_{\alpha, \beta} A(\alpha, \beta|i, j) \\ n_{ij} &= \sum_{\alpha, \beta} A(\alpha, \beta|i, j) [\delta(\alpha|i) + \delta(\beta|j) - \delta(\alpha|j)\delta(\beta|i)]. \end{aligned}$$

Since $E[\delta(\alpha|i)] = p_i$, we have

$$E(n_{ij}) = N_{ij}[p_i + p_j - p_i p_j]$$

and

$$E(\hat{P}_{ij}) = E(n_{ij}/M_{ij}) = N_{ij}/N_i N_j = P_{ij}.$$

A convenient approximation to the distribution of \hat{P}_{ij} is to assume that \hat{P}_{ij} follows a normal distribution with mean P_{ij} and variance $(1 - M_{ij}/N_i N_j)P_{ij}(1 - P_{ij})/M_{ij}$. The first term in the expression for the variance arises from sampling a finite population.

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